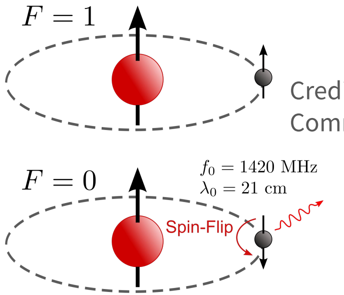
Hydrogen Intensity and Real-time Analysis eXperiment

Primary Beam Systematics and Instrument Simulation¶

SKA Days - Geneva - 02.09.2024

Devin Crichton - ETH Zurich

HI Intensity Mapping Tomography



Credit: Wikimedia Commons

- Hyperfine Hydrogen transition line at 1420.4 MHz
- Probe cosmic dawn and epoch of reionisation at low frequencies and large scale structure at high frequencies
- Low angular resolution, high spectral resolution

$$\nu_{\text{obs.}} = \frac{1420.4\ \text{MHz}}{1+z}$$

- Post-reionisation IM
  - **ν** > 200-300 MHz
  - HI emission acts as biased tracer of large scale structure
  - Large volumes on linear-quasilinear scales

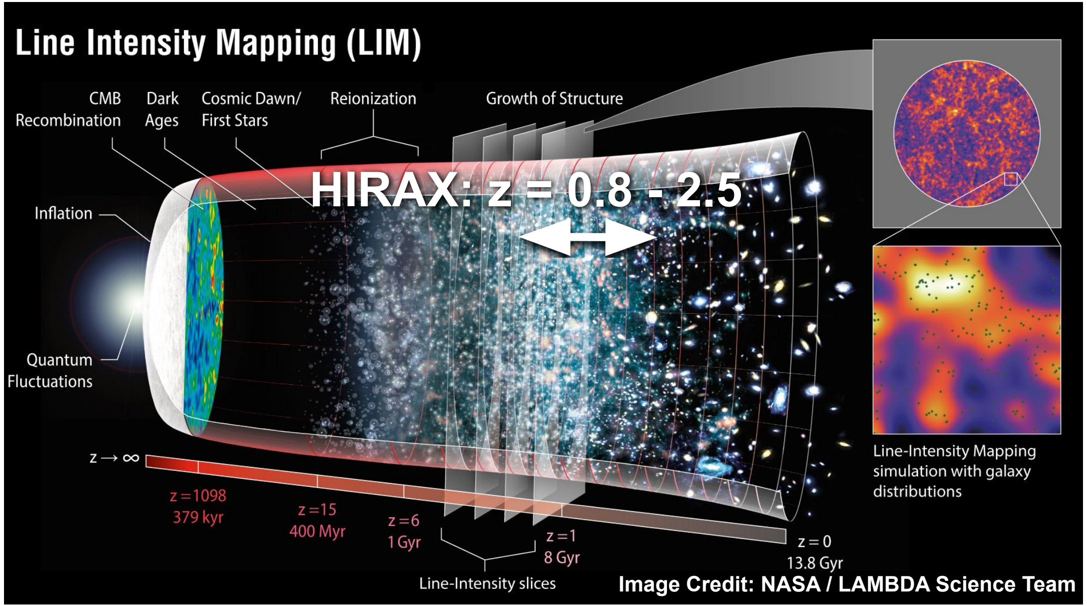


Image Credit: NASA / LAMBDA Science Team

Motivation for Compact Redundant Arrays

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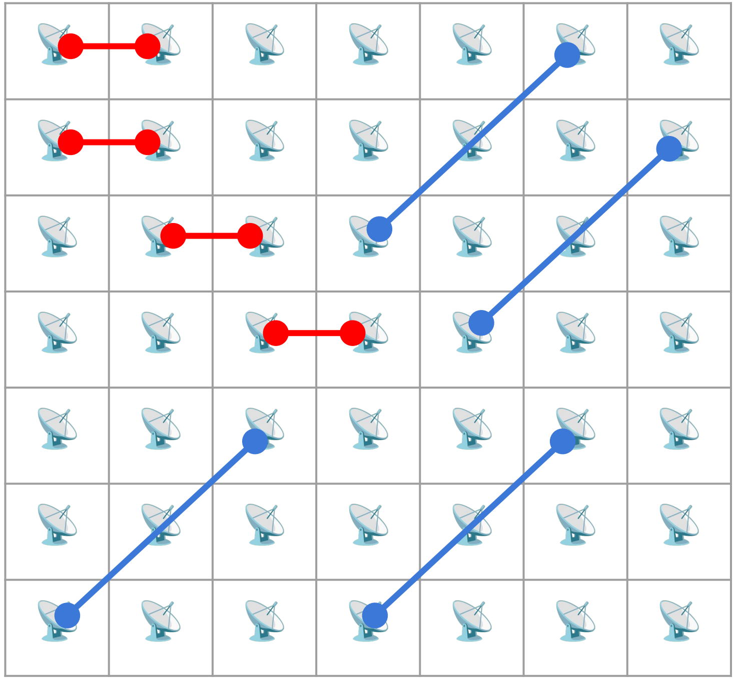
- Compact
  - ***Accessing large, cosmological angular scales***
  - Most weight on short baselines
  - Potential for cross-talk, reflections and impact from array-level effects
- Redundant array
  - **Enhanced sensitivity on sky Fourier modes on interest**
  - Large N with many repeated baselines
  - Internal, redundant, calibration
  - Large grating lobes leads to poor imaging capability
- E.g HIRAX, CHIME, CHORD, HERA, MWA



