BLUEBILD UPDATE

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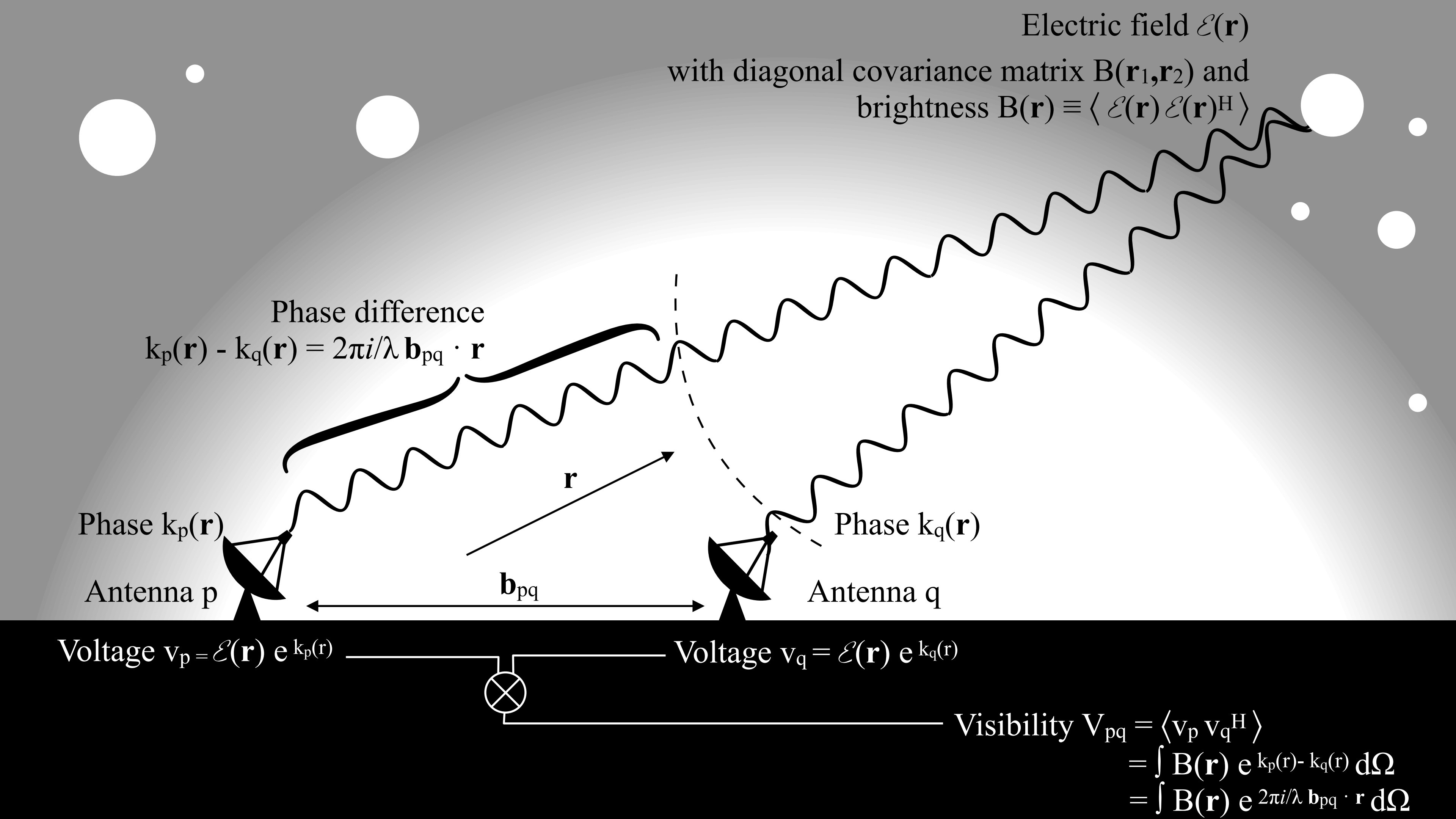
