Schrödinger-Poisson Informed Neural Networks (SPINN)

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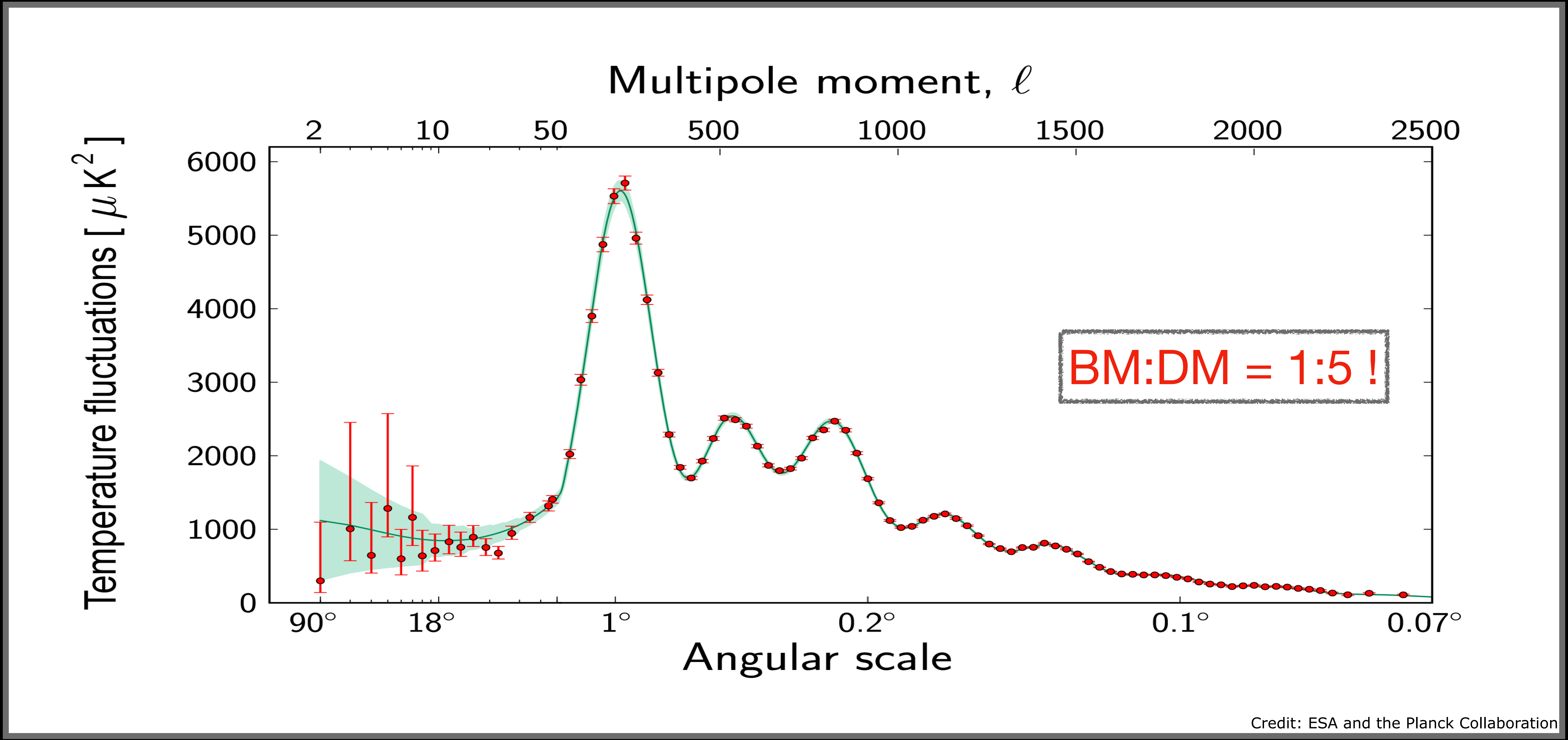
CMB Power Spectrum



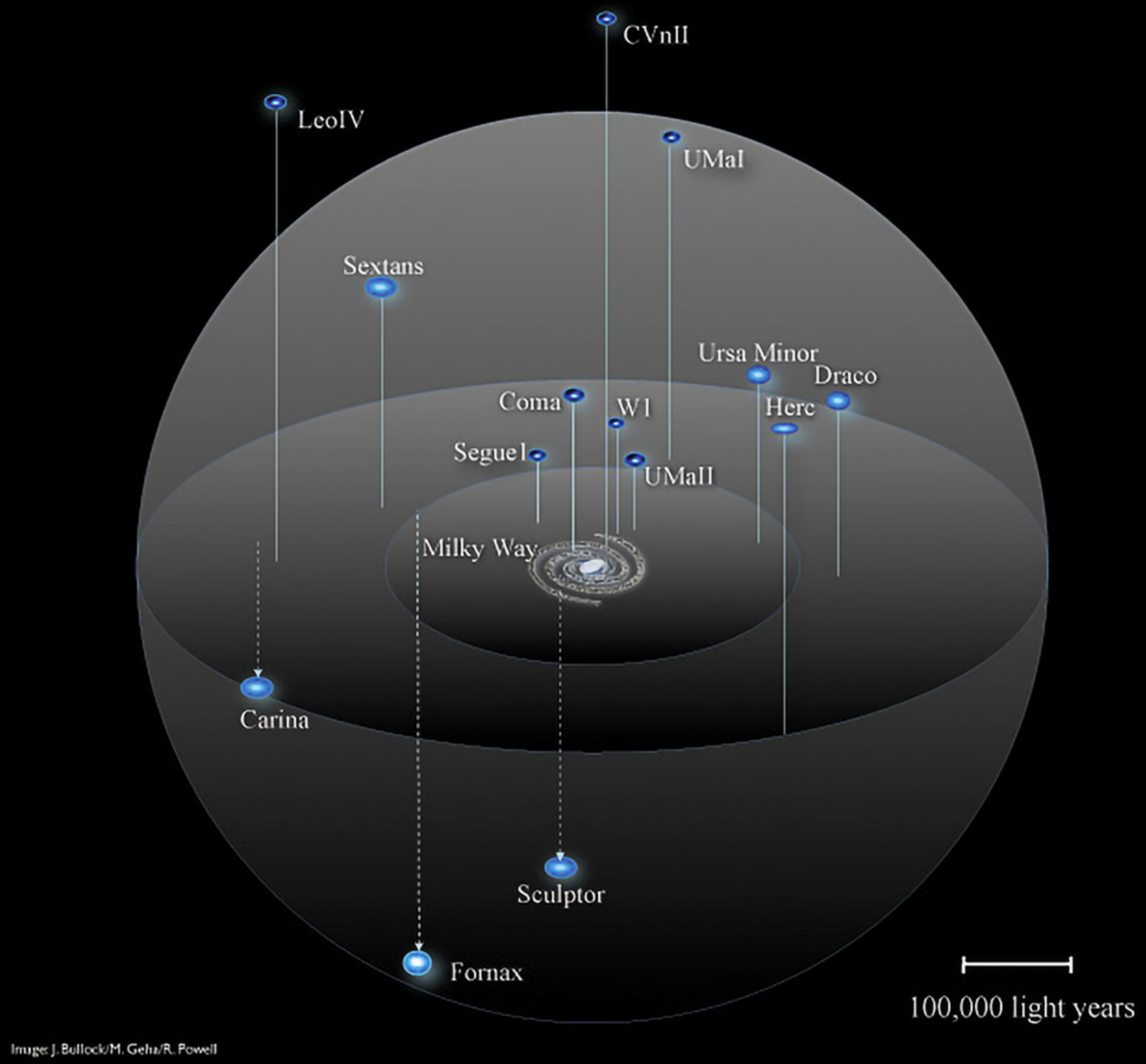
ΛCDM Theoretical Fit: $\Omega_b h^2 \approx 0.024, \quad \Omega_m h^2 \approx 0.14$

Small Scale Challenges in CDM Model

Core-Cusp Problem:
CDM Halo density profiles tend to be 'cuspy'!

Missing Satellite Problem:
#DM subhalos (in simulations) >> #galaxy satellites in Milky Way

'Too big to fail' Problem:
DM subhalos (in simulations) so massive to not have visible stars



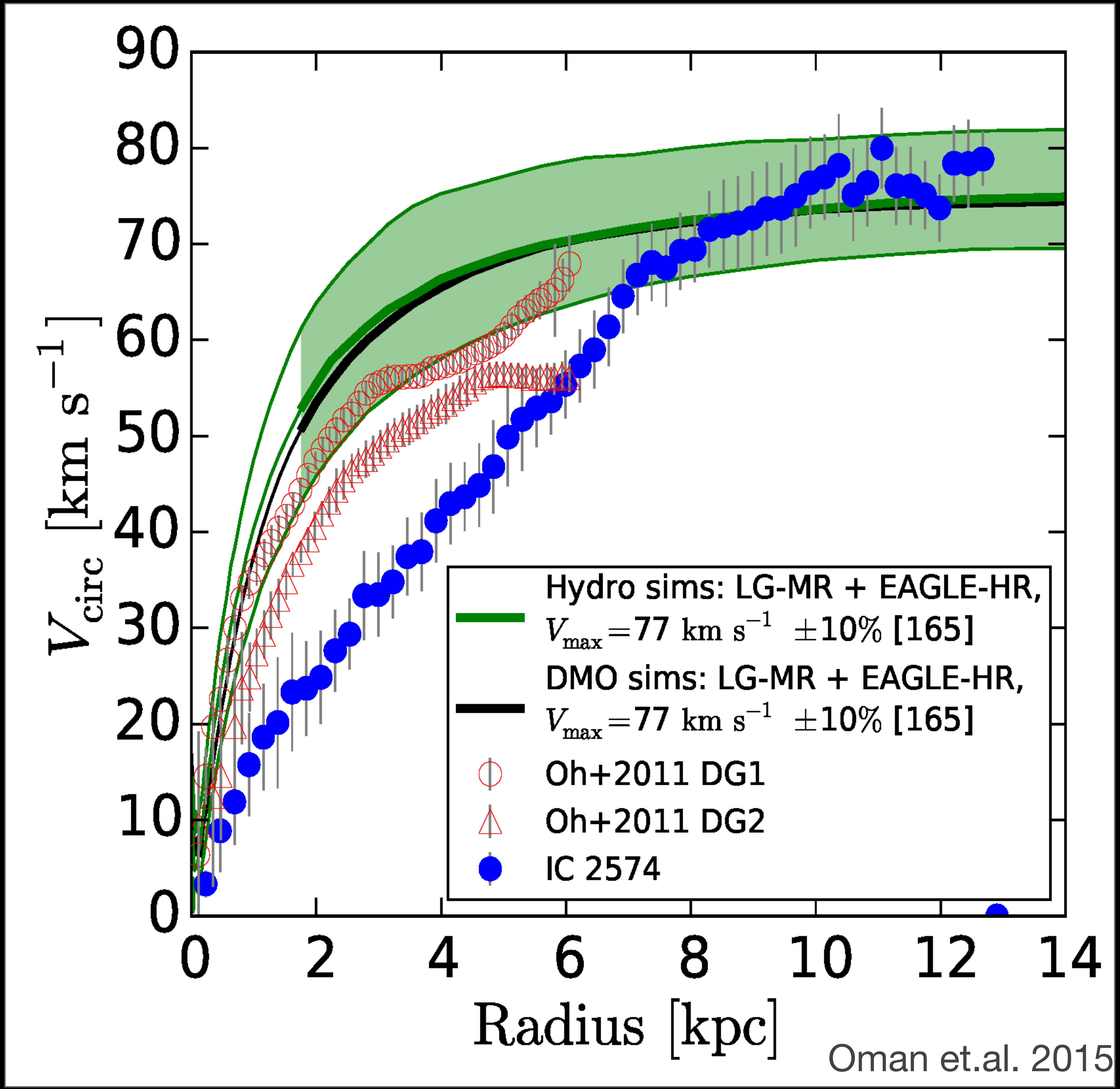
Potential Problem: Absence of **Baryonic Processes** (Feedback, Formation) and/or Nature of **DM**!