Systematics / Chromaticity and Foregrounds

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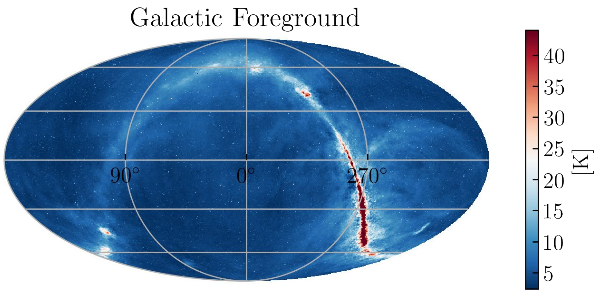
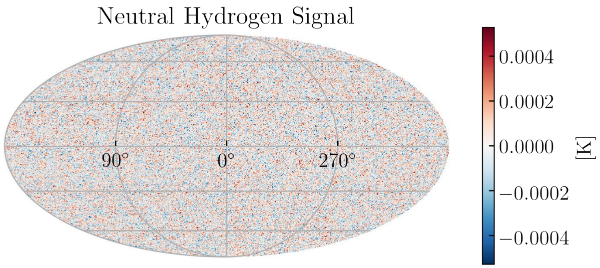
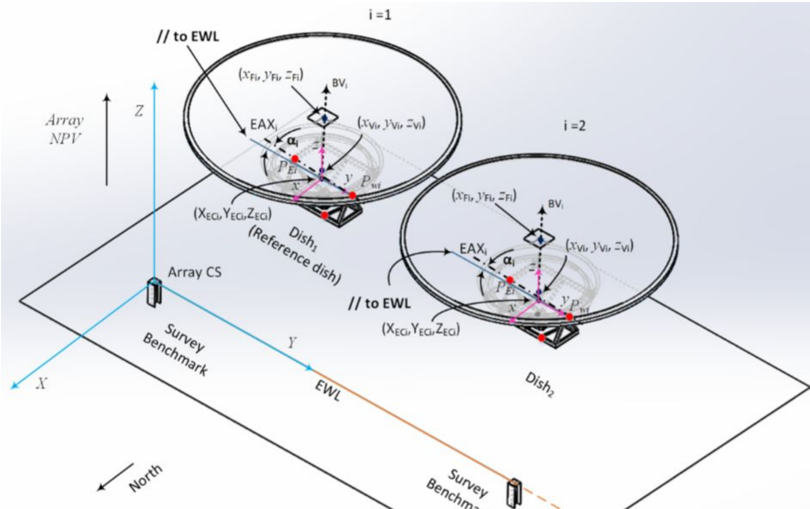
- Foregrounds are the primary challenge for 21cm cosmology
  - Galactic signal brighter by many orders of magnitude
- Signal and Foregrounds have different, *on-sky* properties
  - Galactic emission is:
    - Polarised
    - Strongly correlated over wide frequency bands
    - Structured on the sky in ~known way
  - In principle, there are not many mixed *on-sky* degrees of freedom
- Mode-mixing inherent in measurement is a major issue
  - Instrument has chromatic response *fundamentally* as well as arising from *systematics*
  - With perfect knowledge of the instrument, this can be accounted for, however the large contrast in signal strengths can make small reconstruction residuals a big problem
- **Tight requirements on telescope design**
- **Instrument simulation and characterisation is critical**



