QUANTUM COMPUTING FOR RADIO INTERFEROMETRY

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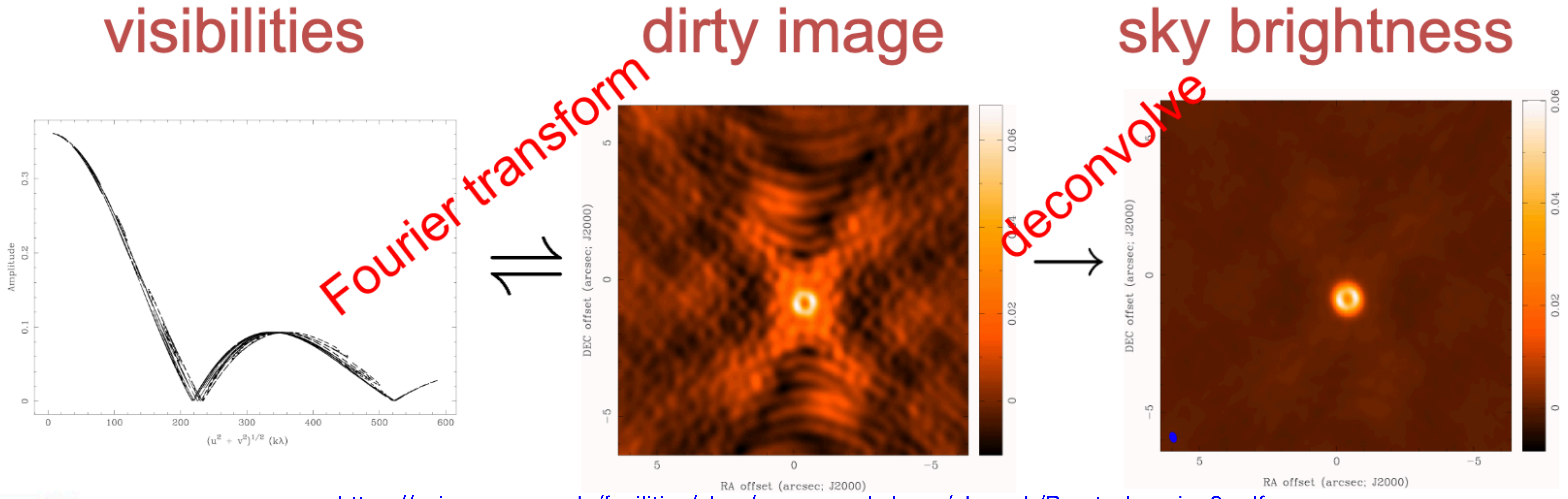
EPFL
ASTRON
Maastricht University

RADIO INTERFEROMETRY

$$I(x,y) = \iint V(u,v) e^{2\pi i(ux+vy)} du dv$$

$$I_D = \iint V(u,v) S(u,v) e^{2\pi i(ux+vy)} du dv$$

How much, if any, of this process can leverage quantum computing? Studied in **arXiv:2310.12084**



https://science.nrao.edu/facilities/alma/naasc-workshops/alma_dr/Braatz_Imaging2.pdf

QUANTUM COMPUTING 101

Orthonormal basis states: $|0\rangle \quad |1\rangle$

Qubit: $|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

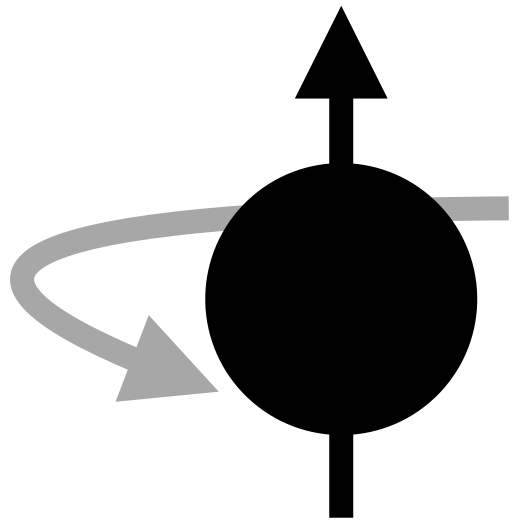
$$P(|0\rangle) = |\langle 0|\Psi\rangle|^2 = |\alpha\langle 0|0\rangle + \beta\langle 0|1\rangle|^2 = |\alpha|^2$$