Baryonic Processes

Strongly model dependent e.g. feedback sensitivity to the gas threshold for galaxy formation.

Very Difficult to disentangle baryonic effects in the Simulations!

Some outliers like IC 2574 still unexplainable with Feedback!



Oman et.al. 2015

Alternative Dark Matter Models

Warm Dark Matter (WDM): favored mass range in tension with Lyα observation & abundance of high-z galaxies

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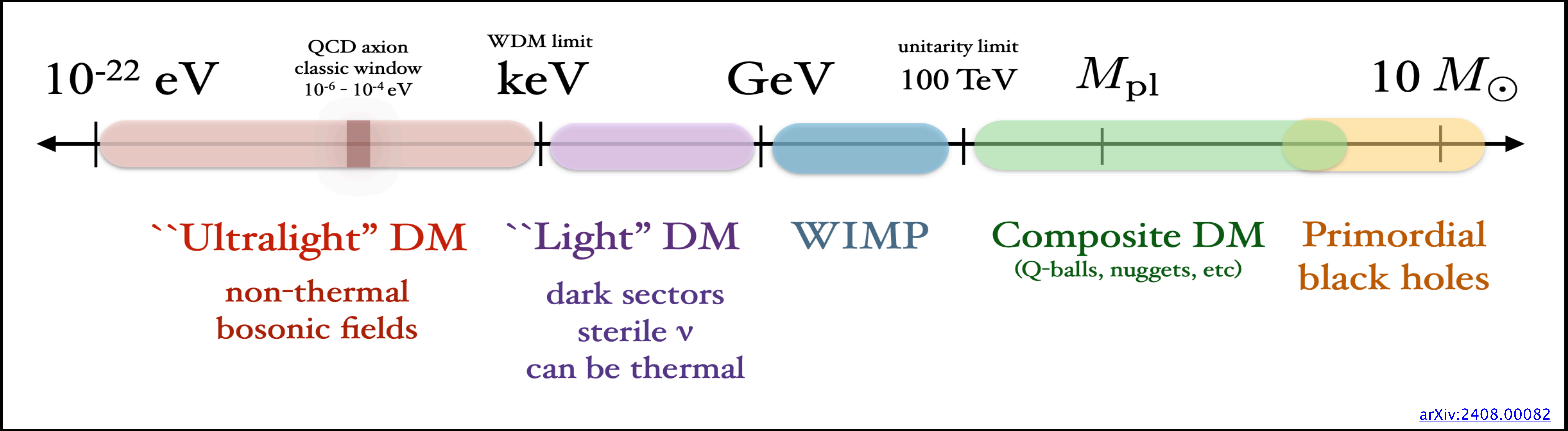

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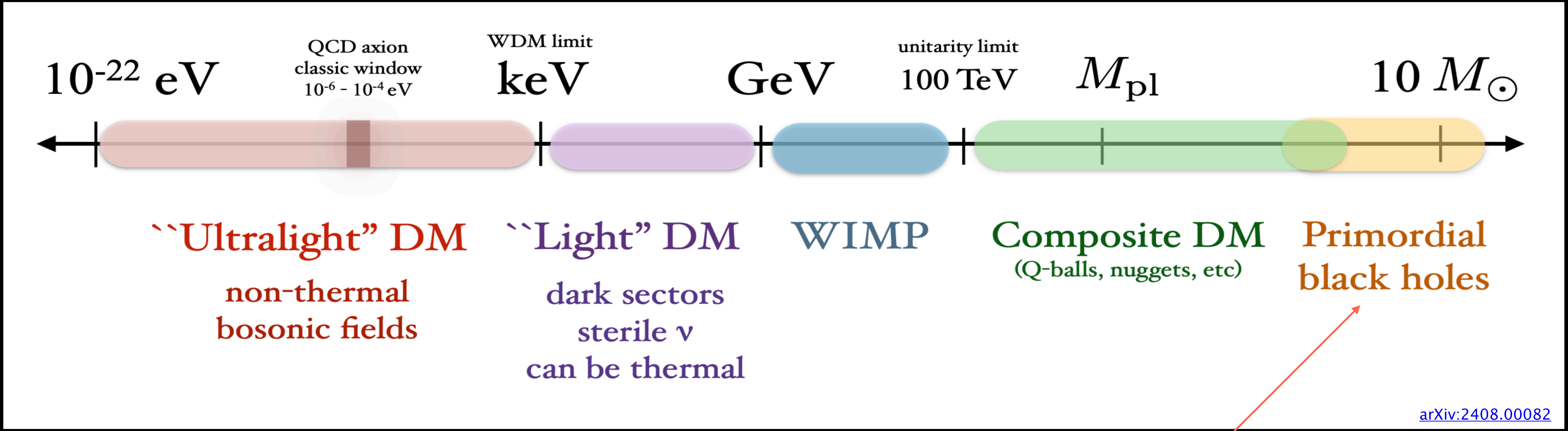

Inconsistent
With microlensing
observations!

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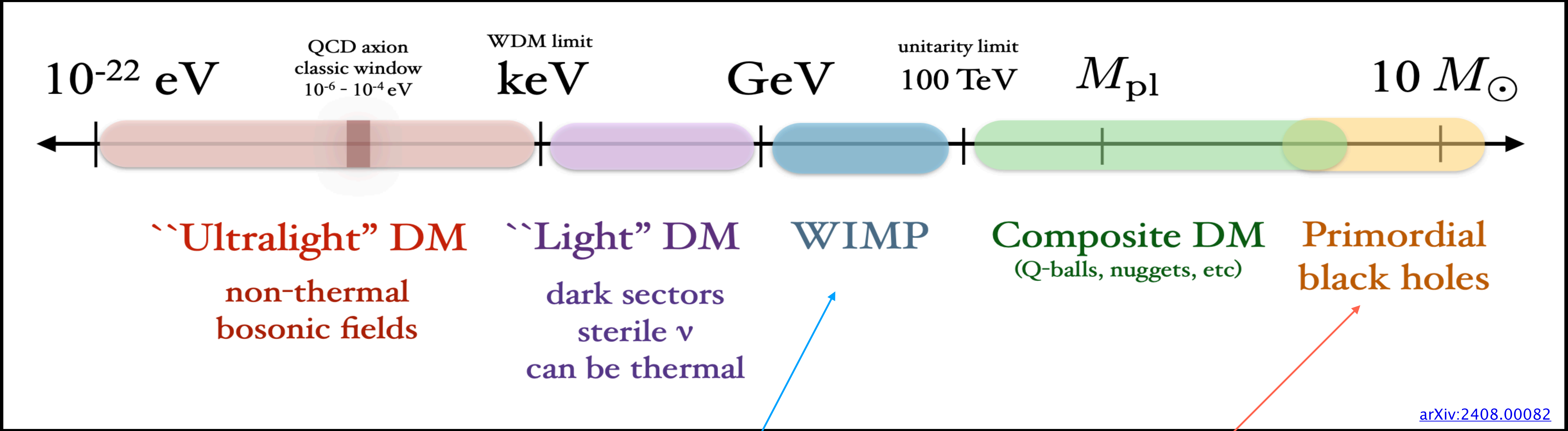

No Empirical Detection so far!

Inconsistent With microlensing observations!

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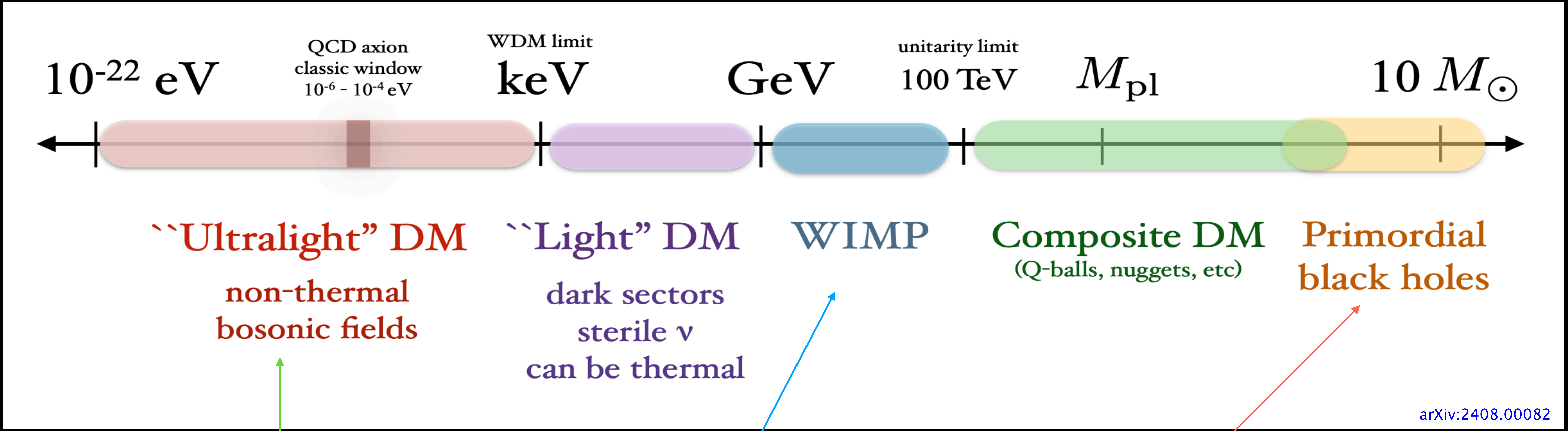


Still a viable window Under observational costraints!

No Empirical Detection so far!

Inconsistent With microlensing observations!

Fuzzy Dark Matter

(F(C)DM, BECDM, ULDM, ELBDM, (ultra-light) axion (-like) DM (ULA, ALP))

- ✦ ***Extremely light*** ***scalar*** **particle (m ~ 10^{-20} - 10^{-22} eV)**
- ✦ **Non-thermally produced (thus not ultra-hot)**
- ✦ **Clumps to form Bose-Einstein Condensate (BEC)!**
- ✦ **Quantum effects counteract gravity at small scales**
- ✦ **Tiny mass**
  - — **large de-broglie wavelength (~ 1/m)**
  - — **macroscopic quantum effects at kpc scales**

Governing Equations

A. Wave Formalism (Schrödinger-Poisson Equations)

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi$$

$$\nabla^2 V = 4\pi G m(|\psi|^2 - |\psi_0|^2)$$

Mean Field Interpretation: Single Macroscopic WF of BEC

B. Madelung Formalism (Fluid Dynamics Representation)

$$\partial_t\rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

$$\partial_t\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{m}\vec{\nabla}\left(V - \underbrace{\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}}_{=\,Q}\right)$$