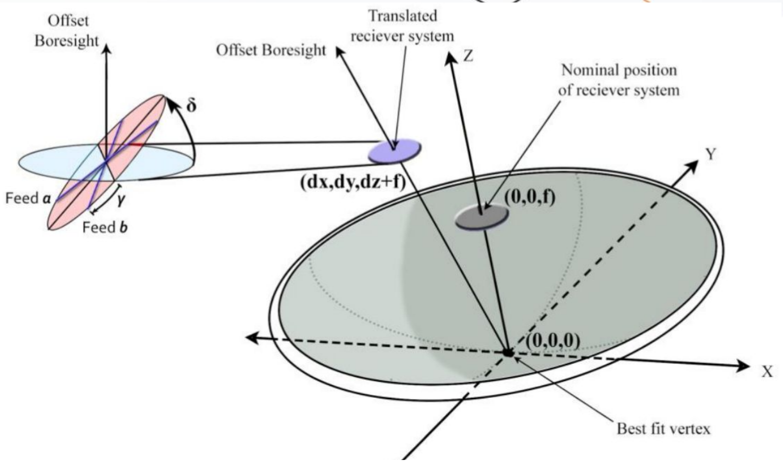


HIRAX Calibration Challenges

- Dishes fixed per elevation pointing
  - **Calibration options limited, pointing etc. needs external verification/measurement**
    - Rely on simulations
- Redundant interferometer
  - Calibration and on-site data compression relies on internal consistency
  - **HW Requirements on precision over accuracy**
- Consistency needs to be verified across array



Islam et al. 2022



Telescope Mechanical Assembly Requirements

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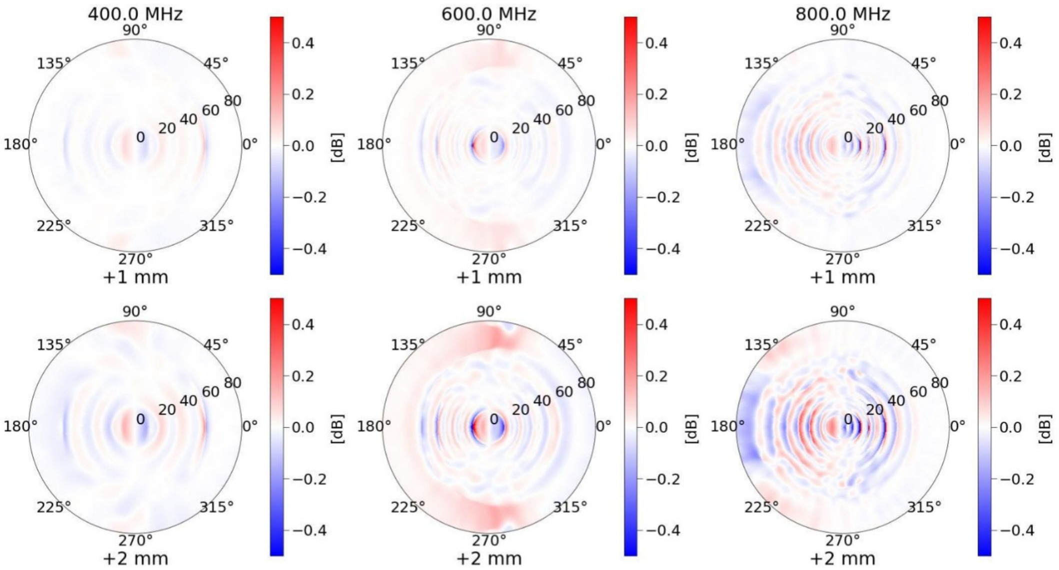
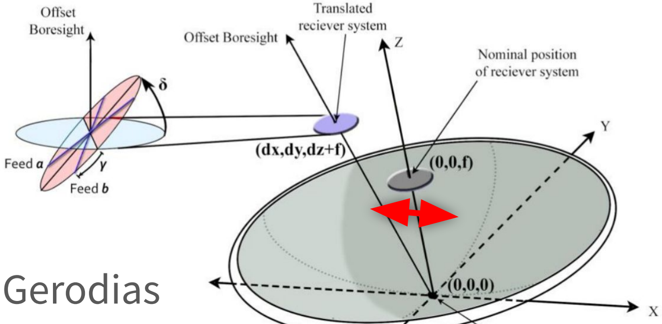


Requirements set with simulations

- λ/100 - λ/50 (< 1 mm)
  - Favour precision over accuracy
- Verified with metrology
  - Laser Tracker and Photogrammetry
  - During manufacture and operation



- Shifts beam centroid/effective pointing
  - Large systematic effect for physical tolerances
- Distribution of mis-pointing across the array is a large systematic concern

