HVOX: Scalable Interferometric Synthesis and Analysis of Spherical Sky Maps

Swiss SKA Days 23

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Analysis and Synthesis for Imaging Purposes

Analysis (a.k.a. **gridding**) and synthesis (a.k.a. **degridding**) are used in radio interferometry to accurately predict visibilities (resp. dirty images) from a given sky distribution (resp. visibility set):

$$\left(V_i = \sum_{\boldsymbol{r}\in\Theta_{\text{pix}}} I(\boldsymbol{r})e^{-j\langle \boldsymbol{r},\boldsymbol{p}_i\rangle}\right)_{i=1,\dots,N_{\text{vis}}} \quad \& \quad \left(I_D(\boldsymbol{r}) = \frac{1}{N_{\text{vis}}}\sum_{i=1}^{N_{\text{vis}}} V_i e^{j\langle \boldsymbol{r},\boldsymbol{p}_i\rangle}\right)_{\boldsymbol{r}\in\Theta_{\text{pix}}}.$$

Direct evaluation of the above expressions has bi-linear complexity $\mathcal{O}(N_{\text{vis}}N_{\text{pix}})$ which represents a major bottleneck for imaging purposes:

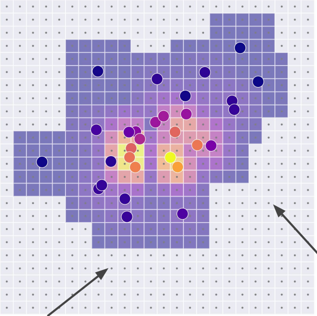
- CLEAN performs **one analysis and synthesis per** major cycle.
- Proximal algorithms typically perform **one analysis and synthesis per** iteration.

The (numerous!) analysis and synthesis steps tend to dominate the overall computational complexity of the imaging task.

We therefore need fast, scalable and accurate analysis and synthesis algorithms.

Standard Gridding

Fourier Domain



Uniform grid
of harmonics

Nonuniform
frequencies

2D NUFFT
(Type 1)

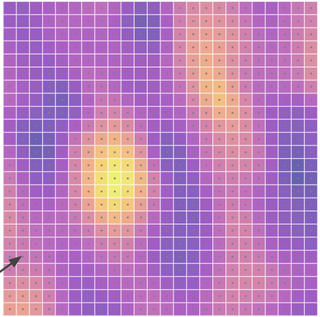
$$\sum_i V_i e^{j(\ell u_i + m v_i)}$$

2D FFT

(uniform to uniform)

$$\mathcal{O}(N_{\text{pix}} \log N_{\text{pix}})$$

Spatial Domain

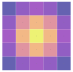


Uniform grid
of *pixels*

Spreading
(nonuniform to uniform)

$$\mathcal{O}(N_{\text{vis}} |\log \epsilon|^2)$$

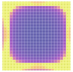
$*$



Tapering
(corrects for spreading)

$$\mathcal{O}(N_{\text{pix}})$$

X

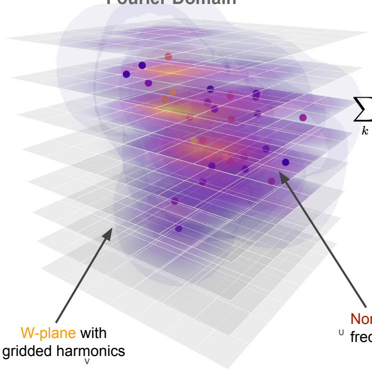


Total Complexity:

$$\mathcal{O}(N_{\text{vis}} |\log \epsilon|^2 + N_{\text{pix}}(\log N_{\text{pix}} + 1))$$

Nifty Gridder (a.k.a. W-gridding)

Fourier Domain



Nifty Gridder
(a.k.a. W-Gridding)

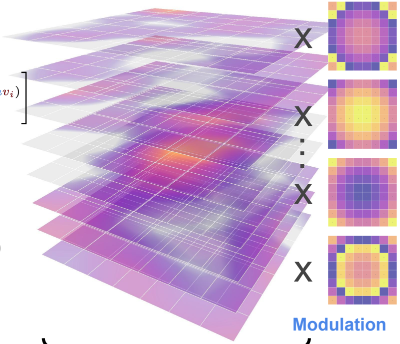
$$\sum_{k} e^{jn(\ell,m)w_k}\left[\sum_{i\in\mathcal{I}_k} V_i e^{j(\ell u_i + m v_i)}\right]$$