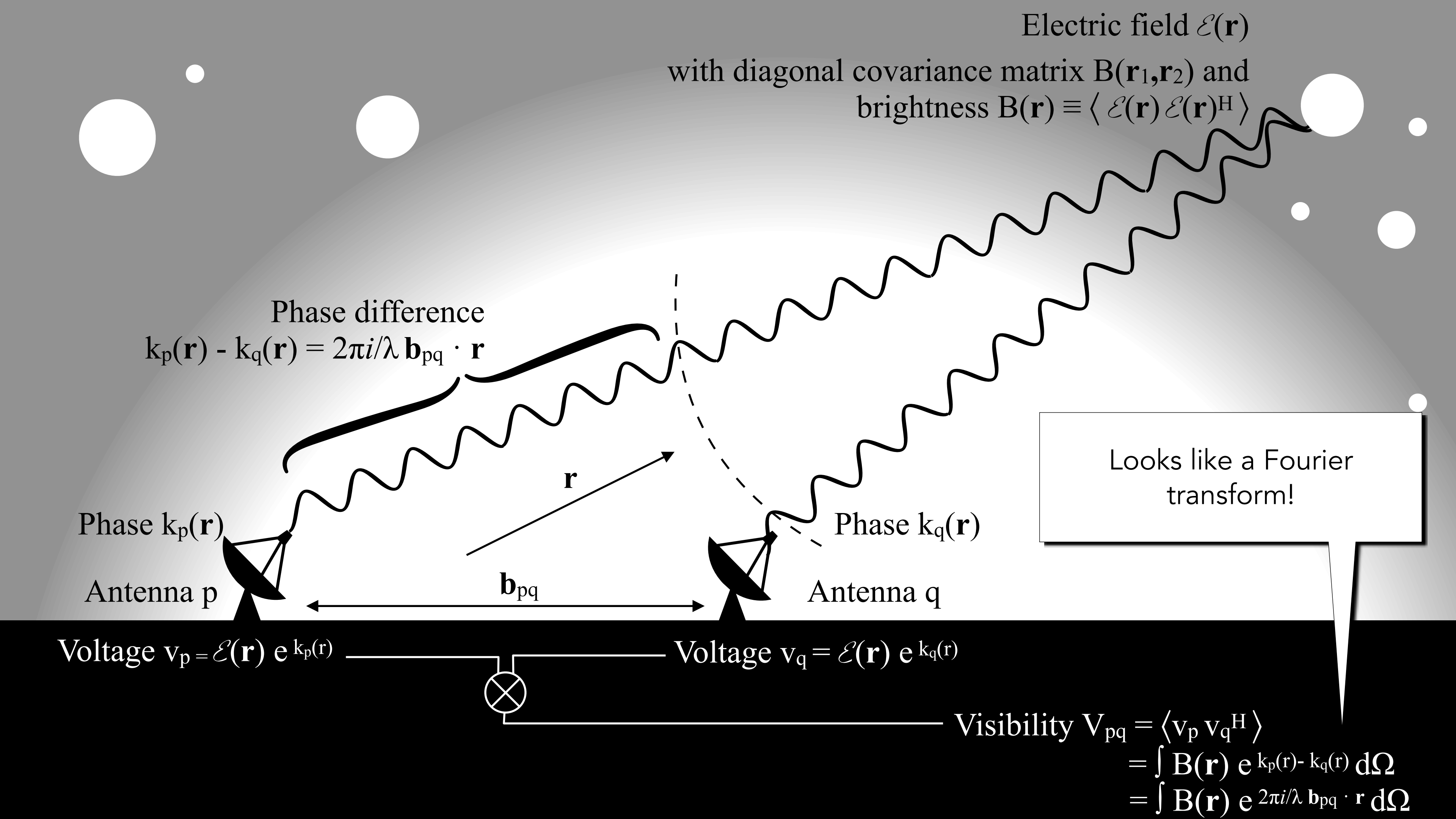
SKACH Winter meeting
January 27, 2025