Beam Simulations: Kit Gerodias

Systematics Propagation to Cosmology

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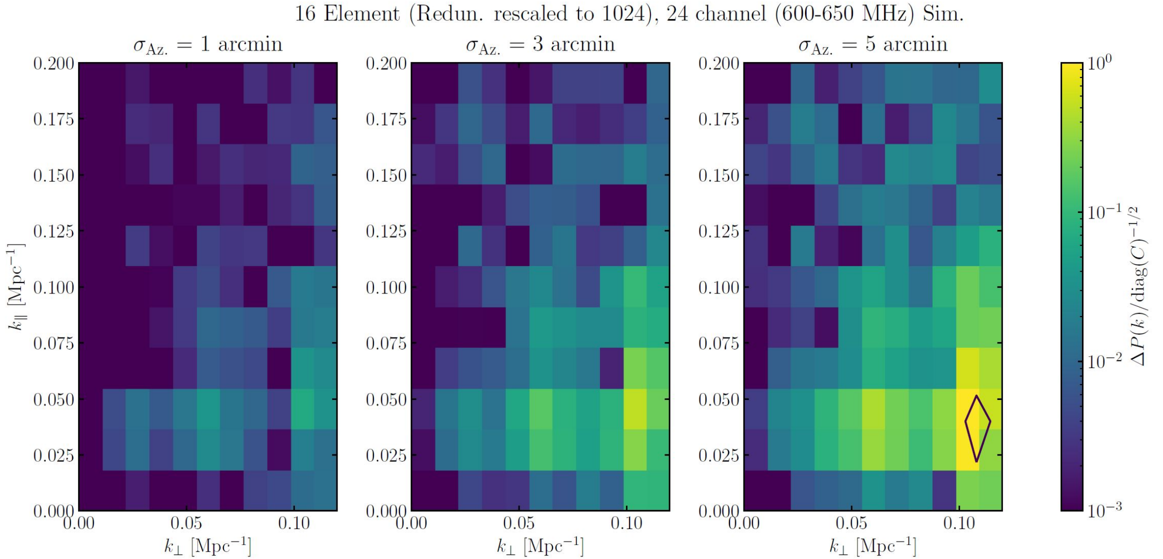
Current dish requirements set by:

- Perturbing per-feed response based on distribution of systematic offsets over array
- Averaging down linearised systematics over redundant baselines
- Propagating residual to foreground filtered power spectrum

Extending with:

- More efficient simulations
- Full rel. and abs. Calibration steps
- Accounting for systematics mitigation through modelling and filtering



Telescope mechanical parameter	Target precision (RMS)
Receiver position relative to focus	0.5 mm
Receiver orientation relative to boresight vector	2.5′ polar and azimuthal
Dish surface deviations	1 mm
Dish vertex position relative to elevation axis	1 mm
Orthogonality of boresight vector and elevation axis	1′
Elevation axis position within the array	0.5 mm in array plane 1 mm out of array plane
Elevation axis alignment within the array	1′
Elevation pointing angle	1′

~ λ/100 - λ/50

Beam Simulations: Ben Saliwanchik, Elizabeth Peters, Kit Gerodias

Visibility synthesis for driftscan telescopes

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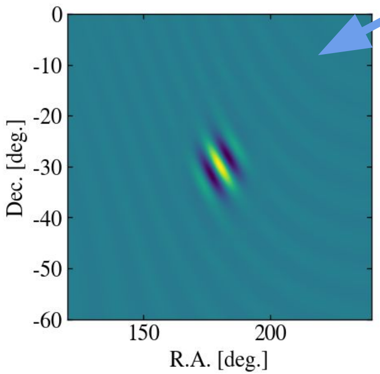
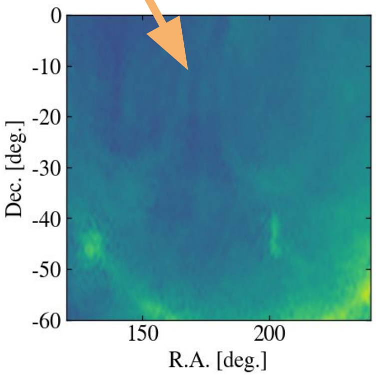
$$\mathcal{V}^{ij\nu}(\phi) = \int d\Omega \, A^{i\nu} A^{j\nu *}(\hat{\boldsymbol{\theta}}) \, \exp\left[-2\pi i \, \boldsymbol{u}^{i-j} \cdot \hat{\boldsymbol{\theta}}\right] T^{\nu}(\hat{\boldsymbol{\theta}}, \phi)$$

SHT of  = B

SHT of  = a

$$\int \frac{\mathrm{d}\phi}{2\pi} \mathcal{V}^{ij\nu}(\phi) e^{-im'\phi} = \sum_{\ell m} B^{ij\nu}_{\ell m} a^{\nu}_{\ell m} \int \frac{\mathrm{d}\phi}{2\pi} e^{i(m-m')\phi}$$

$$\mathcal{V}^{ij\nu}_{m} = \sum_{\ell} B^{ij\nu}_{\ell m} a^{\nu}_{\ell m}$$

M-mode approach:

Integral of two SH's $\rightarrow \delta_{ll'mm'}$
Fourier transform of visibility timestream equates to per-m matrix product.

Visibility synthesis for driftscan telescopes

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M-mode approach for systematics simulation:

- Very efficient and m-separability very useful, particularly for inverse problems
- Instrument model baked in to beam transfer matrices and they must be recomputed if it's updated
- With known instrument, can efficiently predict data as sky varies
  - But hard to vary instrument without recomputing many harmonic transforms
- Requires primary beams evaluated on same grid as the baseline phase term and sky
  - Can't make use of band limits of individual terms separately.
- Can we does something similar with primary beams flexible?