2D FFTs

(for each W-plane)

$$\mathcal{O}(\boxed{N_{w'}} N_{\text{pix}} \log N_{\text{pix}})$$

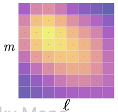
Spatial Domain



Modulation

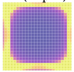
$$\mathcal{O}(\boxed{N_{w'}} N_{\text{pix}})$$

+ Sum



$$\mathcal{O}(N_{\text{pix}})$$

X



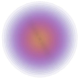
Tapering

W-Spreading

(nonuniform to W-plane)

$$\mathcal{O}(N_{\text{vis}} \boxed{|\log \epsilon|^3})$$

$*$



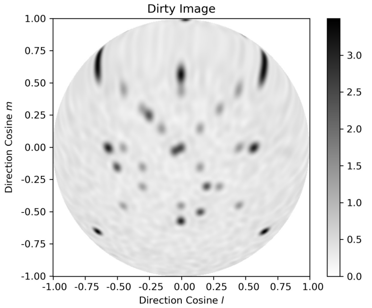
Total Complexity:

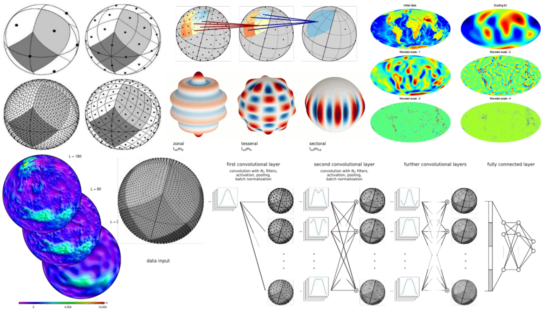
$$\mathcal{O}(N_{\text{vis}} \boxed{|\log \epsilon|^3} + \boxed{N_{w'}} N_{\text{pix}} (\log N_{\text{pix}} + 1))$$

Limitations of Direction Cosine Grids

Current gridding techniques achieve log-linear complexity, but only for direction cosine meshgrids. The latter are ill-suited for large FOVs due to **heavily distorted** or singular/non-feasible pixels away from the center. This significantly complicates (if not *forbids*) the use of traditional **image processing** and **image analysis** tools.

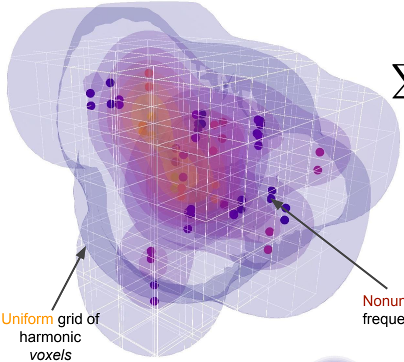
In contrast the HEALPix mesh is tailored to spherical images, and **readily supports** Fourier and wavelet analysis, filtering, CNNs for inference/classification, pattern recognition, noise analysis and more.





Gridding via the 3D NUFFT of Type 3 (HVOX)

Fourier Domain



Uniform grid of harmonic *voxels*

Nonuniform frequencies

3D NUFFT (Type 3)

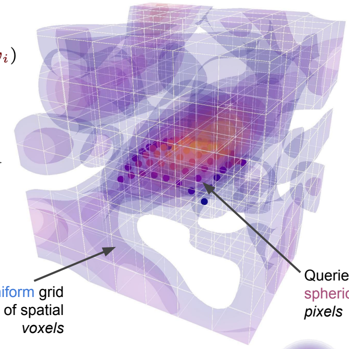
$$\sum_i V_i e^{j(\ell u_i + m v_i + n w_i)}$$

3D FFT

(uniform to uniform)

$$\mathcal{O}(\boxed{N_{\text{mesh}} \log N_{\text{mesh}}})$$

Spatial Domain



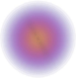
Uniform grid of spatial *voxels*

Queried spherical *pixels*

Spreading (+Tapering)

(nonuniform to uniform)

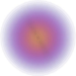
$$\mathcal{O}(N_{\text{vis}} |\log \epsilon|^3 + \boxed{N_{\text{mesh}}})$$

$*$ 

Interpolation (+Tapering)