$$P(|1\rangle) = |\beta|^2$$

$$|\alpha|^2 + |\beta|^2 = 1$$

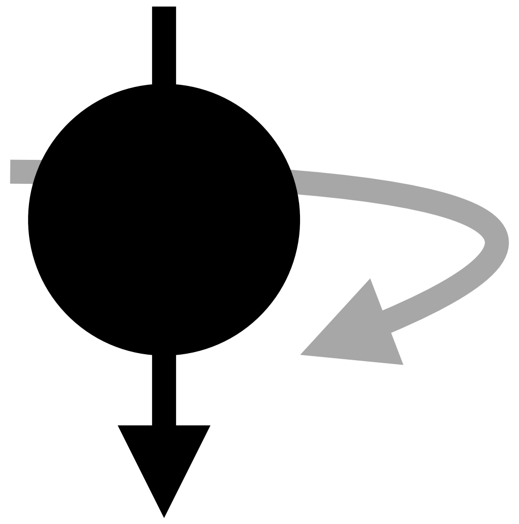
QUANTUM COMPUTING 101

Spin-up in
z-direction



$$|0\rangle$$

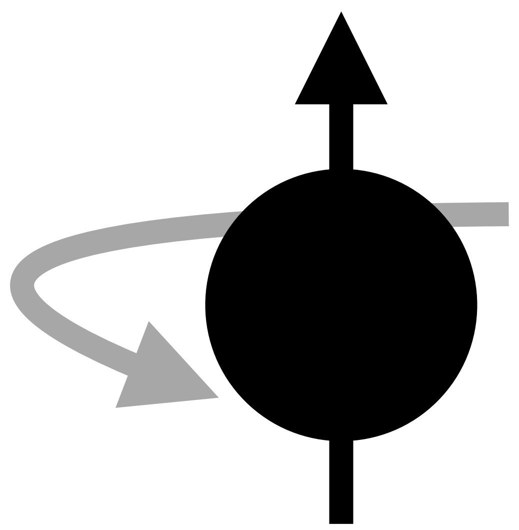
Spin-down in
z direction



$$|1\rangle$$

QUANTUM COMPUTING 101

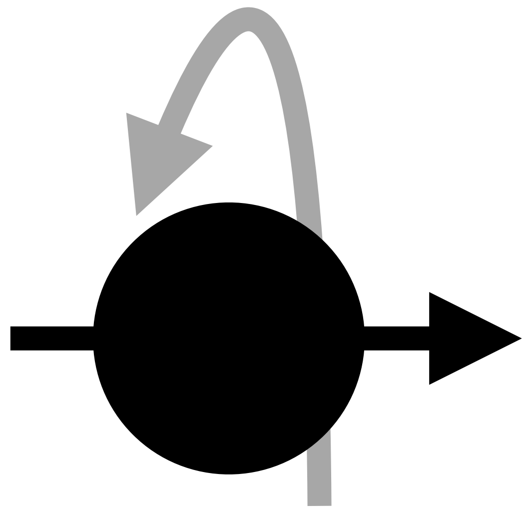
Spin-up in
z direction



$$|0\rangle$$

Precess spin by applying magnetic field

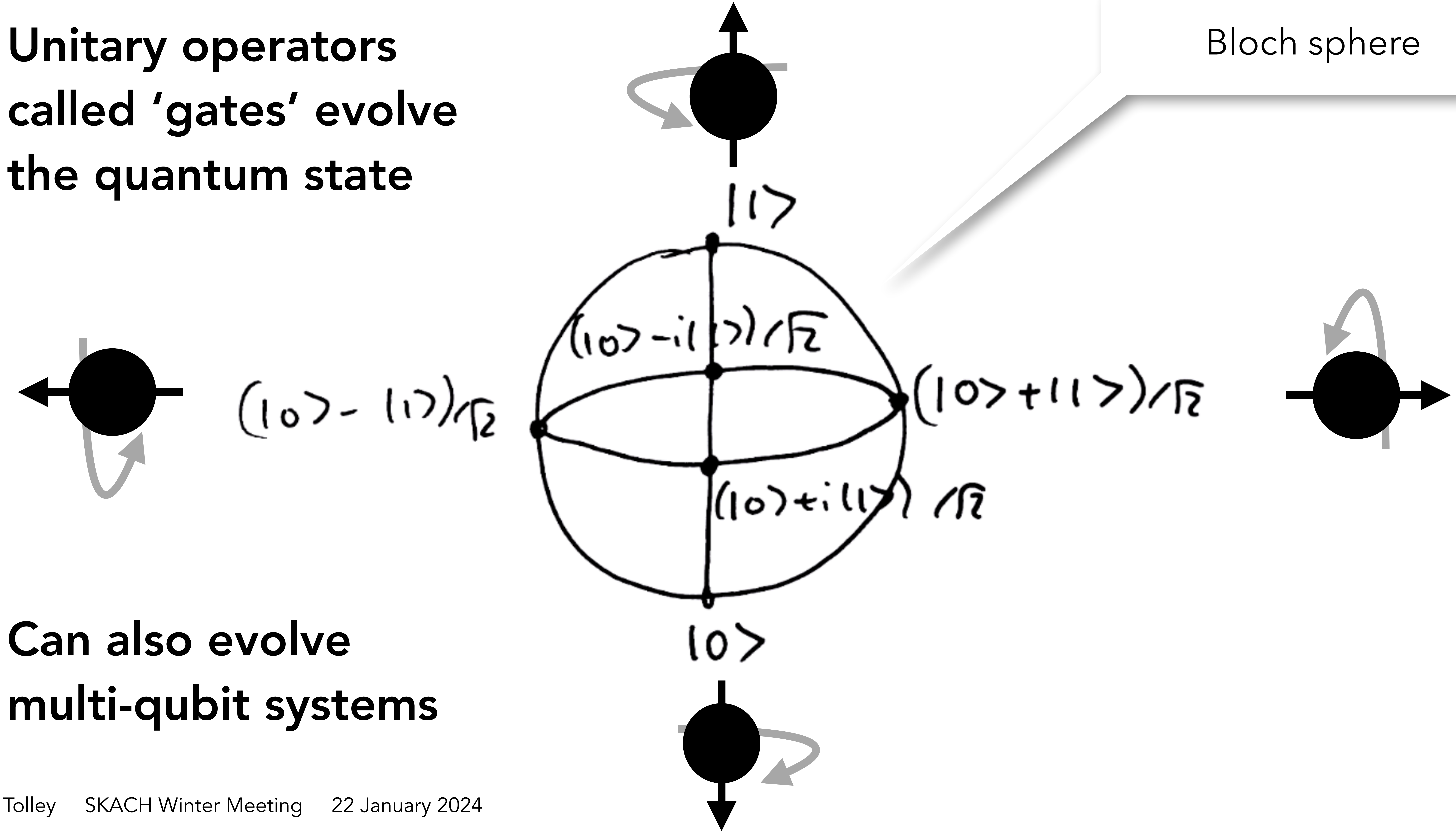
Spin-up in
x direction



$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

QUANTUM COMPUTING 101

Unitary operators called 'gates' evolve the quantum state



Bloch sphere

Can also evolve multi-qubit systems