$$\nabla^2 V = 4\pi G m(\rho - \rho_0)$$

$$\psi = \sqrt{\frac{\rho}{m}}e^{iS}$$

$$\rho = m|\psi|^2$$

$$v = \frac{\hbar}{m}\nabla S$$

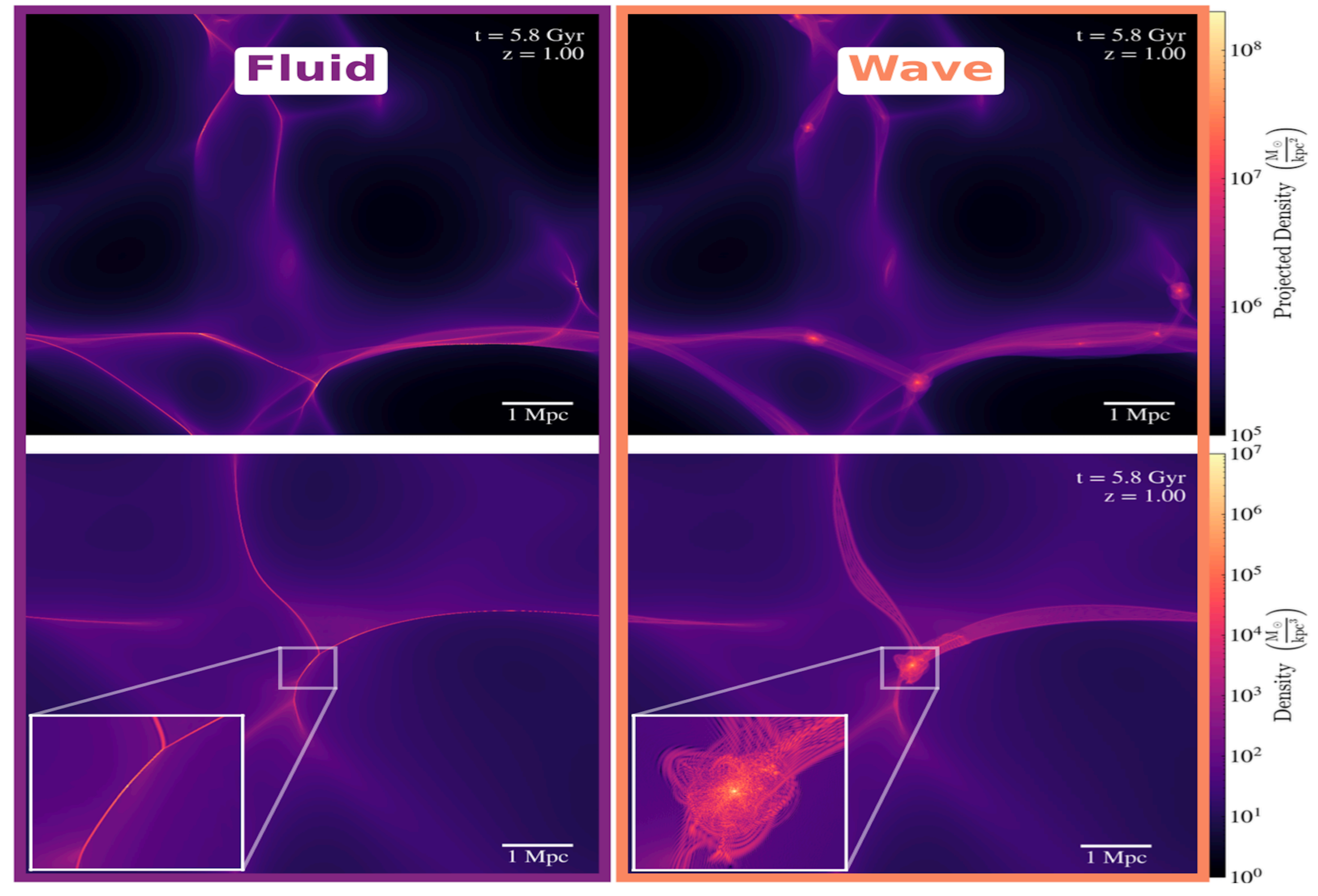
"Quantum Pressure"

Q ill-defined at $\rho = 0$!

Fuzzy Dark Matter Simulations

Fluid Solver unable to capture interference effects!

Stick to SP-Equations for evolution!



A. Kunkel et.al. 2024

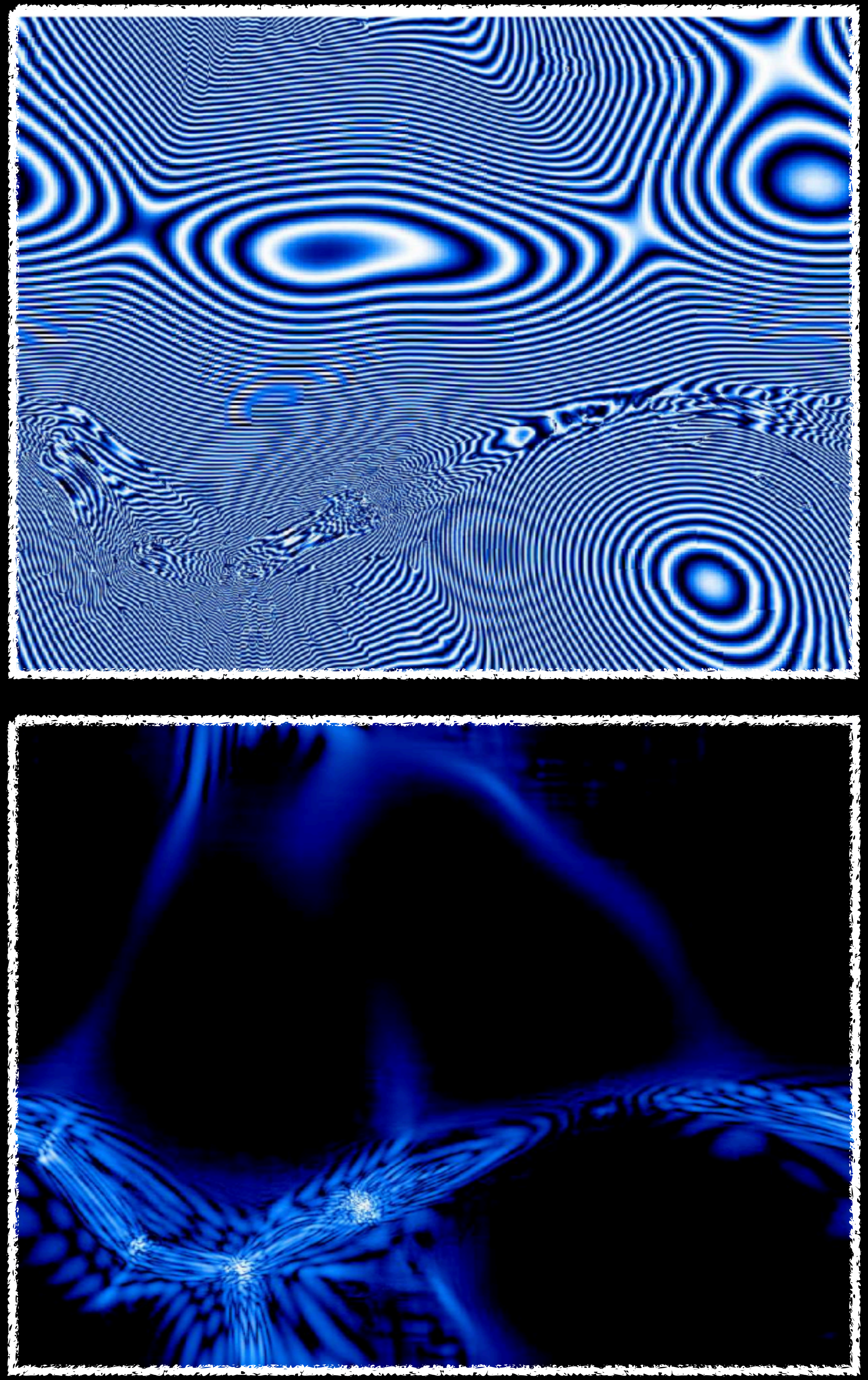
Challenges in Simulating Fuzzy Dark Matter

Both Mpc-scale and kpc-scales need to be Resolved for accurate evolution

Time step scaling: $\Delta t \sim \Delta x^2$

Hydrodynamical codes are used in N-body Simulation (but Fluid Formulation For FDM evolution?)

So far sims. restricted to small box sizes of 10Mpc/h



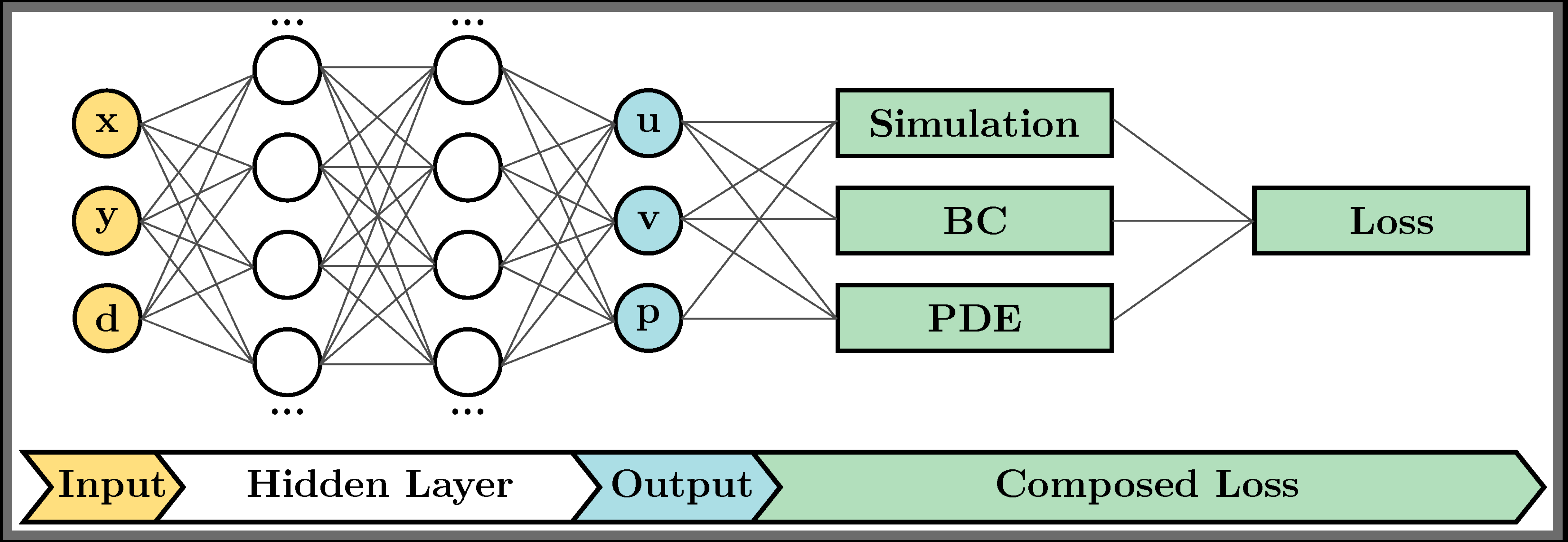
Schive, Chieuh, & Broadhurst (2014)

Physics Informed Neural Networks

General Framework:

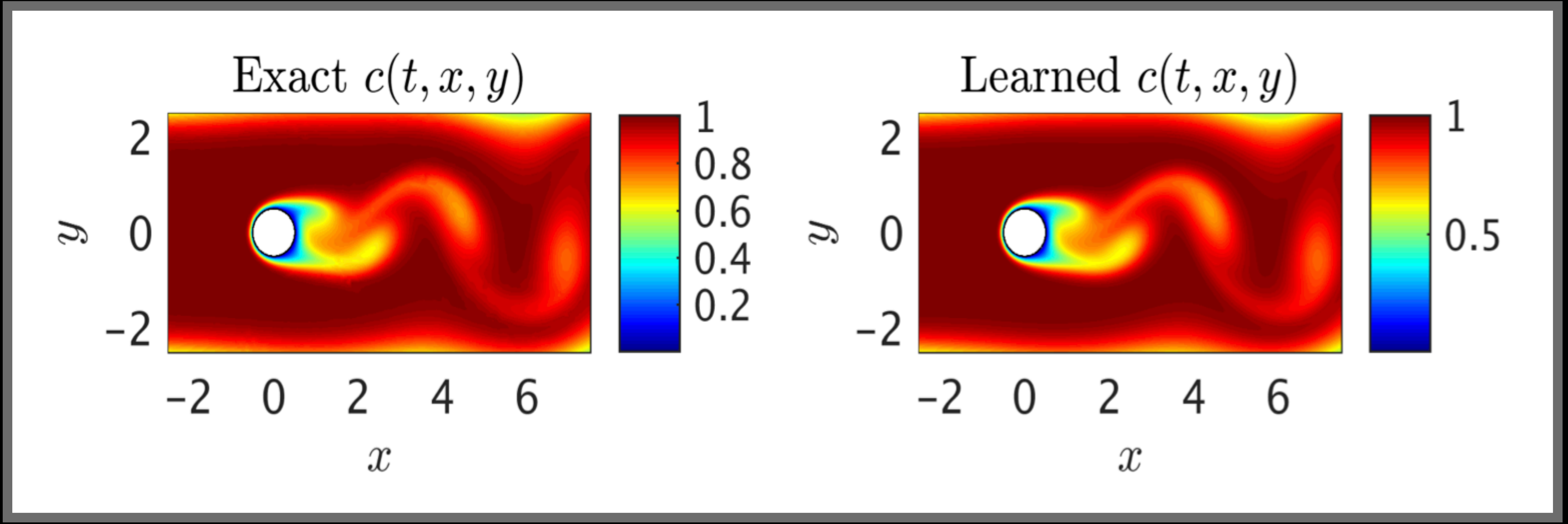
$$\mathscr{D}[NN(X,\theta);\lambda] = f(X), \quad X \in \Omega$$