(uniform to nonuniform)

$$\mathcal{O}(\boxed{N_{\text{pix}} |\log \epsilon|^3 + N_{\text{pix}}})$$

$*$ 

Total Complexity:

$$\mathcal{O}((N_{\text{vis}} + \boxed{N_{\text{pix}}}) |\log \epsilon|^3 + \boxed{N_{\text{mesh}} (\log N_{\text{mesh}} + 1) + N_{\text{pix}}})$$

Scaling up HVOX via Chunked NUFFT Gridding

The NUFFT of Type 3 performs a single 3D FFT with critical size:

$$N_{\text{mesh}} \propto (u_{\max} v_{\max} w_{\max}) \times (\ell_{\max} m_{\max} n_{\max})$$

To help reduce down the FFT sizes, we propose partitioning the uvw and image domains in chunks:

$$\sum_{C \in \mathscr{P}} \sum_{i \in C} V_i e^{j(\ell u_i + m v_i + n w_i)}$$

In both domains, the partitioning is performed via a hierarchical binning to ensure similar workloads in each chunk.

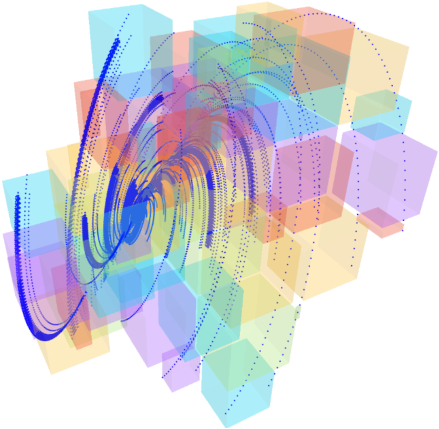
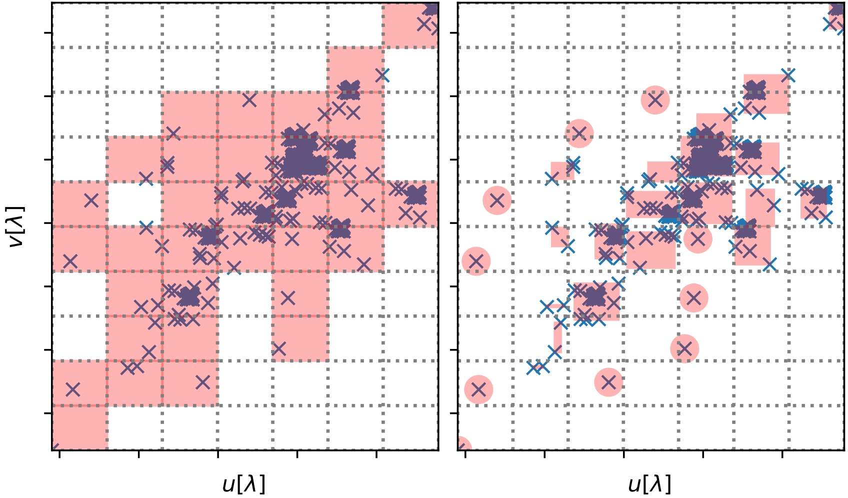
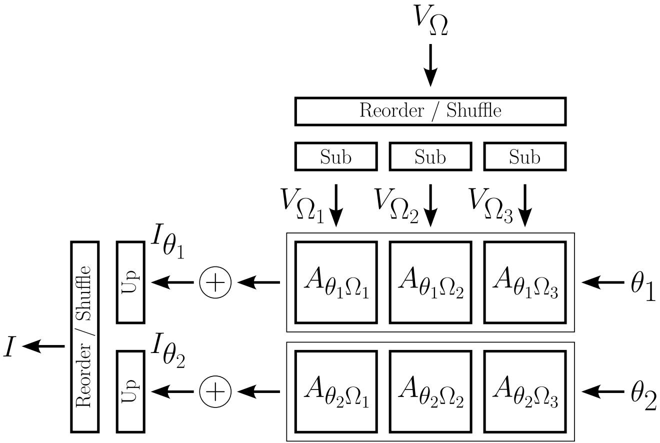


Figure: Example partitioning in the uvw domain.