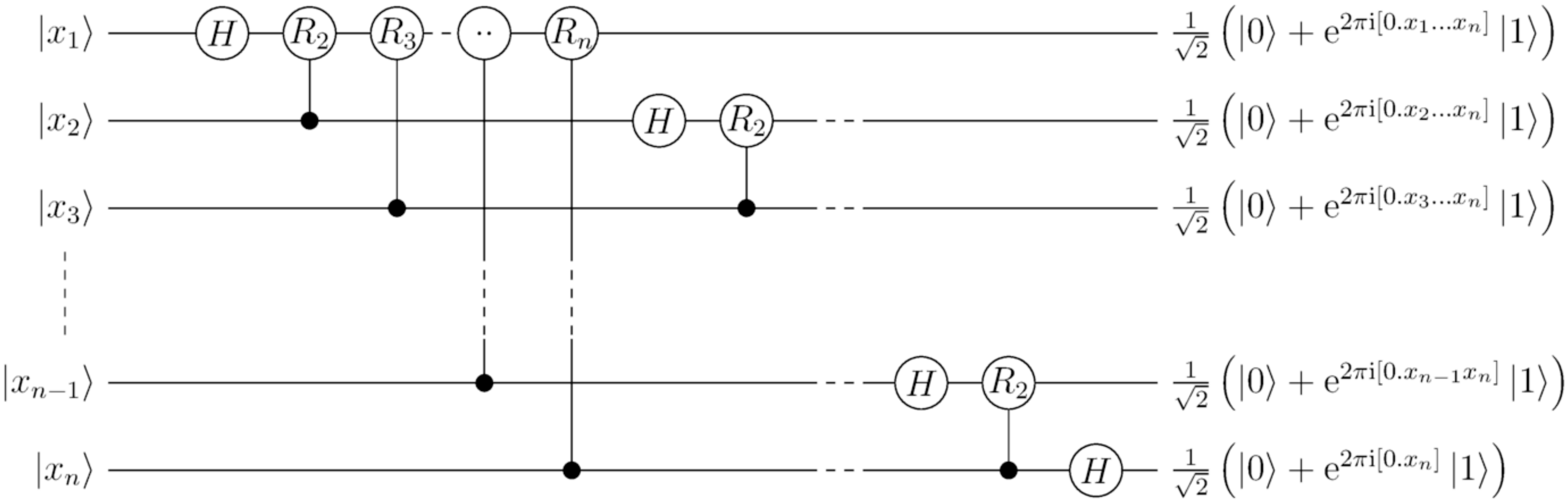
QUANTUM COMPUTING 101

Chain together multiple gates into a quantum circuit

Quantum FT circuit

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{and} \quad R_k = \begin{pmatrix} 1 & 0 \\ 0 & e^{i2\pi/2^k} \end{pmatrix}$$

$$[0.x_1 \dots x_m] = \sum_{k=1}^{m} x_k 2^{-k}.$$



QUANTUM DATA ENCODING

Classical Image

C1	C2
C3	C4

Pixel values Ci are 32-bit floats
Requires 32N^2 bits

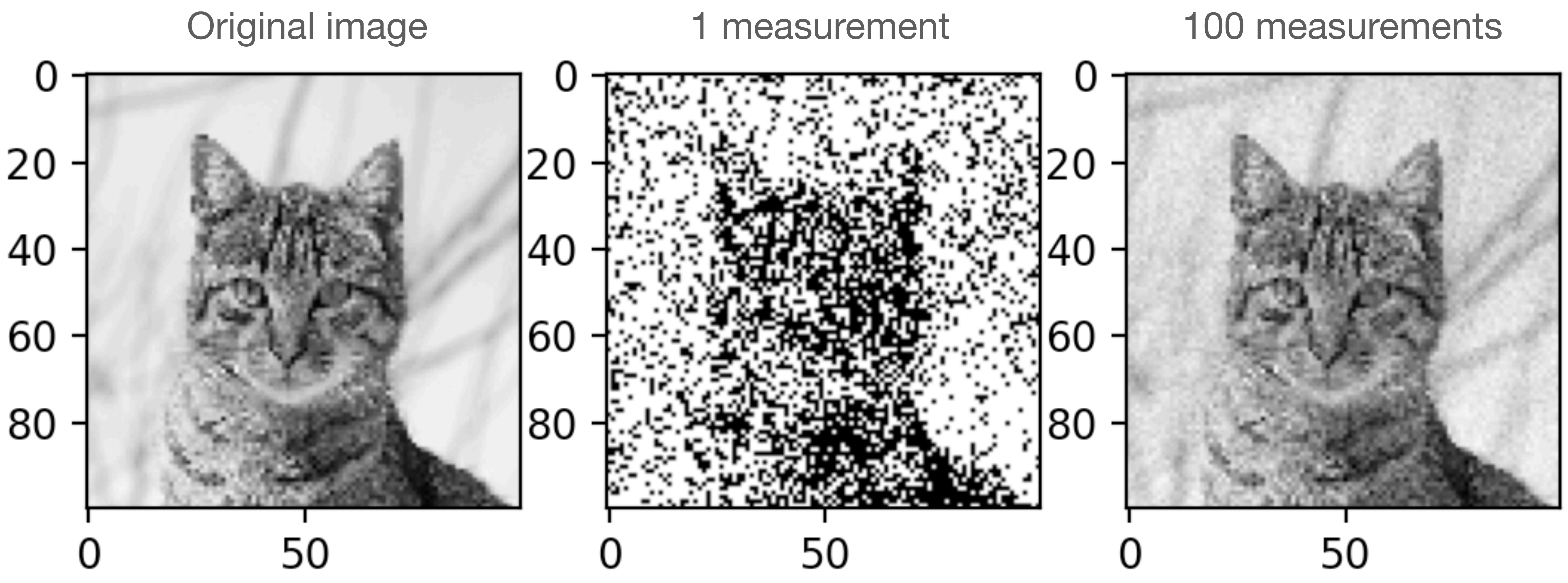
Quantum binary encoding: simply map each bit (0 or 1) to a quit (|0> or |1>) without using any entanglement or superposition
Requires **32N^2 qubits**