Electric field $\mathcal{E}(\mathbf{r})$
with diagonal covariance matrix $B(\mathbf{r}_1,\mathbf{r}_2)$ and
brightness $B(\mathbf{r}) \equiv \langle \mathcal{E}(\mathbf{r})\, \mathcal{E}(\mathbf{r})^H \rangle$
Phase difference
$k_p(\mathbf{r}) - k_q(\mathbf{r}) = 2\pi i/\lambda\, \mathbf{b}_{pq} \cdot \mathbf{r}$
$\mathbf{r}$
Phase $k_p(\mathbf{r})$
Phase $k_q(\mathbf{r})$
Antenna p
$\mathbf{b}_{pq}$
Antenna q
Voltage $v_p = \mathcal{E}(\mathbf{r})\, e^{k_p(r)}$
Voltage $v_q = \mathcal{E}(\mathbf{r})\, e^{k_q(r)}$
Visibility $V_{pq} = \langle v_p\, v_q^H \rangle$
$= \int B(\mathbf{r})\, e^{k_p(r) - k_q(r)}\, d\Omega$
$= \int B(\mathbf{r})\, e^{2\pi i/\lambda\, \mathbf{b}_{pq} \cdot \mathbf{r}}\, d\Omega$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Phase $k_p(\mathbf{r})$
Phase $k_q(\mathbf{r})$
Antenna p
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Looks like a Fourier transform!
Voltage $v_p = \mathcal{E}(\mathbf{r})\, e^{k_p(r)}$
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Visibility $V_{pq} = \langle v_p\, v_q^H \rangle$
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Sampling & Interpolation Operators

Write v and V in a compact form using the sampling operator Ψ^*:

$$v = \Psi^* \mathcal{E} + n, \quad V = \langle v\, v^* \rangle = \Psi^* B \Psi + \eta$$

Backprojection: $B' = \Psi V \Psi^*$

starting point for CLEAN algorithms, but does not not minimize

$$\left\| \Psi^* \widetilde{B} \Psi - V \right\|^2$$

Least-squares: $B' = \Psi (G_\Psi)^{-1} V (G_\Psi)^{-1} \Psi^*$,

where $G_\Psi = \Psi^* \Psi$

Bluebild Algorithm

Find a decomposition of B' in a compact orthogonal basis using functional PCA:

$$B' = \sum_a \lambda_a |\epsilon_a|^2 = \sum_a \lambda_a |\Psi \alpha_a|^2$$

Eigenvectors/values found from the generalized eigenvalue problem:

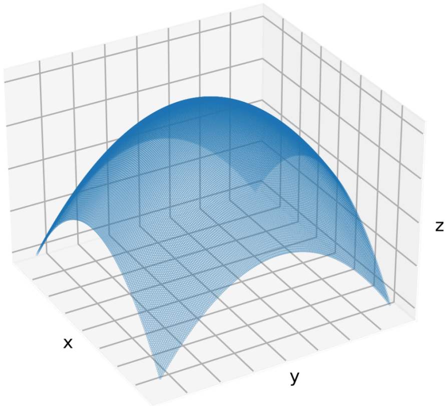
$$V\alpha_a = \lambda_a G_\Psi \alpha_a$$

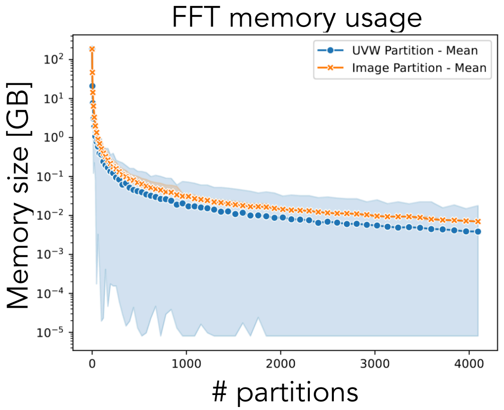
A least-squares solution without inverting G_Ψ