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$$\mathcal{V}^{ij\nu}(\phi) = \int d\Omega \, A^{i\nu} A^{j\nu *}(\hat{\boldsymbol{\theta}}) \, \exp\left[-2\pi i \, \boldsymbol{u}^{i-j} \cdot \hat{\boldsymbol{\theta}}\right] T^{\nu}(\hat{\boldsymbol{\theta}}, \phi).$$

SHTs of

What about?

$$\mathcal{V}^{ij\nu}(\phi) = \sum_{\ell m} B^{ij\nu}_{\ell m} a^{\nu}_{\ell m}(\phi)$$

If we combine the baseline term and sky, we lose m-separability.

- Baseline fixed to terrestrial coordinate systems
- No way keep primary beam flexible while keeping m-mode separability if we only do two spherical harmonic transforms in the integral.
- What about more spherical harmonic transforms?
    - Gets complicated…

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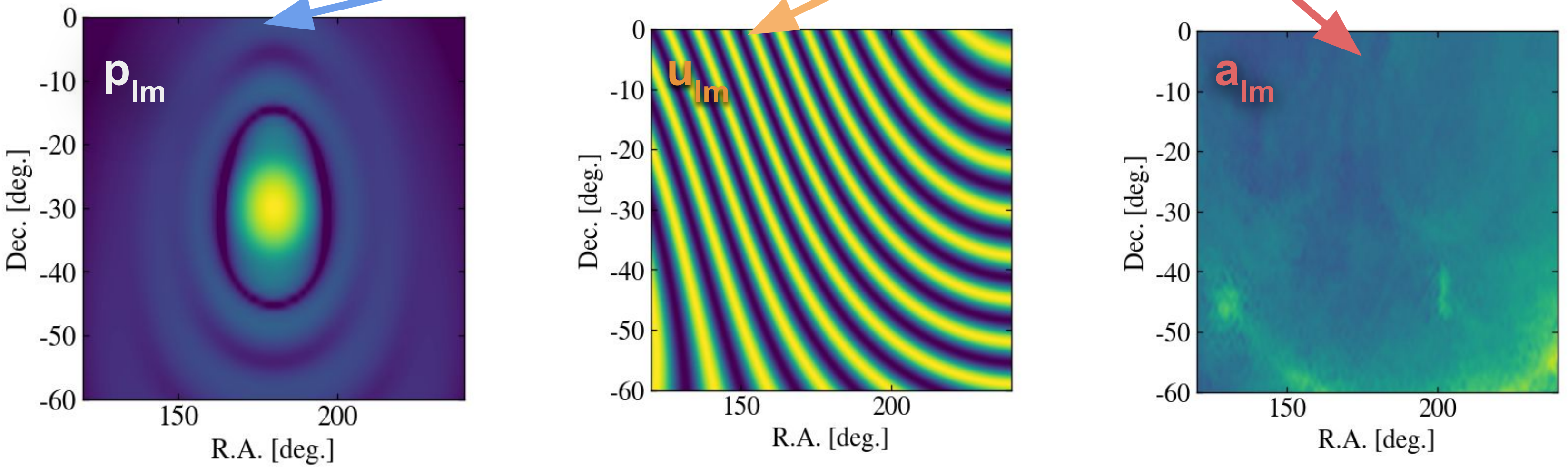

What about integral of three spherical harmonics?

$$\mathcal{V}^{ij\nu}_{m_a} = \sum_{\ell m\text{'s}} p^{ij\nu}_{\ell_p m_p} u^{ij\nu}_{\ell_u m_u} a^{\nu}_{\ell_a m_a} \mathcal{G}^{\ell_p \;\; \ell_u \;\; \ell_a}_{m_p \; m_u \; m_a}$$

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$$\mathcal{V}^{ij\nu}_{m_a} = \sum_{\ell m\text{'s}} p^{ij\nu}_{\ell_p m_p} u^{ij\nu}_{\ell_u m_u} a^{\nu}_{\ell_a m_a} \boldsymbol{\mathcal{G}}^{\ell_p\ \ \ell_u\ \ \ell_a}_{m_p\ m_u\ m_a}$$

Triple integrals of spherical harmonics correspond to Gaunt coefficients

Not as nice as delta function but:

- Extremely sparse:
  - Selection rules on m and l, structured sparsity.
- Highly symmetric
  - Can compute a limited subset of non-zero coefficients and expand with symmetries
- Can use known band-limits in l and m (if in appropriate coordinate systems) to make very compact multilinear map