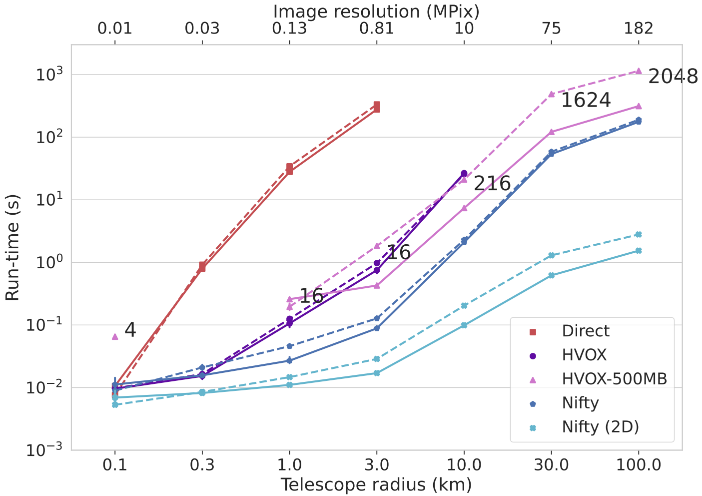
Scaling up HVOX via Chunked NUFFT Gridding (continued)



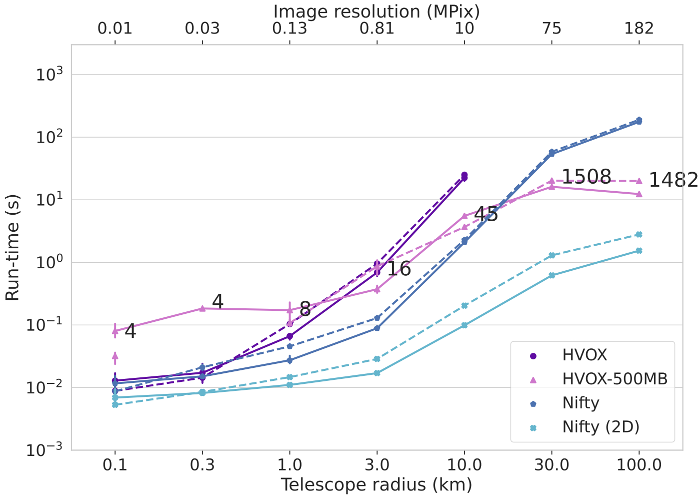
Scaling up HVOX via Chunked NUFFT Gridding (continued)



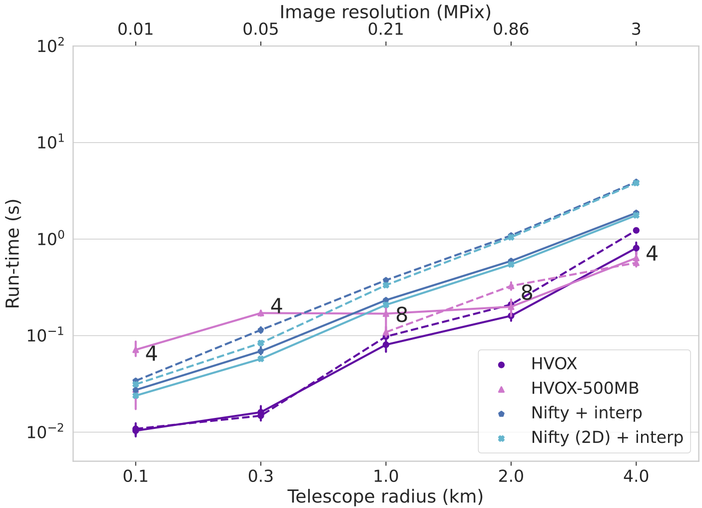
Benchmarking: Run Time (SKA-LOW, 30°, Dense DCOS)



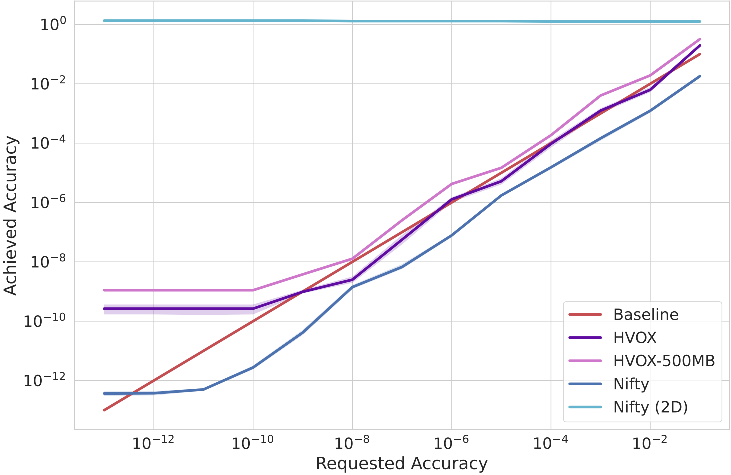
Benchmarking: Run Time (SKA-LOW, 30°, Sparse DCOS)