QUANTUM DATA ENCODING

Quantum lattice: represent each pixel with a single qubit with superposition:

$$|\Psi_k\rangle = \cos\theta_k\,|0\rangle + \sin\theta_k\,|1\rangle \qquad \theta_k = \frac{\pi}{2}c_k$$

Requires only **N^2 qubits**, but compression comes at the cost of additional quantum uncertainty



QUANTUM DATA ENCODING

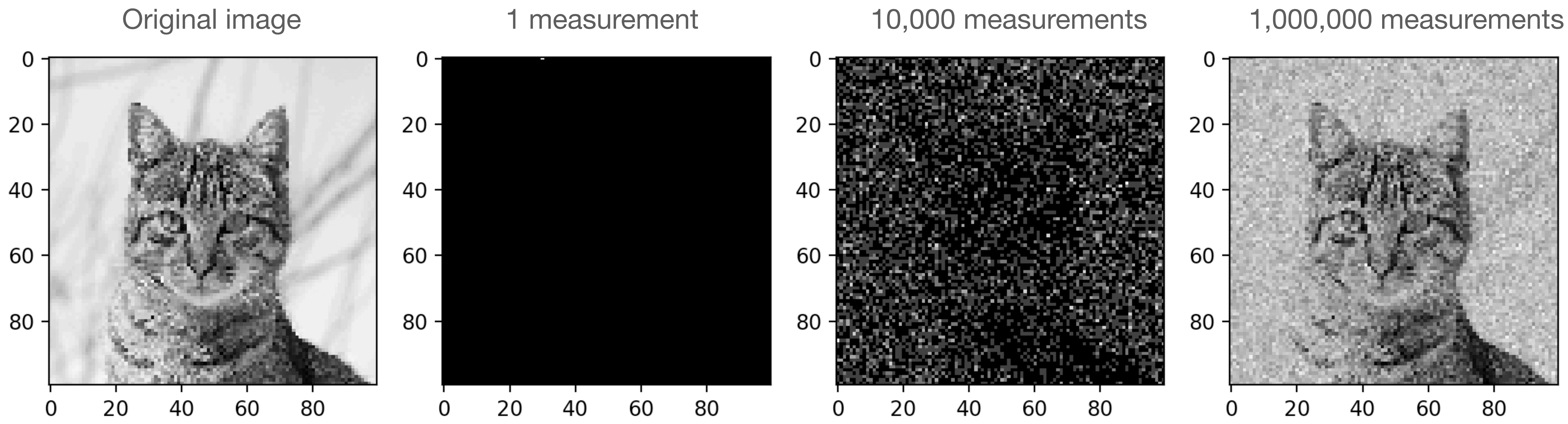
Flexible Representation of Quantum Images (FRQI): encode positional information with with entangled qubits:

$$|\Psi\rangle = \frac{1}{2^n} \sum_{k=0}^{N^2-1} (\cos\theta_k |0\rangle + \sin\theta_k |1\rangle) \otimes |k\rangle$$

Represent pixel coordinates as binary strings, for example:

$$|k = 2\rangle \; = \; |0\rangle \otimes \ldots \otimes |1\rangle \otimes |0\rangle$$

Requires only **log(N^2)+1 qubits**



QUANTUM DATA ENCODING

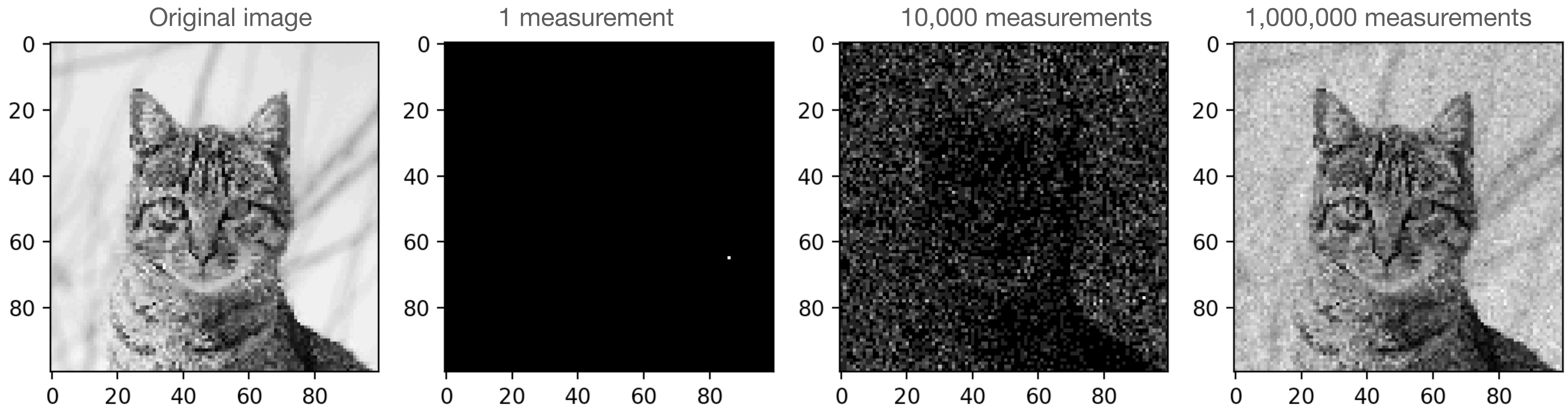
Quantum Probability Image Encoding (QPIE): encode positional information with with entangled qubits:

$$|\Psi\rangle = \sum_{k=0}^{N^2-1} c_k |k\rangle$$

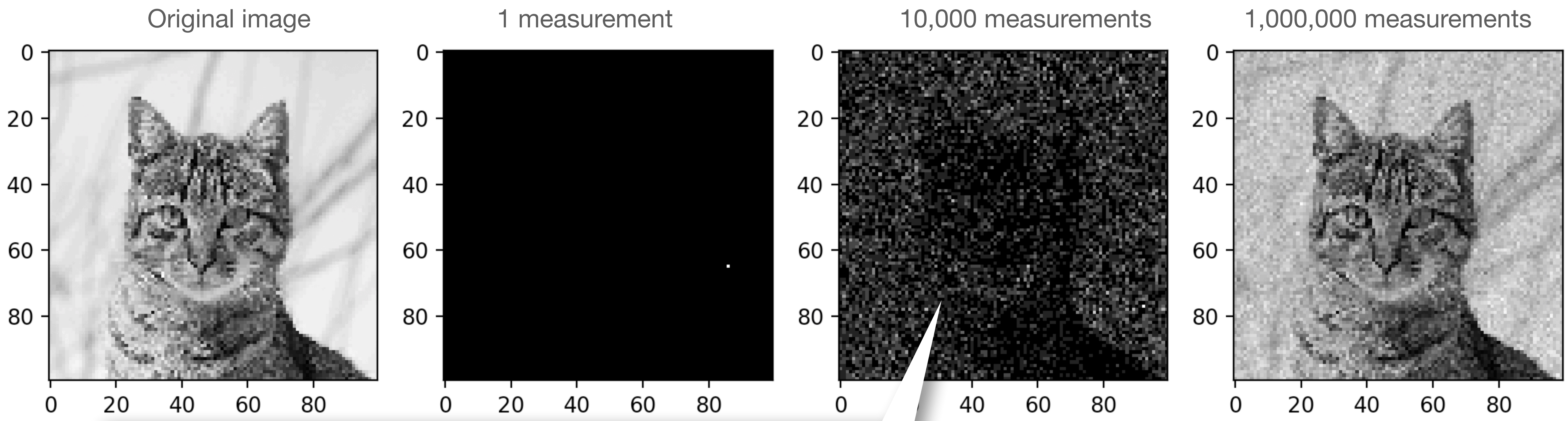
Represent pixel coordinates as binary strings, for example:

$$|k=2\rangle = |0\rangle \otimes \ldots \otimes |1\rangle \otimes |0\rangle$$

Requires only **log(N^2) qubits!!**



QUANTUM DATA ENCODING



Is this additional uncertainty worth it?

Quantum advantage is directly related to this compression factor in quantum computing.

Classical computing Fourier transform on N^2 pixels: $O(N^4)$ or $O(N^2 \log(N^2))$ for FFT

Quantum computing Fourier transform (QFT):

- N^2 pixels represented by $\log(N^2)$ qubits
- QFT requires circuit with $O(\log(N^2)\ \log(N^2))$ gates **=> exponential algorithmic speedup**