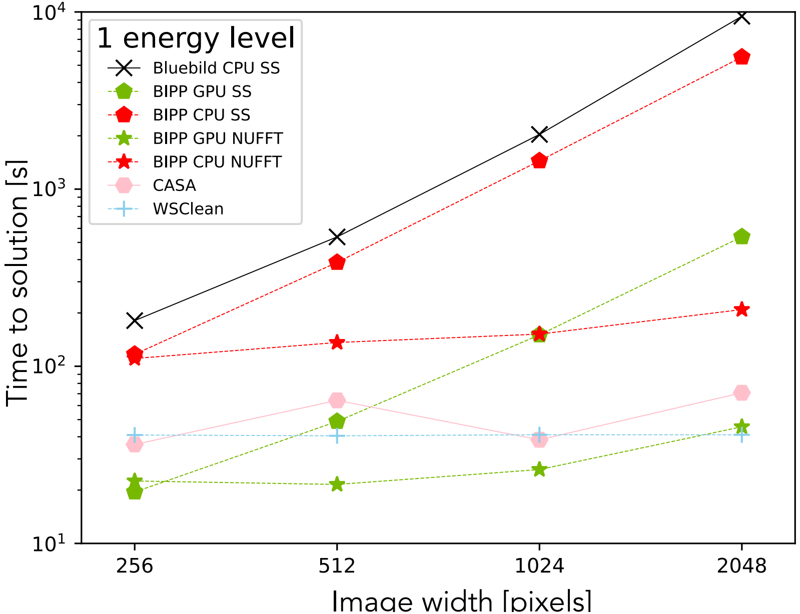
HPC Implementation of Bluebild

Bluebild Imaging++ (BIPP): Reconstruct images on the celestial sphere using fPCA & 3D type-3 non-uniform FFT (NUFFT). CPU & GPU implementation with CUDA & HIP, funded by PASC 2021-2025. Now published: https://doi.org/10.1016/j.ascom.2024.100920

Use **domain partitioning** to take advantage of sparse input & output domains, reduce memory consumption improve performance