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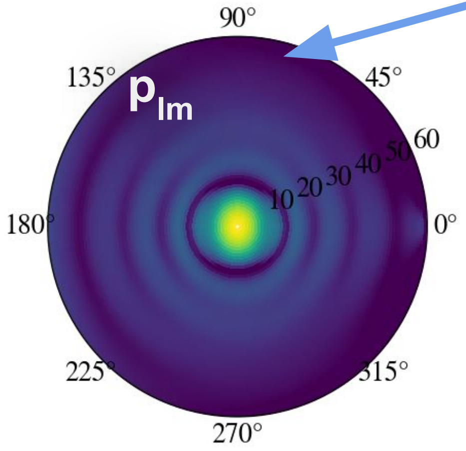
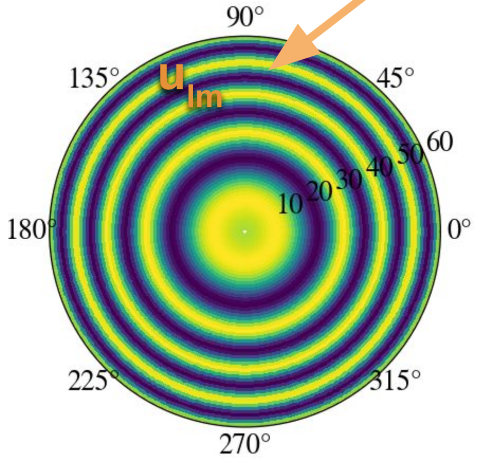
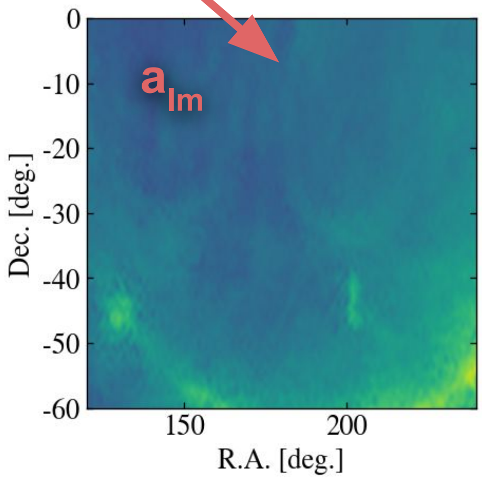
$$\mathcal{V}^{ij\nu}(\phi) = \int d\Omega\, A^{i\nu} A^{j\nu*}(\hat{\boldsymbol{\theta}}) \exp\left[-2\pi i\; \boldsymbol{u}^{i-j}\cdot\hat{\boldsymbol{\theta}}\right] T^{\nu}(\hat{\boldsymbol{\theta}},\phi).$$







$$\mathcal{V}^{ij\nu}_{m_a} = \sum_{\ell m\text{'s}}^{m_{p,\max}, m_u=0} p^{ij\nu}_{\ell_p m_p}\, u^{ij\nu}_{\ell_u}\, a^{\nu}_{\ell_a m_a}\, \boldsymbol{\mathcal{S}}^{\ell_p\ \ell_u\ \ell_a}_{m_p\ 0\ m_a}(\mathrm{P}, \hat{\boldsymbol{u}}_{i-j})$$

- If we do expansions in appropriate coordinates and rotate: compact in m's.

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$$\mathcal{V}^{ij\nu}(\phi) = \int d\Omega\, \boxed{A^{i\nu}}\, \boxed{A^{j\nu *}(\hat{\boldsymbol{\theta}})}\, \exp\left[-2\pi i\, \boldsymbol{u}^{i-j} \cdot \hat{\boldsymbol{\theta}}\right] T^{\nu}(\hat{\boldsymbol{\theta}}, \phi).$$

Can also go a bit further and separately expand in voltage beams

- Still very sparse although a bit more difficult to handle, need product of two sets of gaunt coefficients. Can also rotate to compact form.

$$\mathcal{V}^{ij\nu}_{m_a} = \sum_{\ell m\text{'s}} v^{i\nu}_{\ell_{v_i} m_{v_i}} v^{j\nu}_{\ell_{v_j} m_{v_j}} u^{ij\nu}_{\ell_u m_u} a^{\nu}_{\ell_a m_a} \boldsymbol{\mathcal{G}}^{\ell_{v_i}\ \ell_u}_{m_{v_i}\ m_u} \cdot \boldsymbol{\mathcal{G}}^{\ell_{v_j}\ \ell_a}_{m_{v_j}\ m_a}$$

$$\mathcal{V}^{ij\nu}_{m_a} = \sum_{\ell m\text{'s}}^{m_{v,\max}, m_u = 0} v^{i\nu}_{\ell_{v_i} m_{v_i}} v^{j\nu}_{\ell_{v_j} m_{v_j}} u^{ij\nu}_{\ell_u} a^{\nu}_{\ell_a m_a} \boldsymbol{\mathcal{S}}^{\ell_{v_i}\ \ell_u}_{m_{v_i}\ 0} \cdot \boldsymbol{\mathcal{S}}^{\ell_{v_j}\ \ell_a}_{m_{v_j}\ m_a} (\mathrm{P}, \hat{\boldsymbol{u}}_{i-j})$$