However…

IMAGE RECONSTRUCTION ACCURACY

Reconstruction accuracy of random images

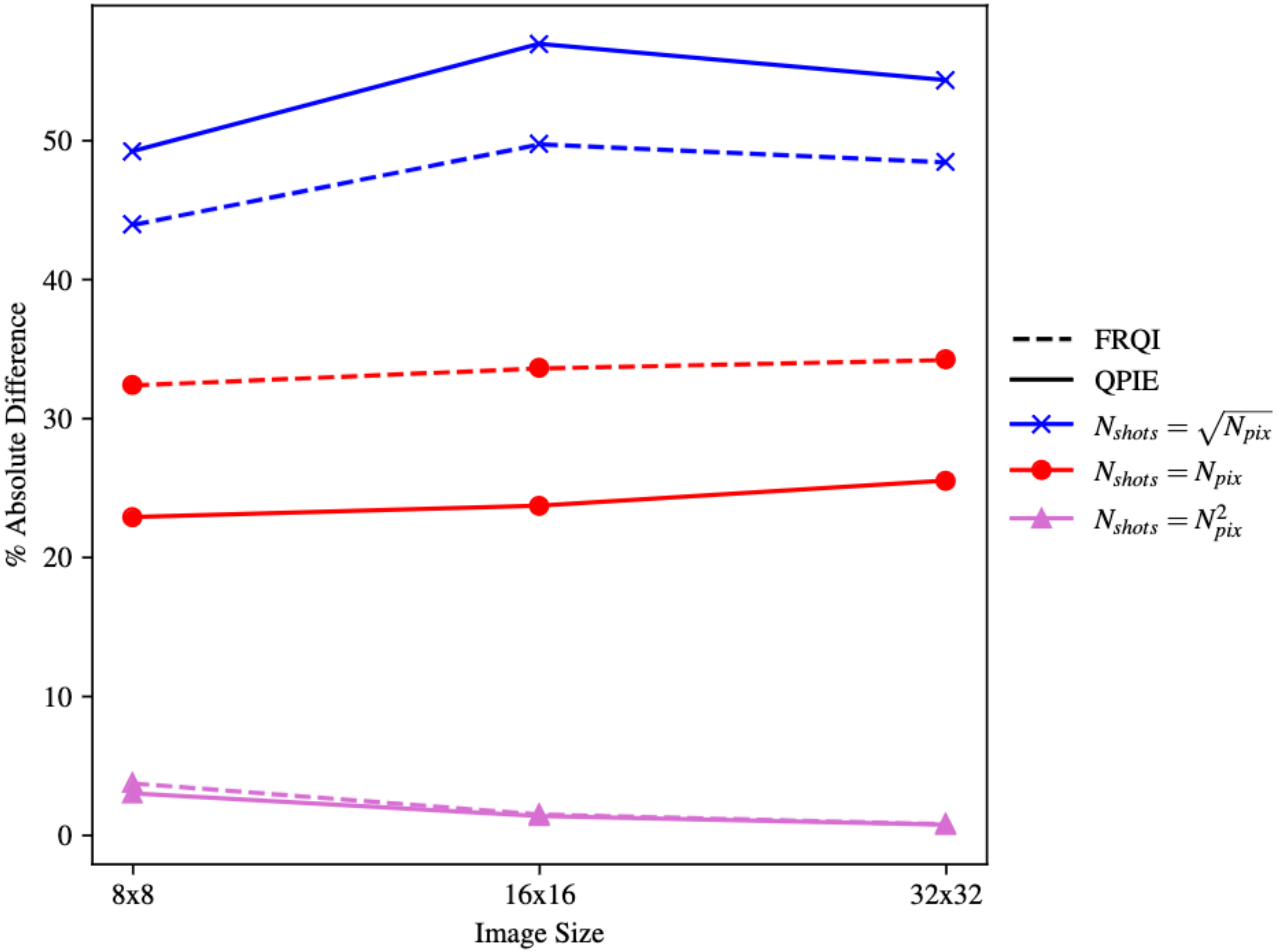
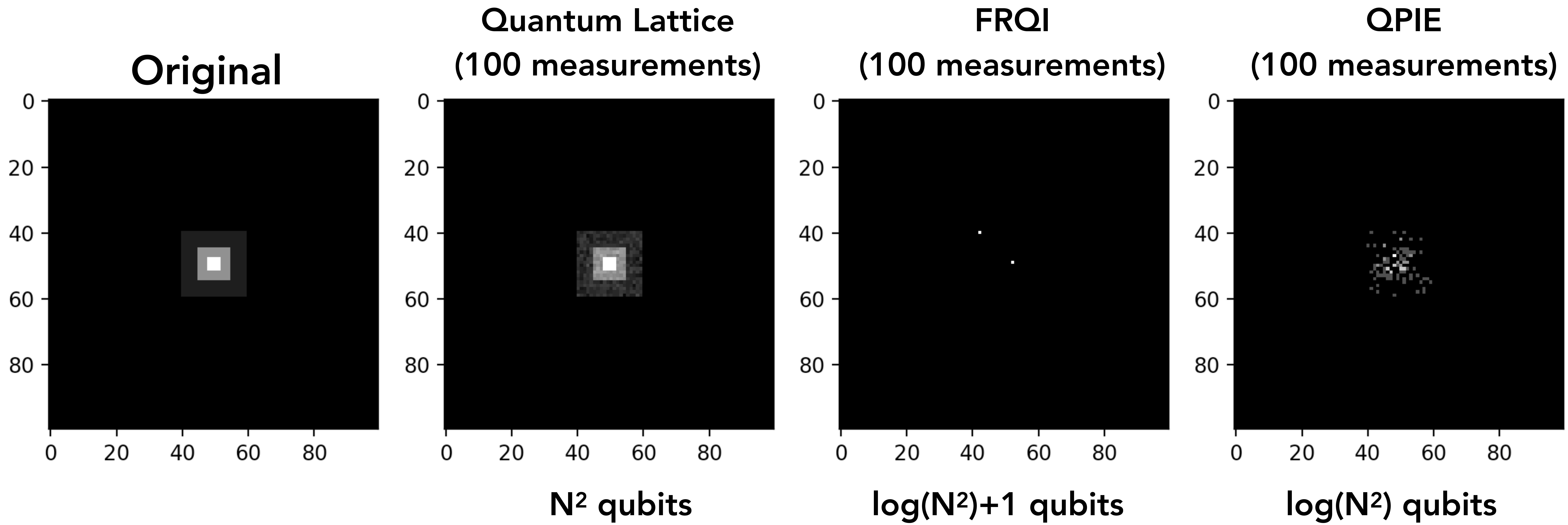