$$\mathscr{B}[NN(X,\theta);\,] = g(X) \quad X \in \partial\Omega$$



Adapted from F. Pioch et.al.2023

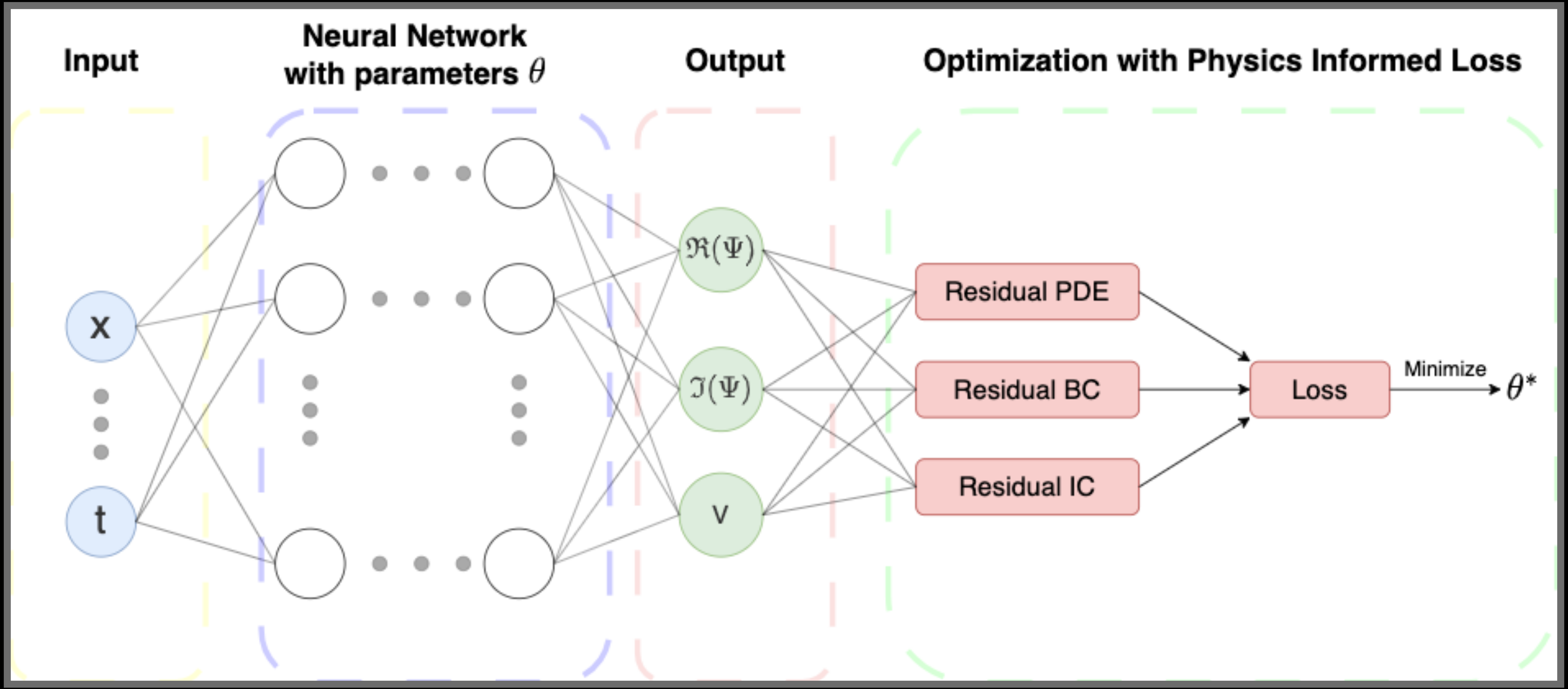
Custom Loss Function: with PDE and boundary conditions as additional constraints



Raissi, Yazdani, Karinadakis 2020

Pretty Successful in Fluid and Climate Simulations!

Schrodinger-Physics Informed Neural Networks (SPINN)



$$\{x,y,z,t\} \rightarrow NN(X;\theta) \equiv \{\Re(\Psi), \Im(\Psi), V\}$$

Schrodinger-Poisson Equations used

$$\lambda = \frac{\hbar}{m} \implies$$

$$i\frac{\partial}{\partial t}\Psi(\boldsymbol{x},t) = \left(-\frac{\lambda}{2}\nabla^2 + \frac{1}{\lambda}V[\Psi(\boldsymbol{x},t)]\right)\Psi(\boldsymbol{x},t)$$

$$\nabla^2 V[\Psi(\boldsymbol{x},t)] = (|\Psi(\boldsymbol{x},t)|^2 - 1)$$

$\frac{1}{\lambda}$: the strength of potential

$\lambda \to 0,$ Gravitational Potential Term is dominant in the SP Equations!

$\lambda \to \infty,$ Gravitational Potential Term vanishes, Free Schrodinger Equation representing diffusion!

$\lambda = 1$ throughout this work!

Architecture & Optimization

Network: Simple Multi-Layered Perceptrons (MLPs)

Activations: Sinusoidal Functions in 1D (Siren)

in 3D, Sine + Wavelet- New Adaptive Activation (PINNsformer)

Optimization: Minimize Total Loss through backpropagation as usual to obtain optimized Network,

$$\theta^* = \arg\min_{\theta} \left(MSE_{PDE} + MSE_b + MSE_i \right)$$

(MSE: Mean Squared Error)

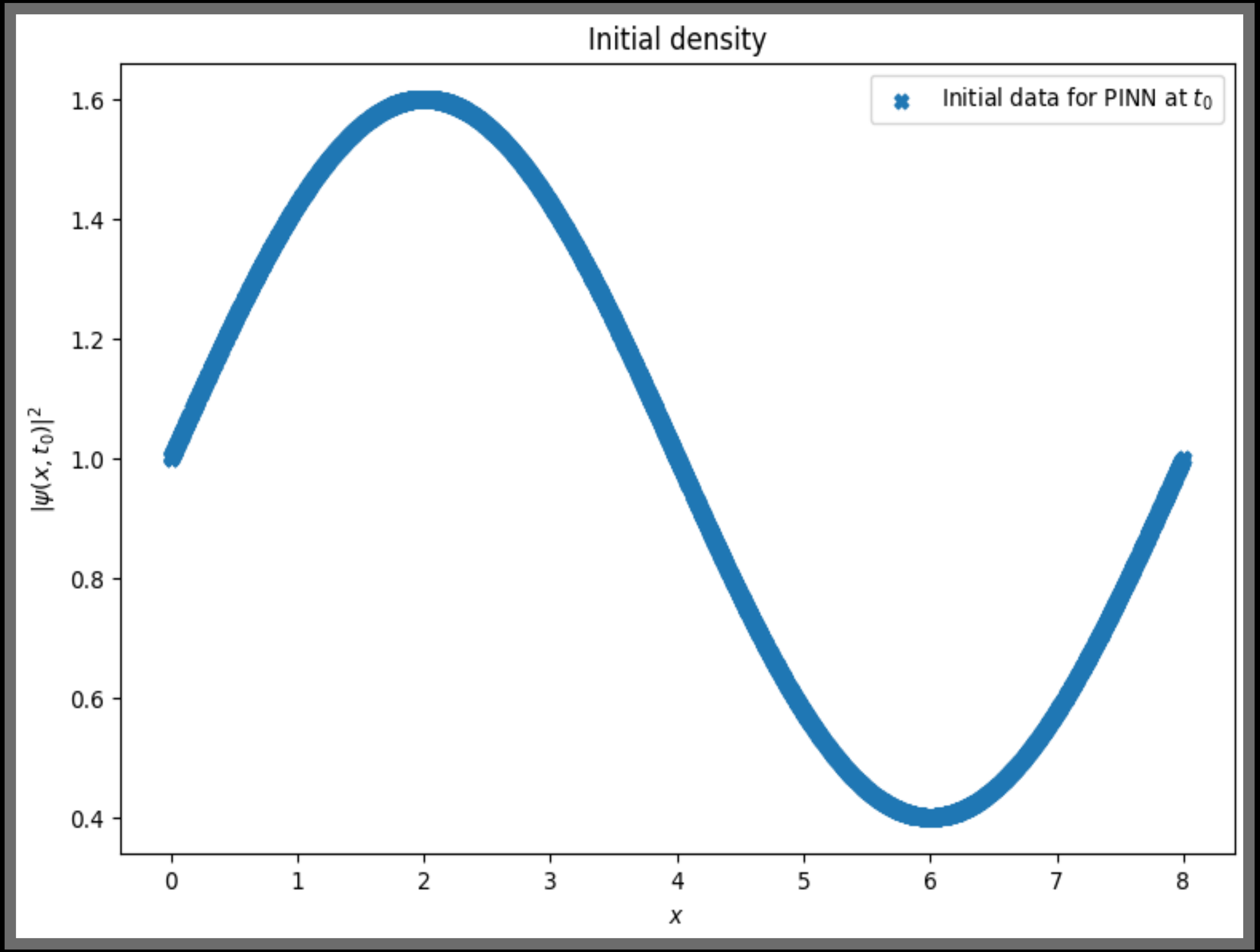
Initial Functions Used

1D Test Function:

$$\psi(x,0) = \sqrt{1 + 0.6\sin\left(\frac{\pi x}{4}\right)}$$

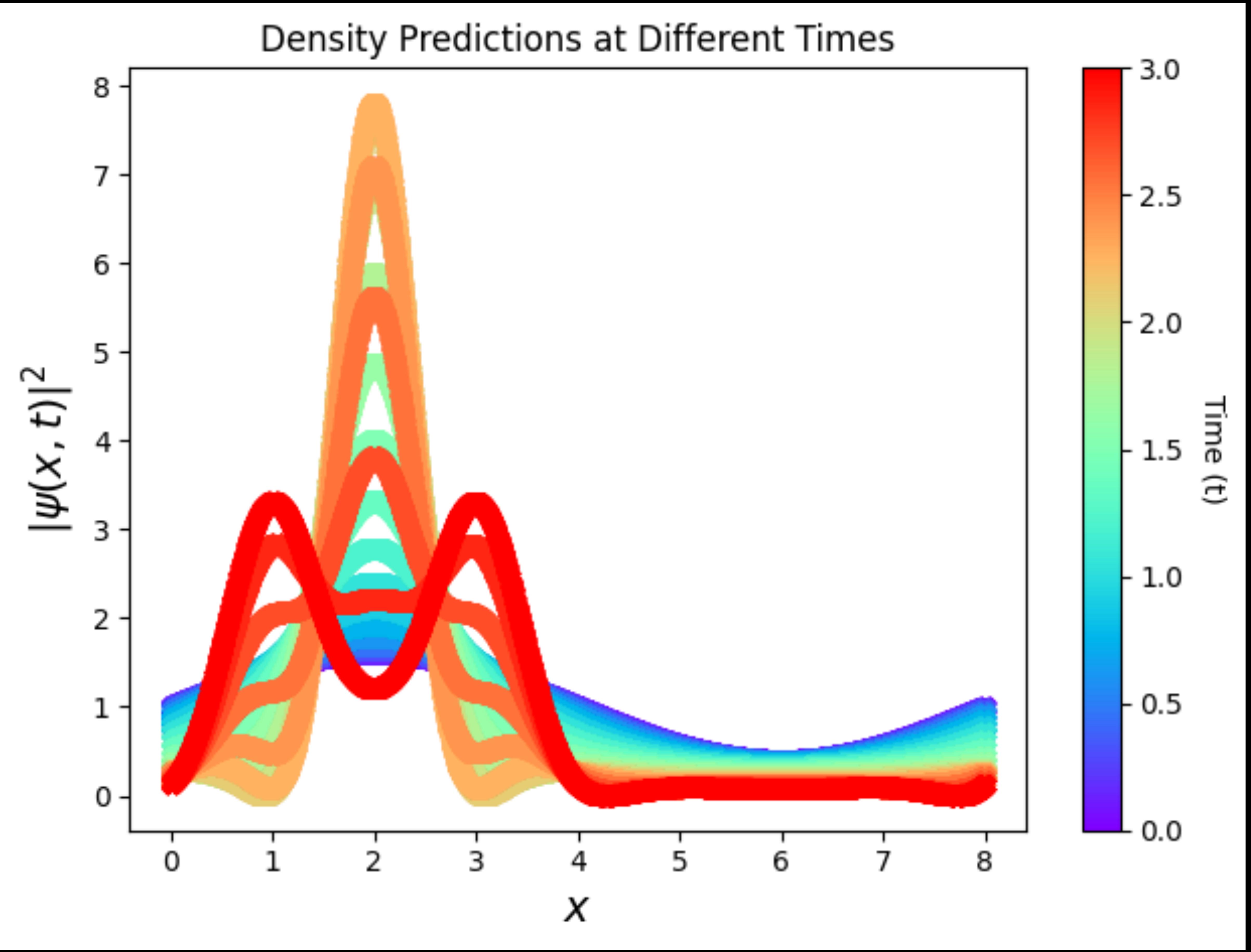
3D Test Function:

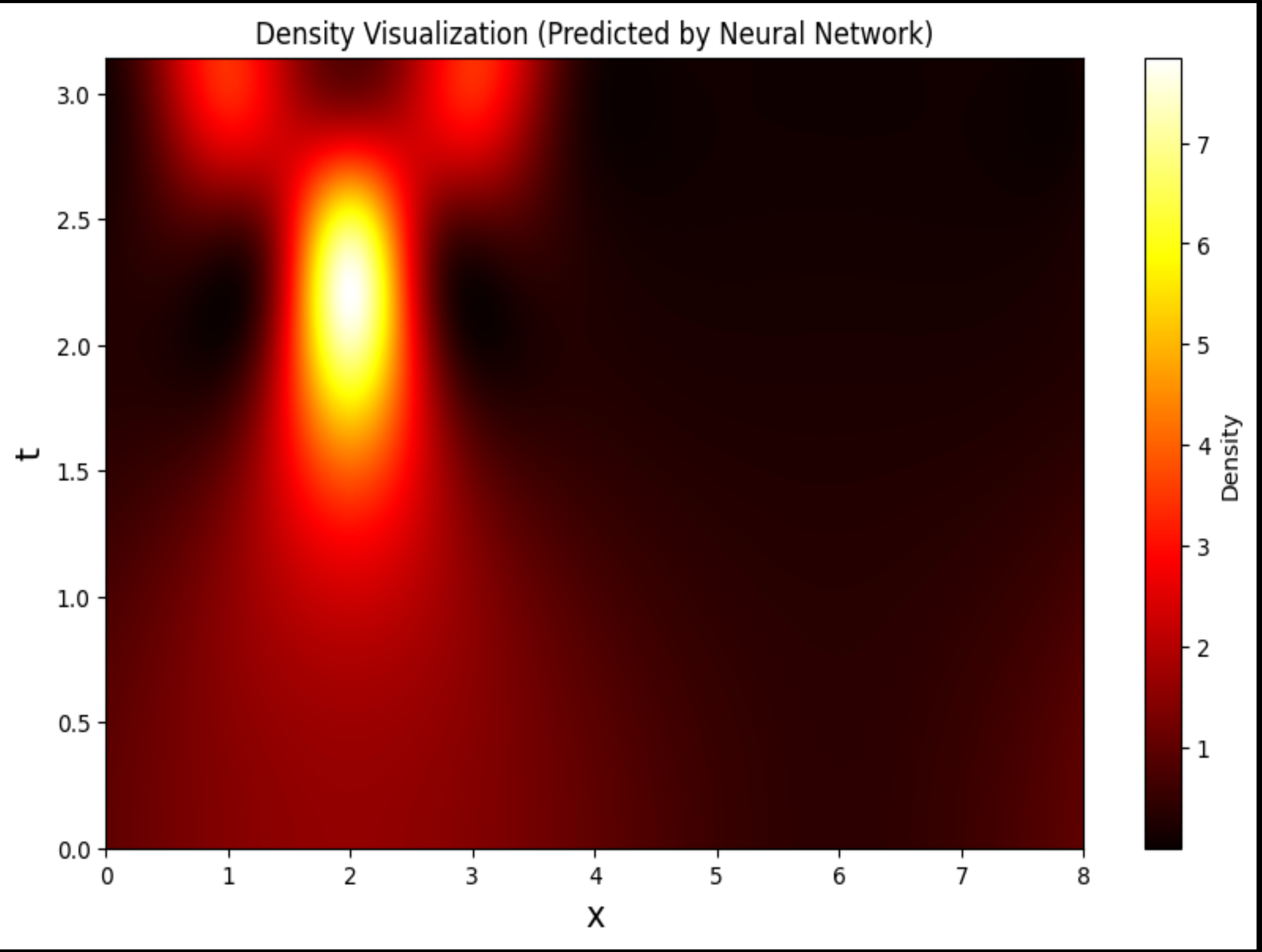
$$\psi(\vec{x},0) = \sqrt{1 + 0.6\sin\left(\frac{\pi x}{4}\right)\sin\left(\frac{\pi y}{4}\right)\sin\left(\frac{\pi z}{4}\right)}$$



Results

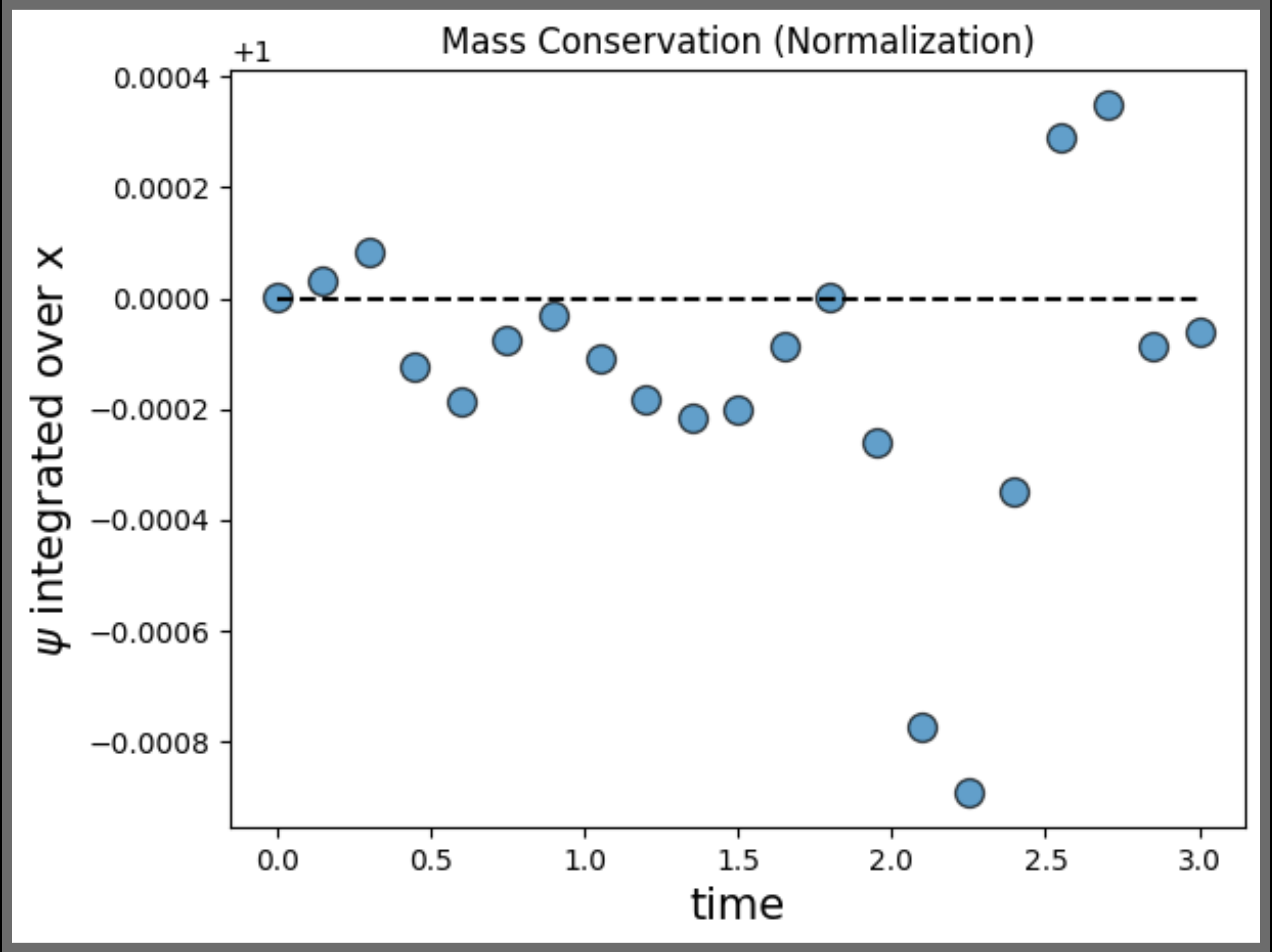
Density Predictions in 1D

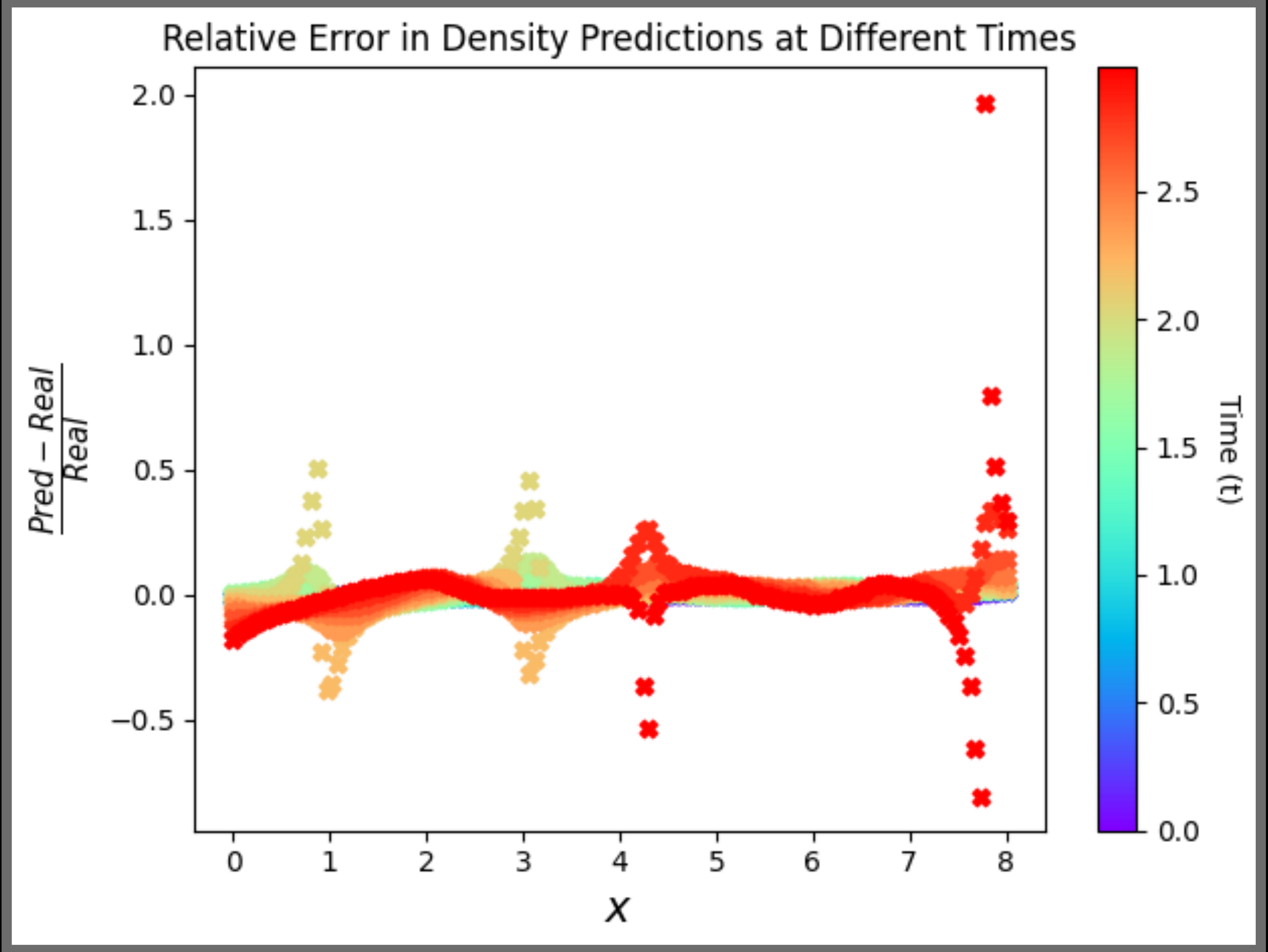




Overdensities collapse as expected!