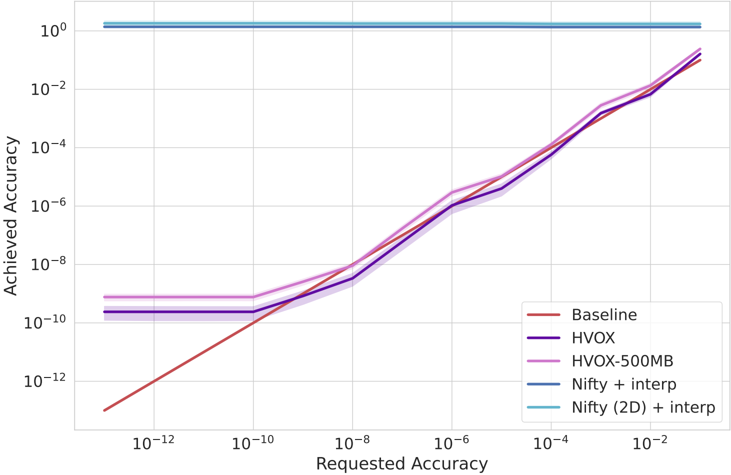
Benchmarking: Run Time (SKA-LOW, 30°, Sparse HEALPix)



Benchmarking: Accuracy (SKA-LOW, 30°, $r_{\text{max}} = 1$ km, DCOS)



Benchmarking: Accuracy (SKA-LOW, 30°, $r_{\text{max}} = 1$ km, HEALPix)



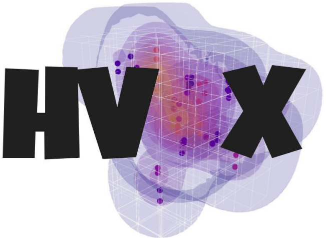
Conclusion

We have proposed the HVOX gridder, a new wide-field (de)gridding algorithm for radio interferometry with:

- support for arbitrary spherical meshes (including **HEALPix**),
- similar accuracy and speed than the **Nifty gridder** (s.o.a) (faster for sparse sky images),
- good scaling behaviour thanks to a chunking strategy (opens up the way to stochastic optimisation too...).
- Pre-release available on GitHub: `https://github.com/matthieumeo/hvox`. Good integration with RASCIL, implementation based on Pyxu (facilitates computational imaging).
- Pre-print available on arXiv: `https://arxiv.org/abs/2306.06007`.

Next steps:

- deploy and test in a production environment,
- add GPU support,
- extend the algorithm to support A-terms and compare to Image Domain Gridding (IDG).



Backup Slides

Comparison with Nifty Gridder: Complexity

The total complexity of the Nifty gridder is given by:

$$\mathcal{O}(N_{\text{vis}}|\log\epsilon|^3 + N_{w'}N_{\text{pix}}(\log N_{\text{pix}} + 1))$$

while that of HVOX is given by:

$$\mathcal{O}((N_{\text{vis}} + N_{\text{pix}})|\log\epsilon|^3 + N_{\text{mesh}}(\log N_{\text{mesh}} + 1))$$

Nifty's inner 2D FFTs are performed at the size of the final image. This represents a memory bottleneck: FFTs are **notoriously hard to distribute** and require the processed array to fit in memory.