Visibility synthesis for driftscan telescopes

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Advantages of these approaches:

- Allows fixing of sky while varying baseline term
  - Or doing contractions in any order with cached intermediate results
- Maintains sky m-mode separability
- Don’t have to recompute intermediate products with primary beam model updates
- Can compute derivatives of data e.g. with respect to beams, fixing sky

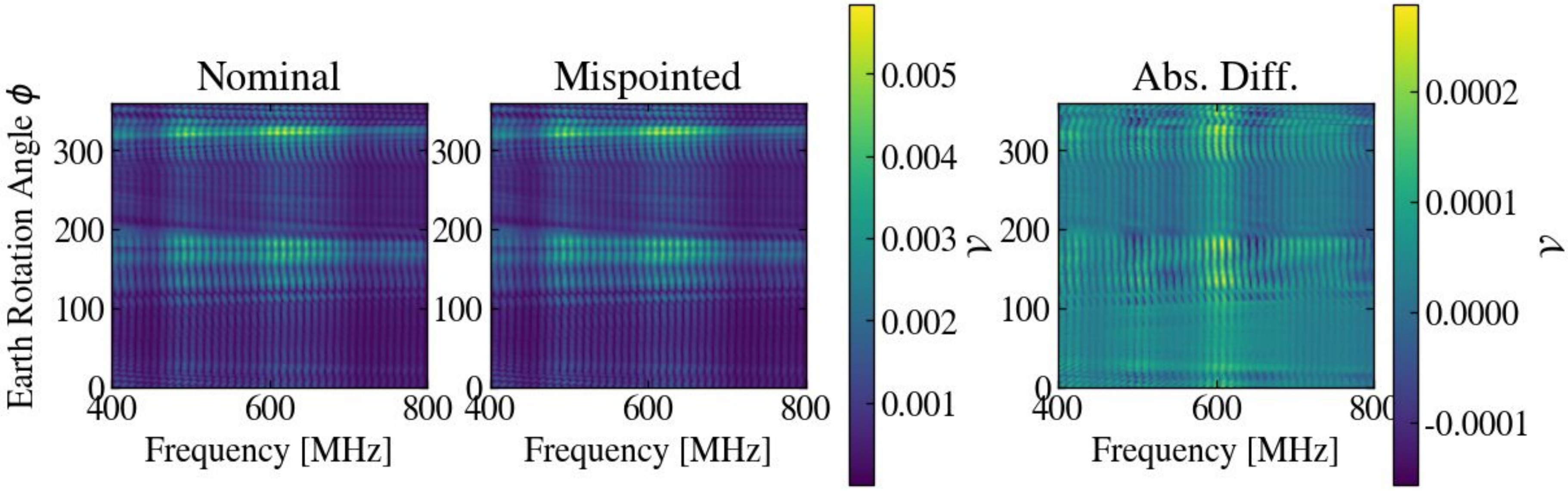
But:

- Need to compute and perform tensor products on large sparse objects
- Although after e.g. fixing a sky model or baseline, can get a compact tensor that that can use accelerate dense linear algebra operations.

Currently only have early prototype proof-of-concept code but extending soon

Visibility synthesis for driftscan telescopes

Example results: visibility waterfalls for perturbed primary beam.