Checks on Density Predictions

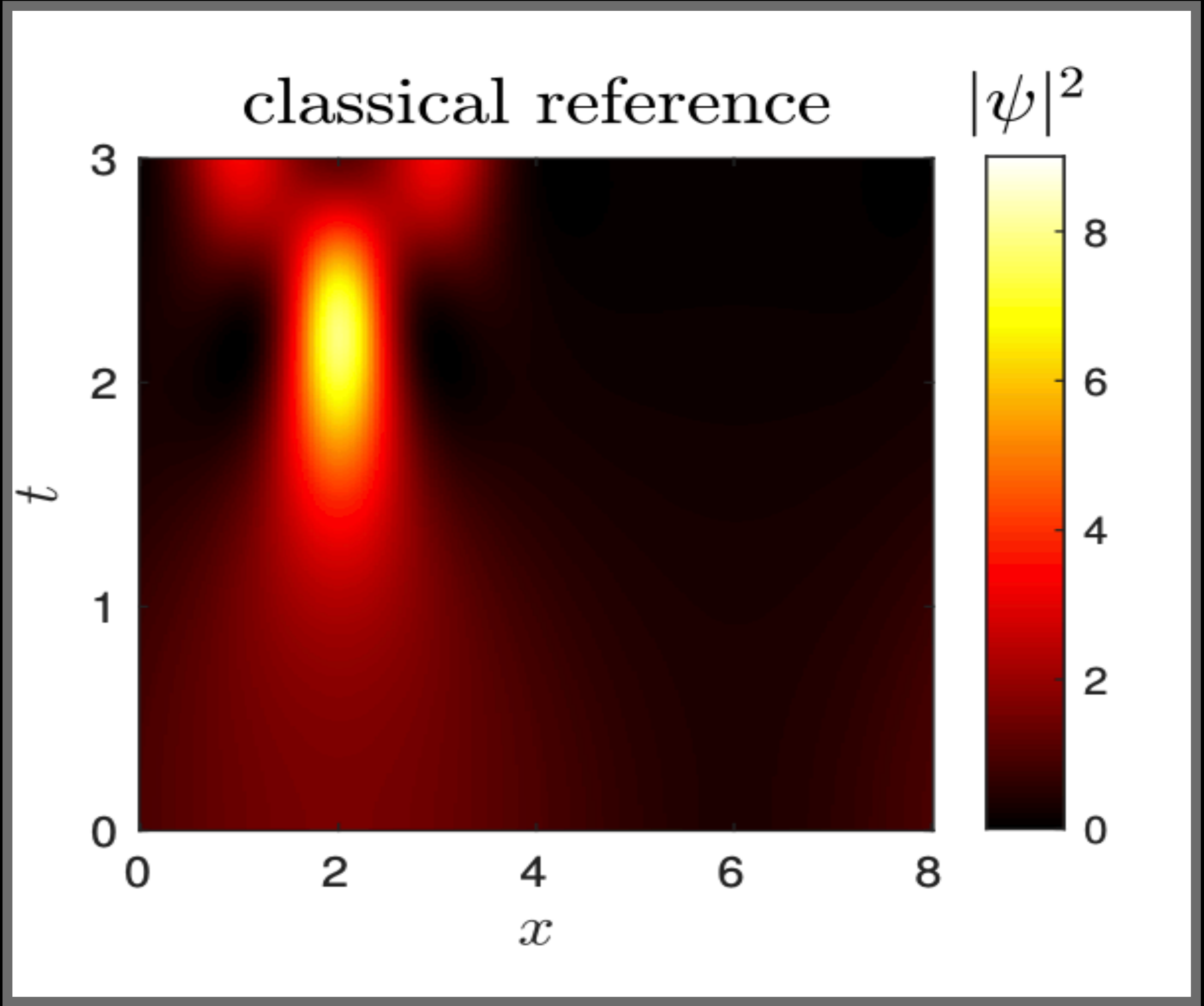


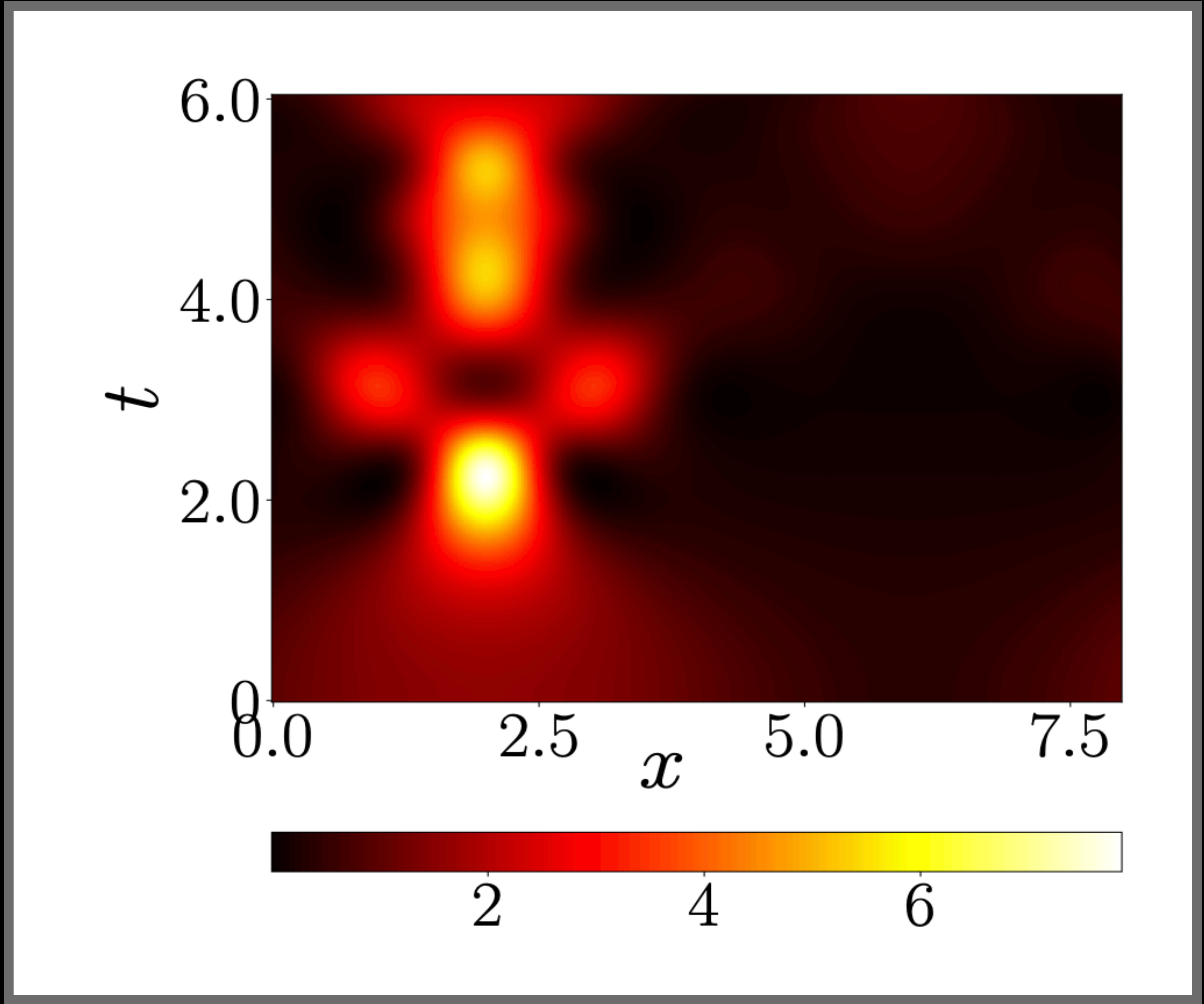


Mass is largely conserved ✓

Decent Match with Spectral Method ✓

Comparison with existing references



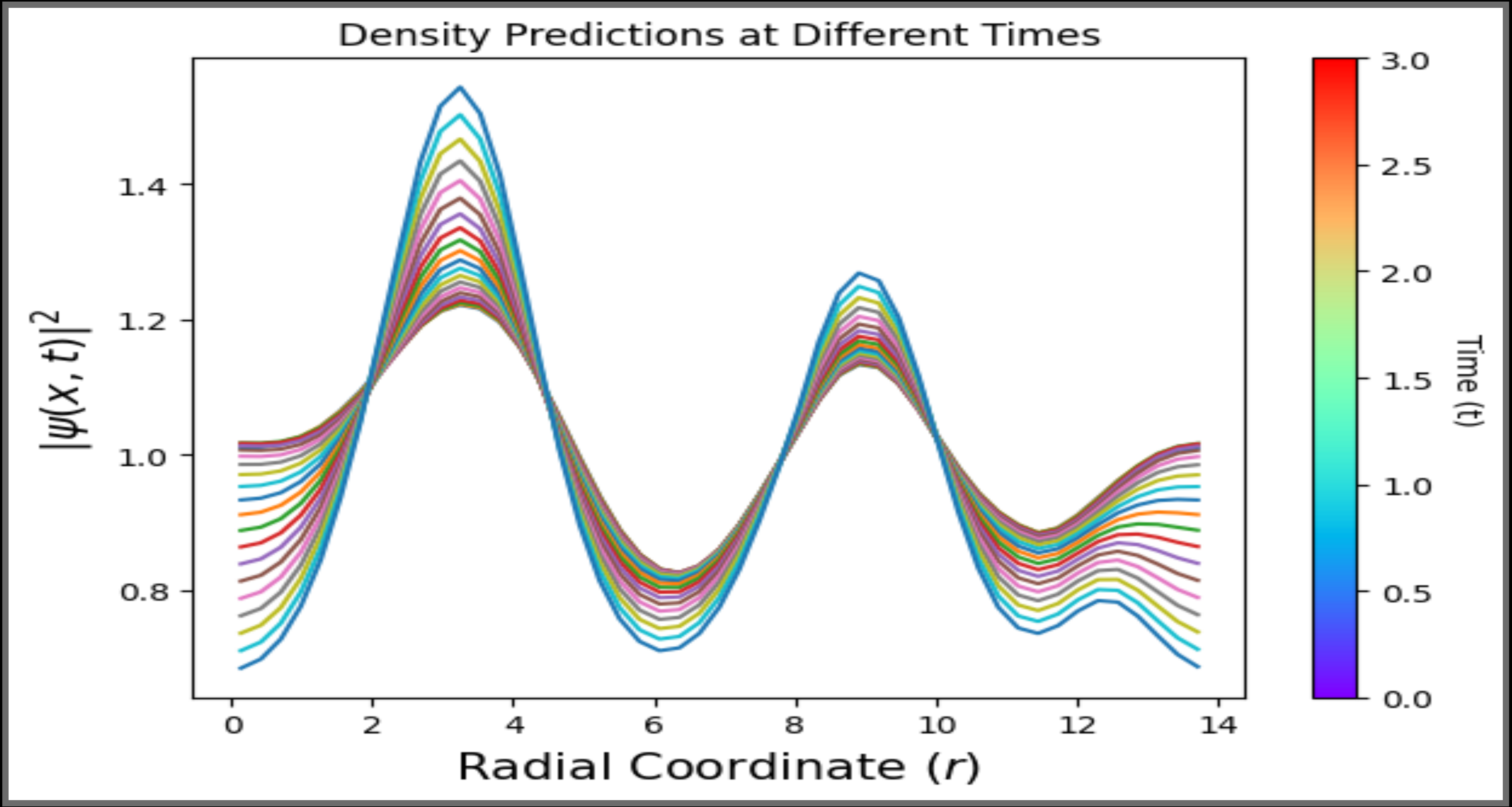


arXiv:2101.05821

arXiv:2307.06032