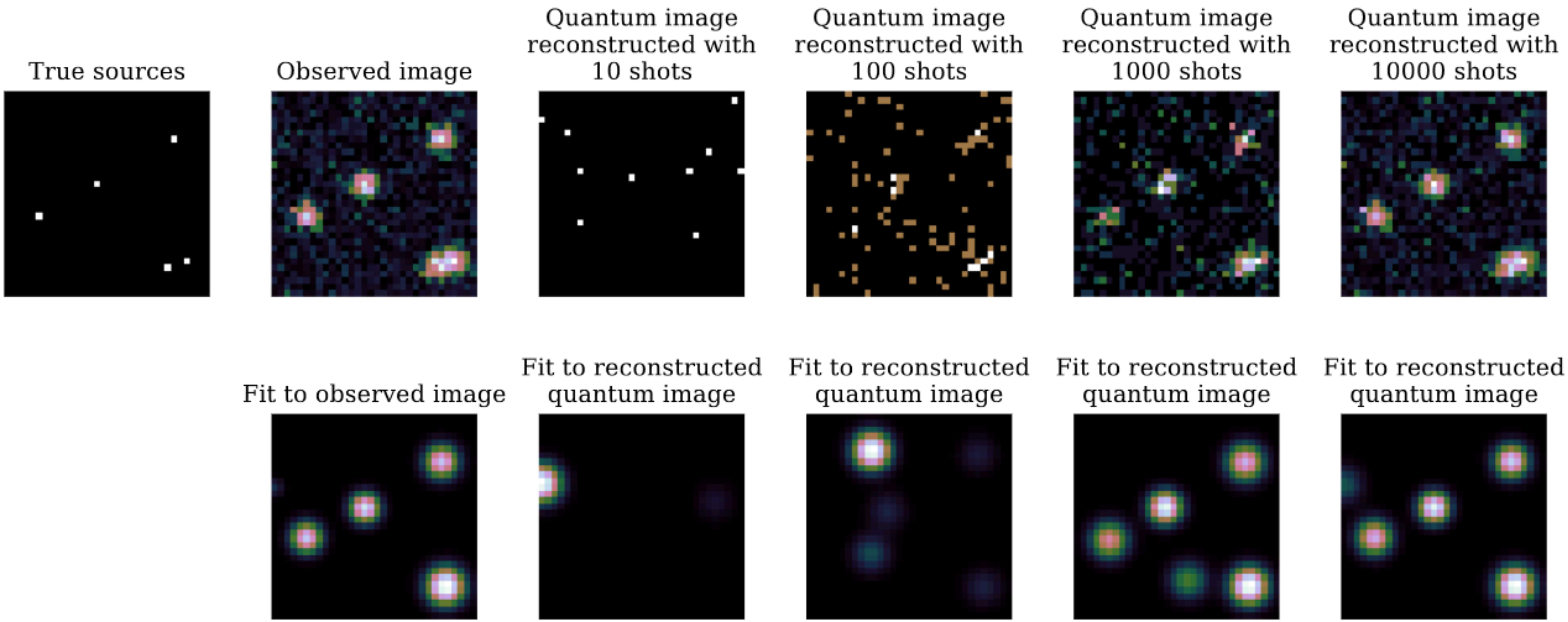
IMAGE RECONSTRUCTION ACCURACY

Reconstruction accuracy of sparse images



SOURCE RECONSTRUCTION ACCURACY

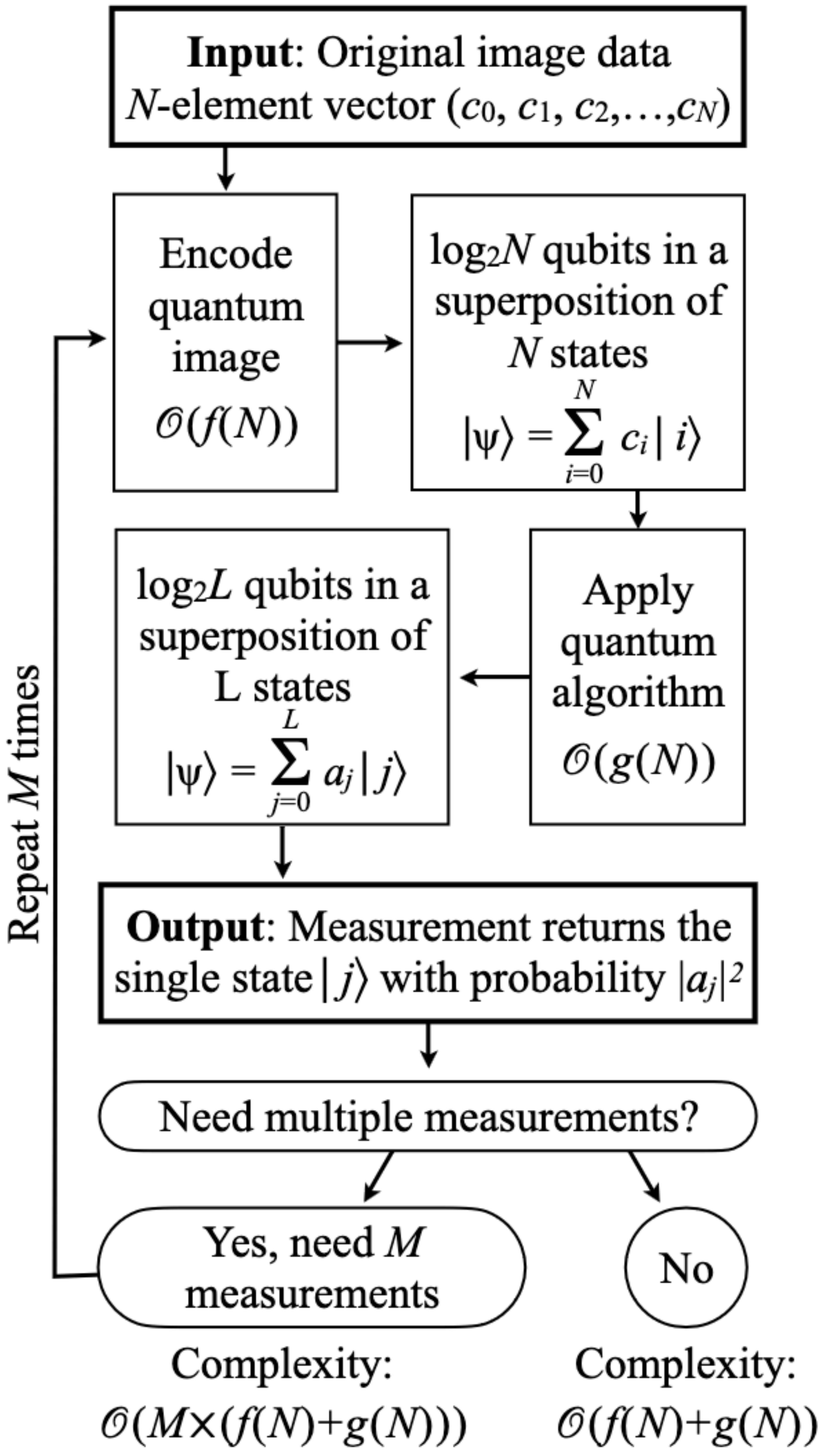
Toy source reconstruction pipeline



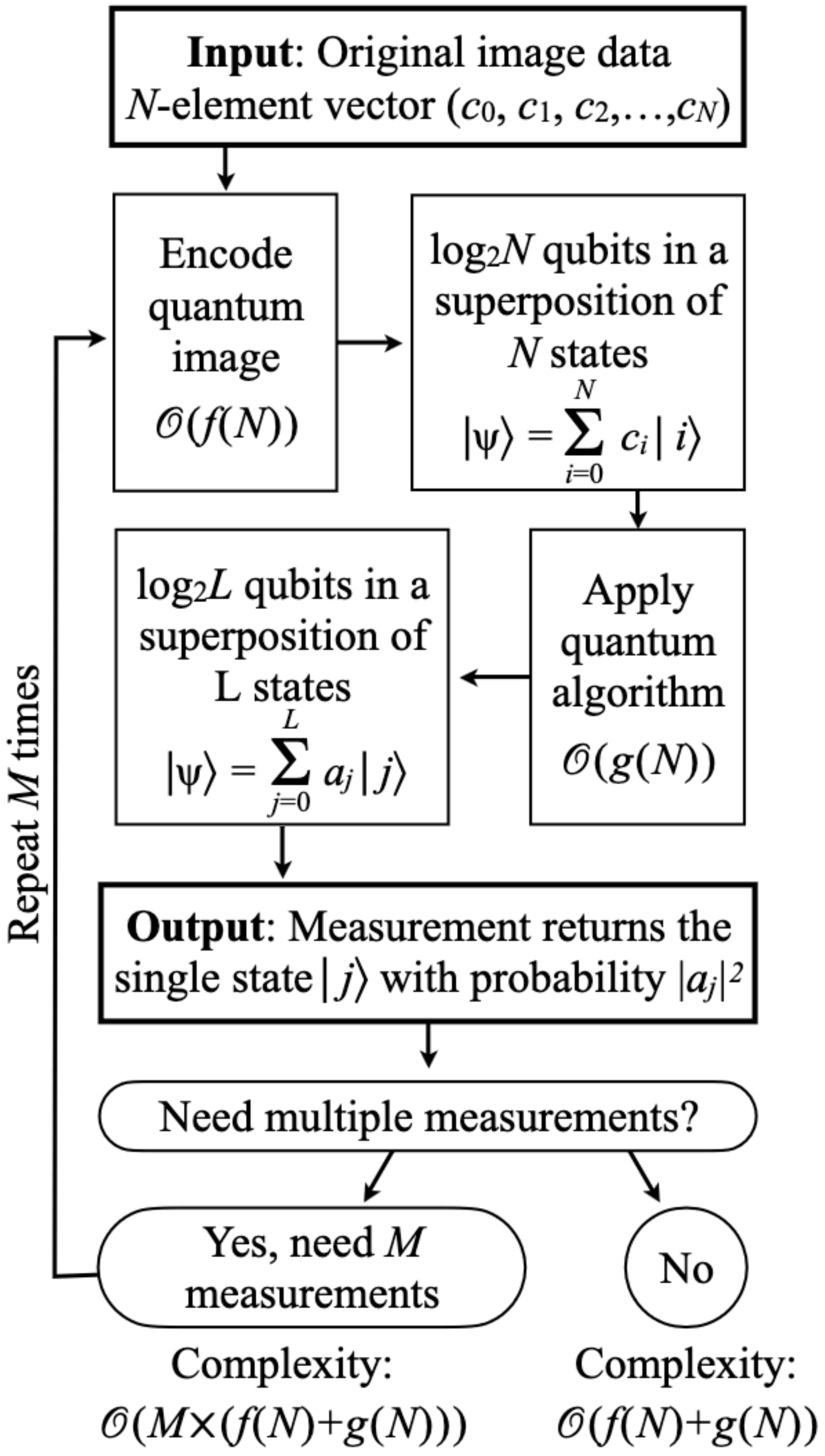
Try to fit 2D Gaussian profiles to these images, and measure reconstruction efficiency

SOURCE RECONSTRUCTION ACCURACY

QUANTUM ADVANTAGE



QUANTUM ADVANTAGE



Optimal QPIE circuit depth:

$$\mathcal{O}(f(N^2)) = \mathcal{O}(\log_2 N \times \log_2 N)$$

Complexity of QFT is:

$$\mathcal{O}(\log_2 N \times \log_2 N)$$

Total complexity is:

$$\mathcal{O}(N_{\text{shots}} \times \log_2 N \times \log_2 N)$$

For SNR=10 source reconstruction: $N_{\text{shots}} = \mathcal{O}(N^2)$

Beats classical O(N^4) **FT but not** O(N^2 log (N^2)) **FFT**

For SNR=100 source reconstruction: $N_{\text{shots}} = \mathcal{O}(N)$

Exponential speedup over FFT!

However…

QUANTUM ERROR

Real quantum computers are quite noisy, gates and circuits can be corrupted

Assuming a uniform gate error rate of:

$$\epsilon = P(\text{gate fails})$$

Then a circuit with depth D will have a global failure rate of:

$$\begin{aligned} P(\text{at least 1 gate fails}) &= 1 - P(\text{all gates succeed}) \\ &= 1 - (1 - \epsilon)^{D_{\text{circ}}} \end{aligned}$$

Thus ϵ or D need to be quite small…

QUANTUM ERROR

Can measure ϵ and D on the publicly available IBM quantum computers

ϵ is gate and hardware-dependent, but typical values on current hardware are 10^{-2} - 10^{-5}

Using a recursive initialization algorithm from Shende et al. (2006) to build the QPIE initialization circuit

4x4 image initialized with 74 gates: **~15% failure rate**

256x256 image initialized with > 20,000 gates: **~100% failure rate**

OUTLOOK

Lower ϵ:

- Rapid developments in the field of quantum hardware, which may improve the quantum error situation
- Quantum error correction can flag corrupted circuits

Lower D:

- QRAM: Store data instead of re-encoding it each time
- Improved algorithms for data encoding (interesting proposal in Zhang et al (2021) for encoding a 1024×1024 image with ~ 100 gates)
- Can decompose image to run more, smaller circuits

FUTURE DIRECTIONS

- Quantum error: should be mitigated by continuous developments in the field of QC
- Quantum uncertainty: can only be mitigated by **algorithmic developments**
- Quantum FFT? Quantum gridding/degridding? Quantum NU-FFT? Quantum deconvolution?
- Similar study for time-domain/pulsar searches?
- Quantum variational circuits & quantum machine learning