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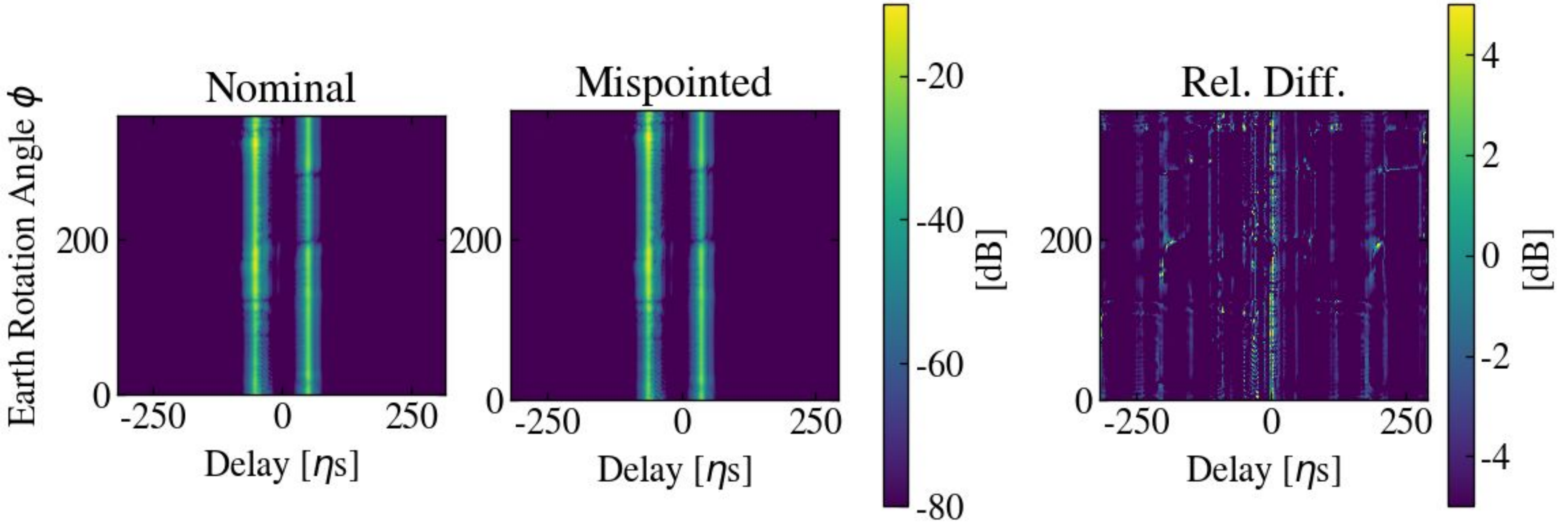
Example results: visibility waterfalls for perturbed primary beam.



Conclusions

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- 21cm intensity mapping provides access to large cosmological volumes over mostly linear scales
- BAO can be targeted with dedicated, compact interferometers, relying on redundancy
- Overcoming systematics/foregrounds challenge is difficult and requires a controlled and well-characterised instrument model.
- Static dishes cannot be easily calibrated directly, requires reconstruction and verification with system measurements.
- Simulating systematics associated with the primary beams varying across the array efficiently is a challenge at large-N
- Extended spherical expansions are an exciting approach to this problem.

Thanks!

Backup Slides

Visibility synthesis for driftscan telescopes

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$$\int \mathrm{d}\Omega\ Y_{\ell_1 m_1}(\hat{\boldsymbol{\theta}}) Y_{\ell_2 m_2}(\hat{\boldsymbol{\theta}}) Y_{\ell_3 m_3}(\hat{\boldsymbol{\theta}}) = \sum_{\ell m} \mathcal{G}^{\ell_1\ \ell_2\ \ell}_{m_1 m_2 m} \int \mathrm{d}\Omega\ Y_{\ell_3 m_3}(\hat{\boldsymbol{\theta}}) Y^*_{\ell m}(\hat{\boldsymbol{\theta}})$$

$$= \sum_{\ell m} \mathcal{G}^{\ell_1\ \ell_2\ \ell}_{m_1 m_2 m} \delta_{\ell \ell_3} \delta_{m m_3}$$

$$= \mathcal{G}^{\ell_1\ \ell_2\ \ell_3}_{m_1 m_2 m_3}$$

Visibility synthesis for driftscan telescopes

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$$\int \mathrm{d}\Omega\ Y_{\ell_1 m_1}(\hat{\boldsymbol{\theta}}) Y_{\ell_2 m_2}(\hat{\boldsymbol{\theta}}) Y_{\ell_3 m_3}(\hat{\boldsymbol{\theta}}) Y_{\ell_4 m_4}(\hat{\boldsymbol{\theta}})$$

$$= \sum_{\ell m} \sum_{\ell' m'} \mathcal{G}^{\ell_1\ \ell_2\ \ell}_{m_1 m_2 m} \mathcal{G}^{\ell_3\ \ell_4\ \ell'}_{m_3 m_4 m'} \int \mathrm{d}\Omega\ Y^*_{\ell m}(\hat{\boldsymbol{\theta}}) Y^*_{\ell' m'}(\hat{\boldsymbol{\theta}})$$

$$= \sum_{\ell m} \sum_{\ell' m'} \mathcal{G}^{\ell_1\ \ell_2\ \ell}_{m_1 m_2 m} \mathcal{G}^{\ell_3\ \ell_4\ \ \ell'}_{m_3 m_4 -m'} (-1)^{m'} \int \mathrm{d}\Omega\ Y^*_{\ell m}(\hat{\boldsymbol{\theta}}) Y_{\ell' m'}(\hat{\boldsymbol{\theta}})$$

$$= \sum_{\ell m} \mathcal{G}^{\ell_1\ \ell_2\ \ell}_{m_1 m_2 m} \mathcal{G}^{\ell_3\ \ell_4\ \ \ell}_{m_3 m_4 -m} (-1)^{m}$$

$$\equiv \mathcal{G}^{\ell_1\ \ell_2}_{m_1 m_2} \cdot \mathcal{G}^{\ell_3\ \ell_4}_{m_3 m_4}.$$