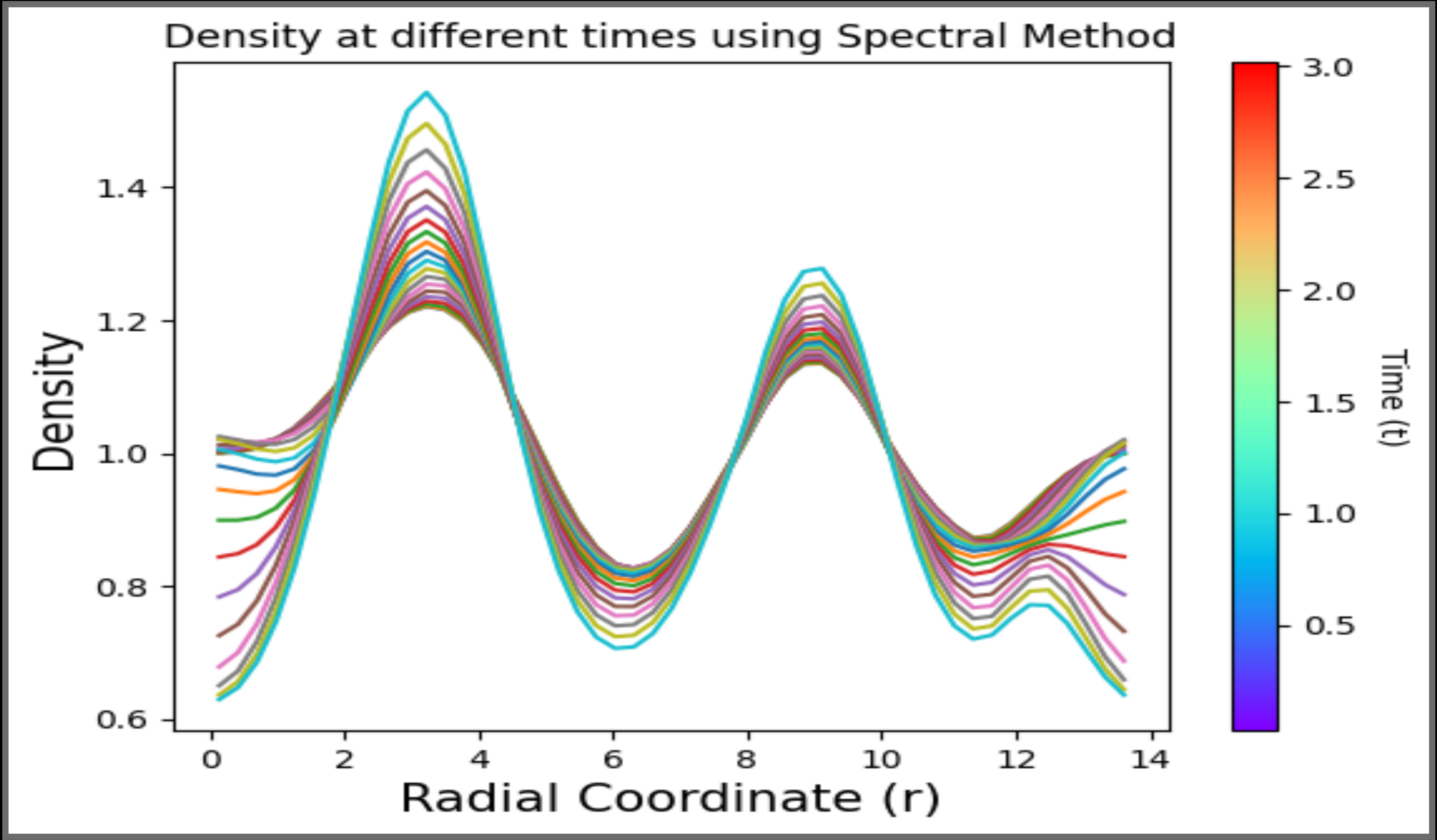
Well agrees with existing works!

Density Predictions in 3D



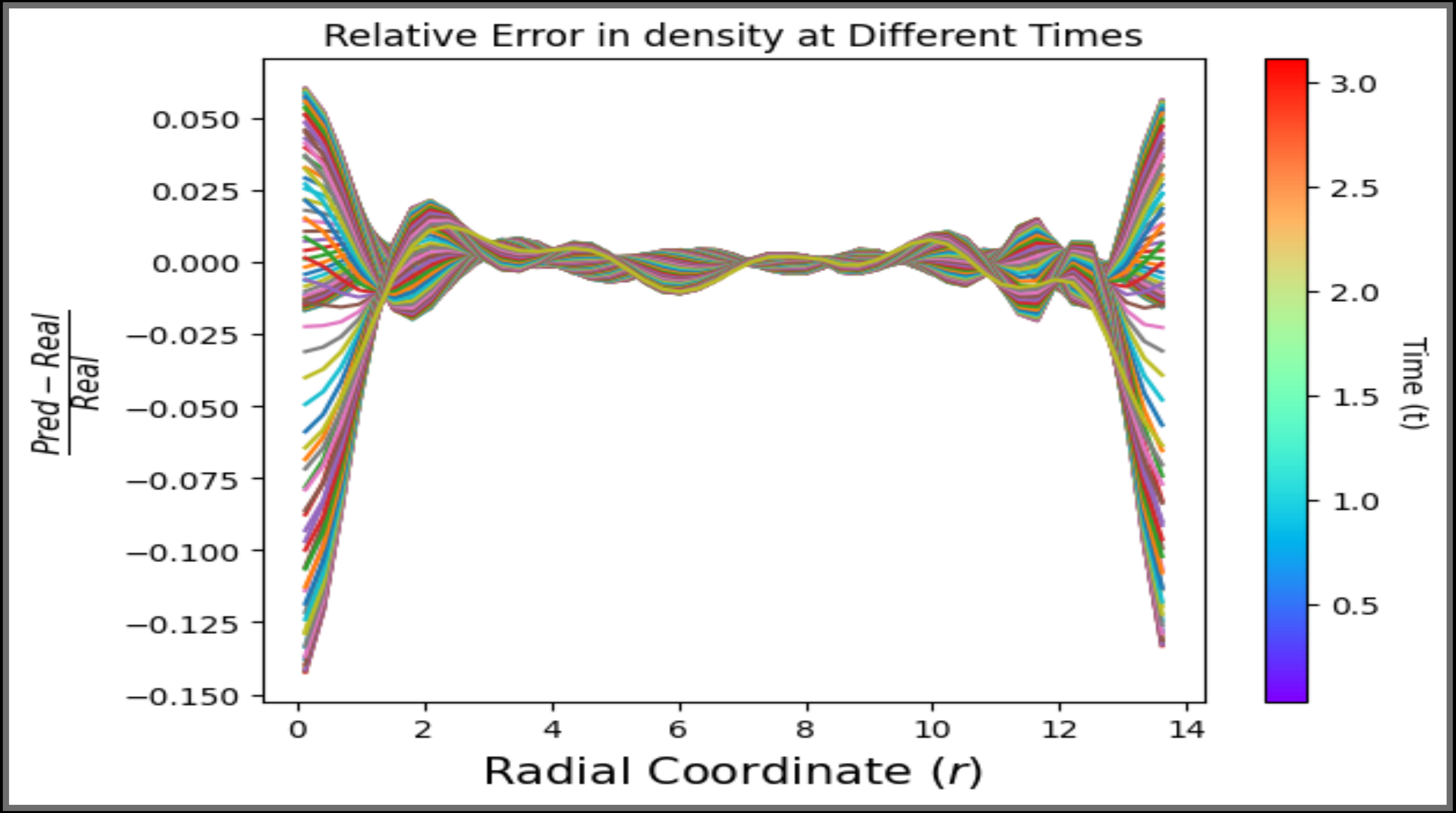
Overdensity collapse, well extends to 3D!

Comparison with Analytical Result



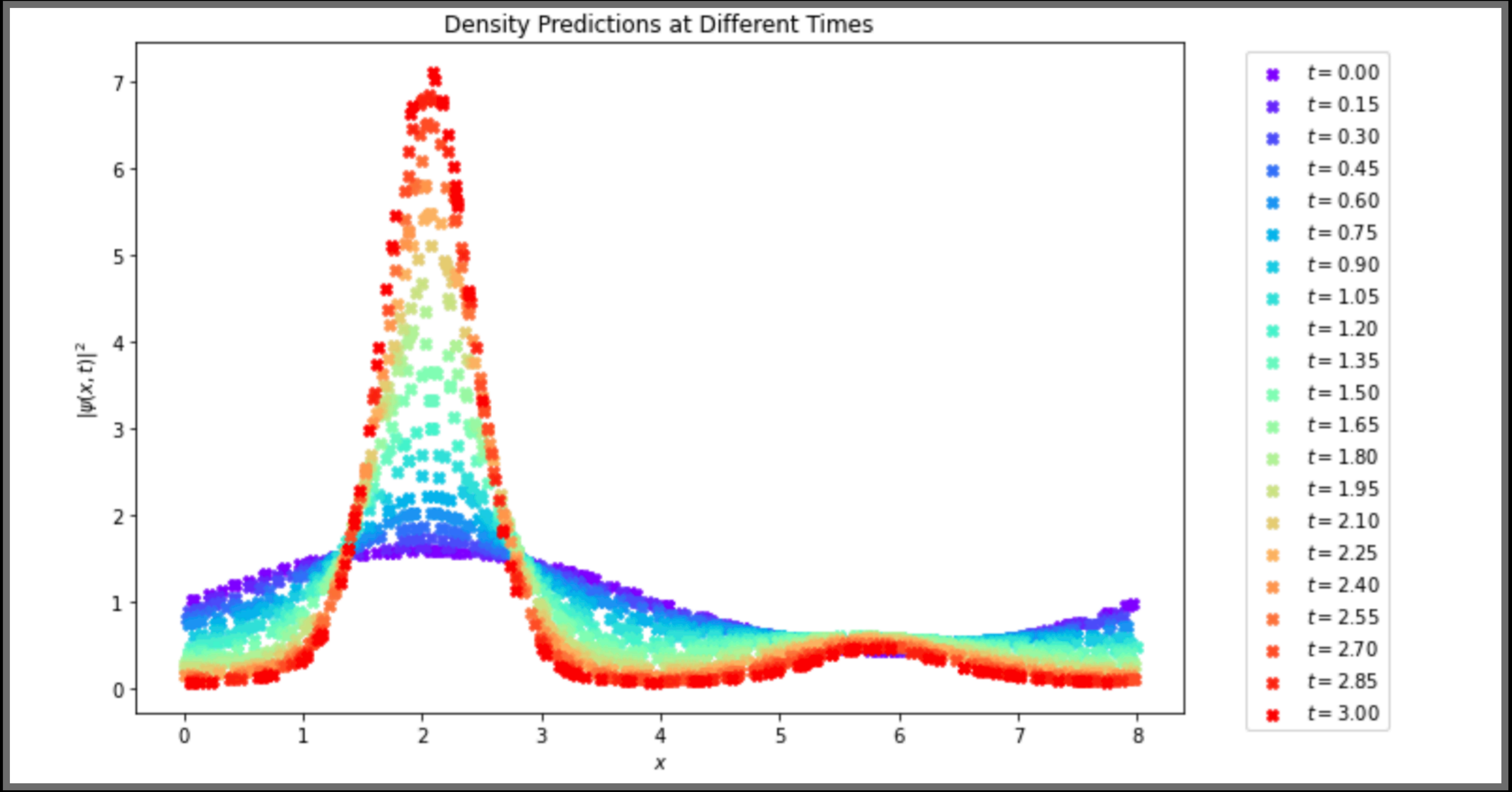
Spectral Method results indicate the same !