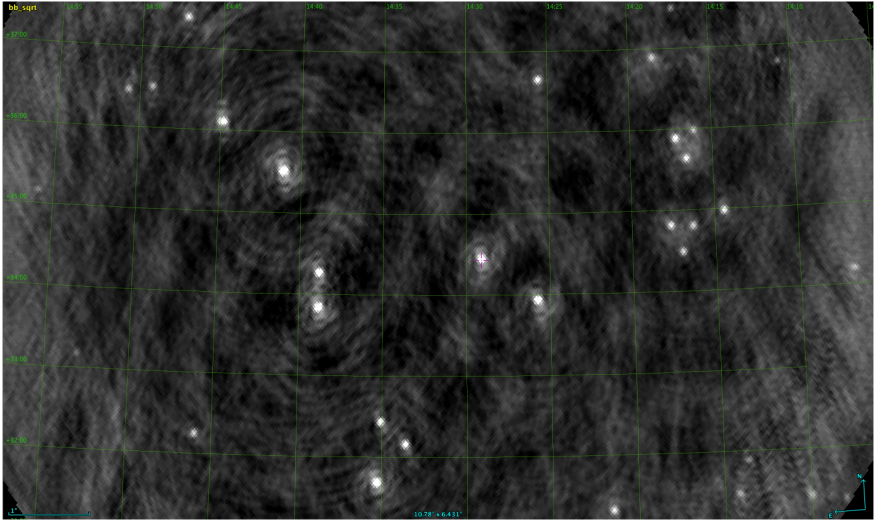
In contrast, the NUFFT of Type 3 performs a single 3D FFT with critical size N_{mesh} determined by the space-frequency product of the data bounding boxes.

We have typically

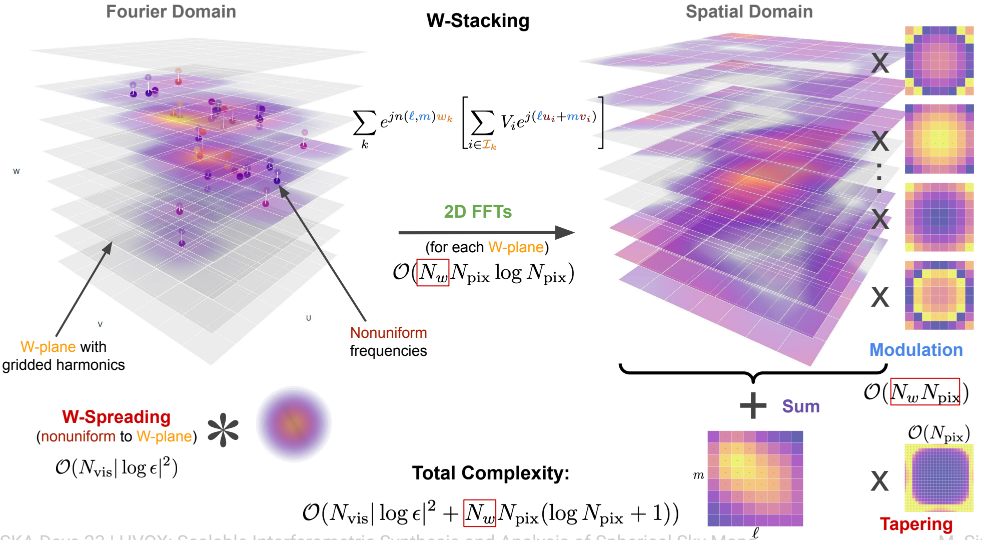
$$[5-10] \times N_{\text{mesh}} \simeq N_{w'}N_{\text{pix}}.$$

The price to pay is however an extra $N_{\text{pix}}|\log\epsilon|^3$ term (so hard to judge performances from **big-O analysis**).

HEALPix Synthesis via HVOX



W-Stacking


Fourier Domain
W-Stacking
Spatial Domain
$\sum_k e^{jn(\ell,m)w_k}\left[\sum_{i\in\mathcal{I}_k} V_i e^{j(\ell u_i + m v_i)}\right]$
w
v
u
2D FFTs
(for each W-plane)
$\mathcal{O}(\boxed{N_w} N_{\text{pix}}\log N_{\text{pix}})$
X
X
X
X
W-plane with gridded harmonics
Nonuniform frequencies
Modulation
$\mathcal{O}(\boxed{N_w N_{\text{pix}}})$
Sum
W-Spreading
(nonuniform to W-plane)
$\mathcal{O}(N_{\text{vis}}|\log\epsilon|^2)$
$\mathcal{O}(N_{\text{pix}})$
m
X
ℓ
Tapering
Total Complexity:
$\mathcal{O}(N_{\text{vis}}|\log\epsilon|^2 + \boxed{N_w} N_{\text{pix}}(\log N_{\text{pix}} + 1))$