Relative Error



Slightly high relative error at boundaries (need more collocation points there)!

Results with Madelung Formalism



Doesn’t learn the interference features !

Work in Progress!
(Still to scale to larger times)

01 **Unsupervised FDM PINNs with Intial conditions same as CDM case**

02 **Supervised PINNs using large-scale CDM simulations as additional data constraint**

03 **Generative Models for painting-in small-scale features**

04 **Reproducing Core-Halo Relations for FDM with PINNs**

THANK YOU!

Question?

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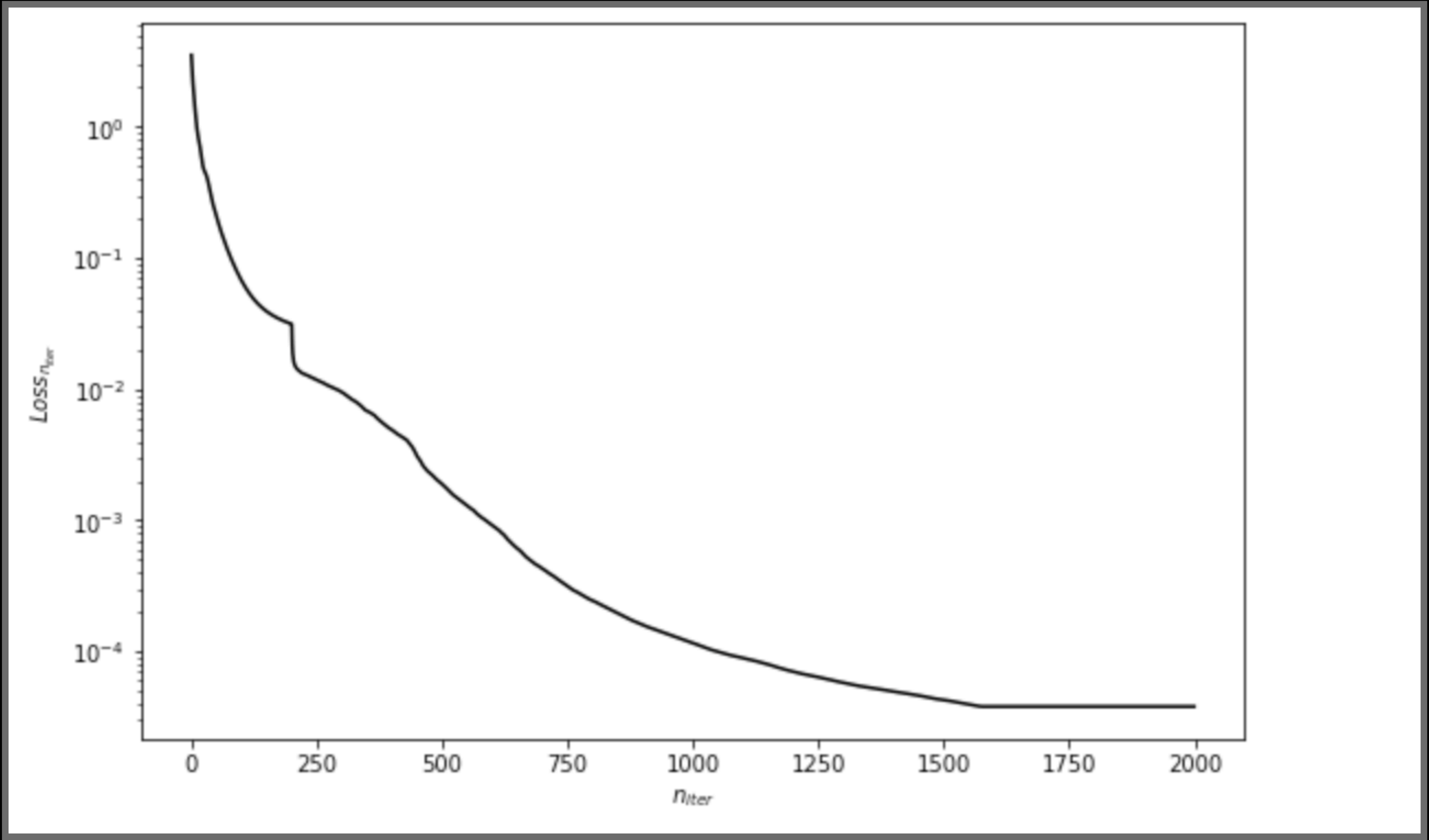
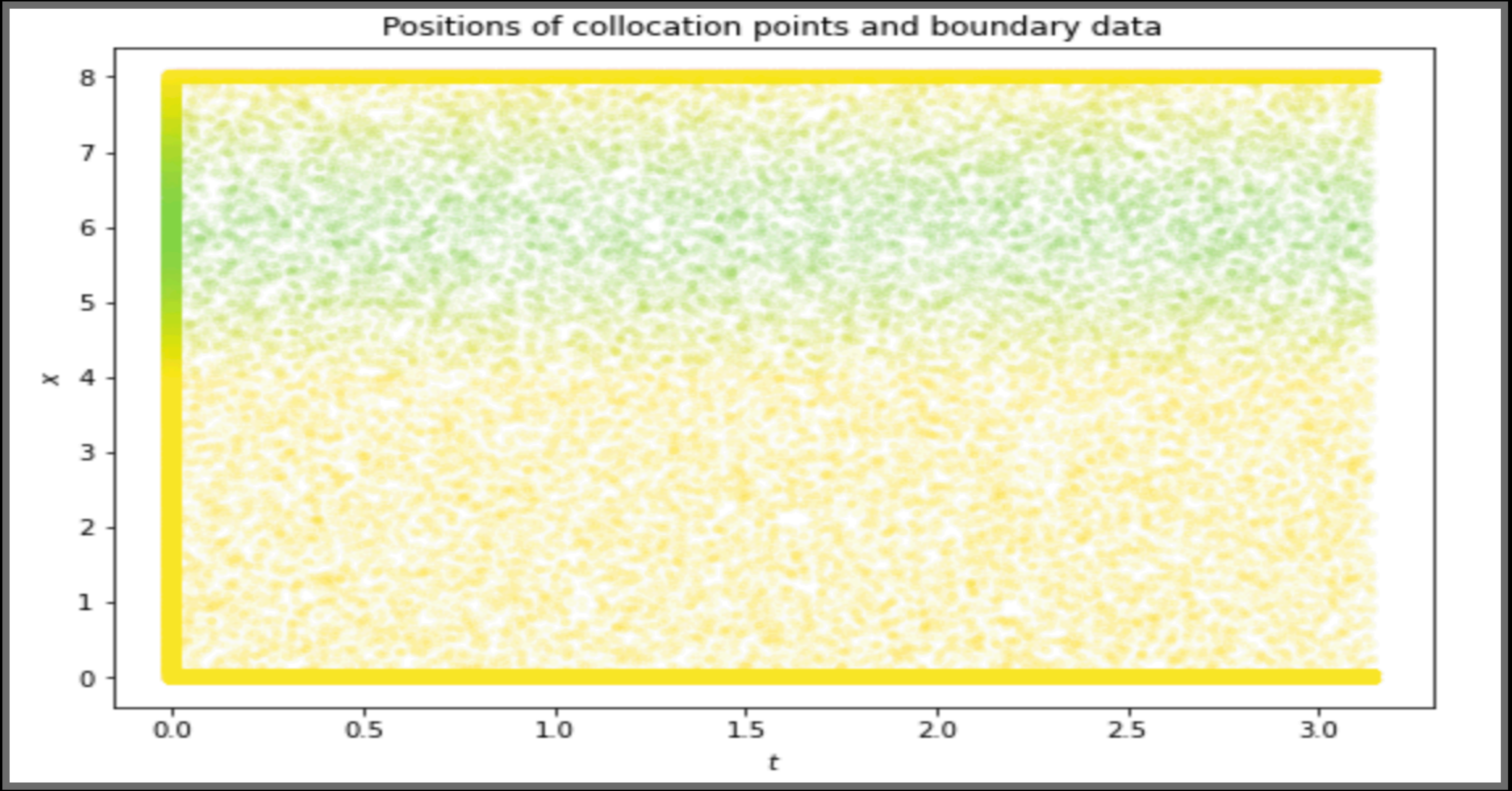
Results : Loss Curve



Training is stopped as soon as total Loss Saturates!

Collocation Points



$x \in [0,8]$ $\quad t \in [0,\pi]$

Dense enough to learn the solution!

Periodic Boundary Conditions

Example in x-direction for Real part of Wavefunction and Potential:

Periodicity for Real Part:

$$\Re(\psi)(x=0, y, z, t) = \Re(\psi)(x=L, y, z, t)$$

$$\partial_x \Re(\psi)(x=0, y, z, t) = \partial_x \Re(\psi)(x=L, y, z, t)$$

Periodicity for Potential:

$$V(x=0, y, z, t) = V(x=L, y, z, t)$$

$$\partial_x V(x=0, y, z, t) = \partial_x V(x=L, y, z, t)$$

Loss Term for Boundary

$$MSE_b(\theta) = \frac{1}{N_b} \sum_{n=1}^{N_b} \left[\left| \Re_\theta(\Psi)(X_n^b) - \Re_b(\Psi)(X_n^b) \right|^2 + \left| \Im_\theta(\Psi)(X_n^b) - \Im_b(\Psi)(X_n^b) \right|^2 + \left| V_\theta(X_n^b) - V_b(X_n^b) \right] \right.$$

Residual Functions

Residual Contributions for Schrodinger + Poisson equations

$$\mathscr{R}_{\mathfrak{R}(\Psi)}(X) = \partial_t \mathfrak{R}_\theta(\Psi) + \frac{1}{2}\left(\sum_{i=1}^{d} \partial_{x_i}^2 \mathfrak{I}_\theta(\Psi)\right) - V_\theta \cdot \mathfrak{I}_\theta(\Psi)$$

$$\mathscr{R}_{\mathfrak{I}(\Psi)}(X) = \partial_t \mathfrak{I}_\theta(\Psi) - \frac{1}{2}\left(\sum_{i=1}^{d} \partial_{x_i}^2 \mathfrak{R}_\theta(\Psi)\right) + V_\theta \cdot \mathfrak{R}_\theta(\Psi)$$

$$\mathscr{R}_V(X) = \sum_{i=1}^{d} \partial_{x_i}^2 V_\theta - \left((\mathfrak{R}_\theta(\Psi)^2 + \mathfrak{I}_\theta(\Psi)^2) - 1.0\right)$$

Loss Term for Enforcing PDEs

$$MSE_{PDE}(\theta) = \frac{1}{N_r}\sum_{n=1}^{N_r}\left[\left|\mathscr{R}_{\mathfrak{R}(\Psi)}(X_n^r)\right|^2 + \left|\mathscr{R}_{\mathfrak{I}(\Psi)}(X_n^r)\right|^2 + \left|\mathscr{R}_V(X_n^r)\right|^2\right]$$

Numerical Method (Mocz et. al. 2017)

2nd Order Unitary Spectral Method

- ✦ Calculate potential:

$$V = \text{IFFT}\left(-\frac{1}{k^2}\text{FFT}\left(4\pi Gm(|\psi|^2 - |\psi_0|^2)\right)\right)$$

- ✦ Half-Step 'Kick':

$$\psi \leftarrow exp[-i(m/\hbar)(\Delta t/2)V]\psi$$

Kick

- ✦ Full-Step 'Drift' in Fourier Space:

$$\psi \leftarrow \textbf{IFFT}\left(exp[-i\Delta t(\hbar/m)k^2/2]\textbf{FFT}(\psi)\right)$$

Drift

- ✦ Update the potential:

$$V \leftarrow \textbf{IFFT}\left(-\frac{1}{k^2}\textbf{FFT}\left(4\pi Gm(|\psi|^2 - |\psi_0|^2)\right)\right)$$

- ✦ Another Half-Step 'Kick':

$$\psi \leftarrow exp[-i(m/\hbar)(\Delta t/2)V]\psi$$

Kick