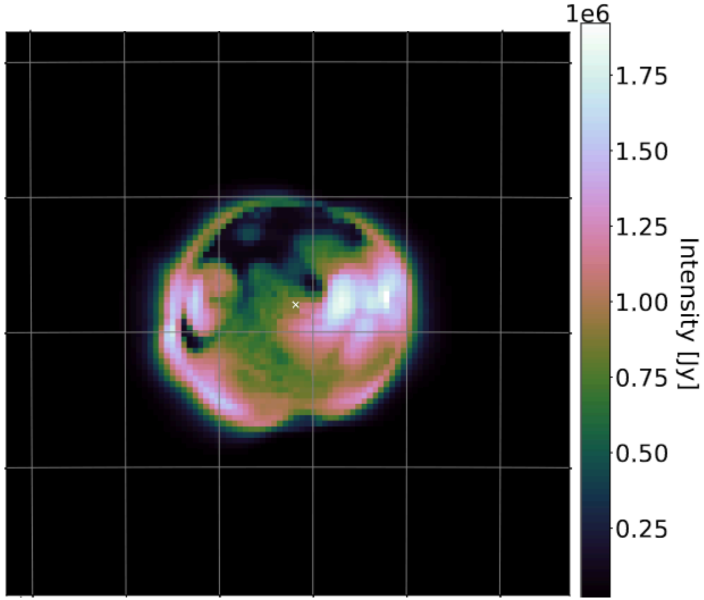


fPCA & energy levels

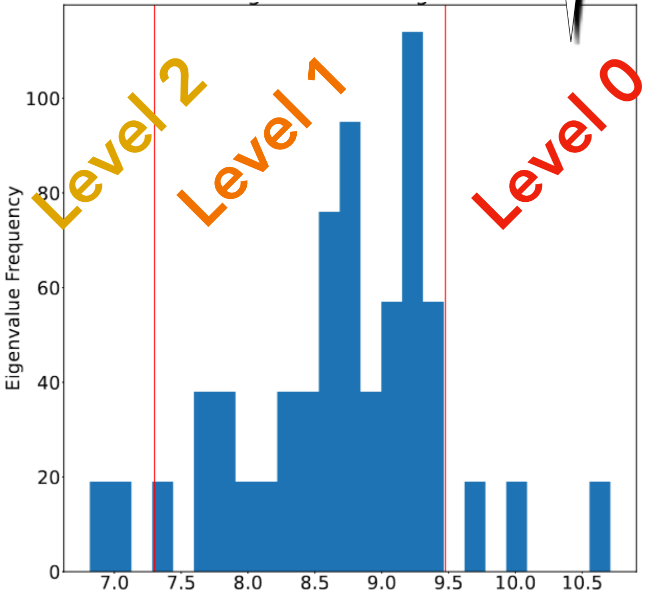
$$B' = \sum_a \lambda_a |\epsilon_a|^2 = \sum_a \lambda_a |\Psi \alpha_a|^2$$

Cluster the eigenvectors/ eigenimages based on eigenvalue

Simulation



Eigenvalues



$\log_{10}(\lambda_a)$

Plots by Shreyam Krishna

fPCA & energy levels

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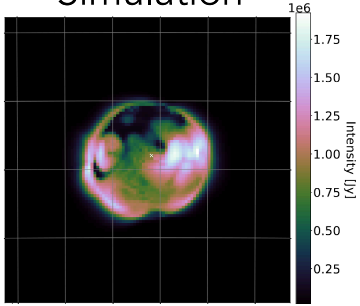
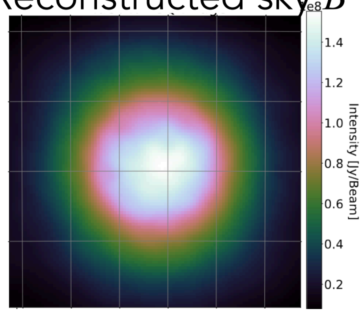
Simulation



Reconstructed sky B'

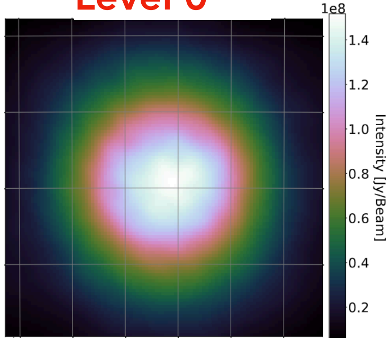


Cluster the eigenvectors/ eigenimages based on eigenvalue

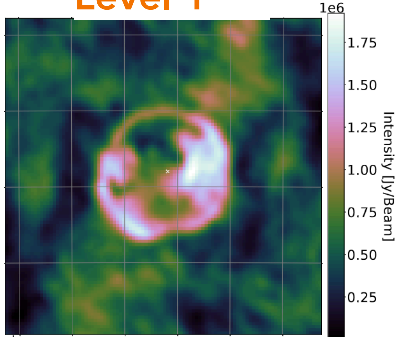
Diffuse structure becomes apparent

Plots by Shreyam Krishna

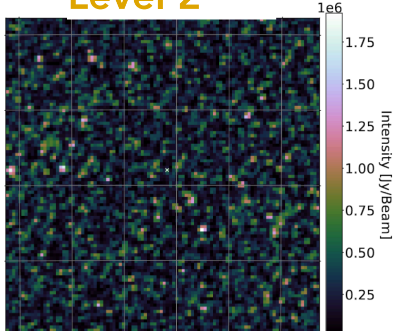
Level 0



Level 1



Level 2



A trivial example

- Assume that we have a sky composed only of point sources, with # sources = # antenna elements = N. In this case:

- Voltages $v_p = \sum_{\text{sources } s} \mathcal{E}_s \, e^{k_p(\mathbf{r}_s)}$, $v = \Psi^* \mathcal{E}$, Ψ is an NxN square matrix with element at index (p,s) given by $e^{k_p(\mathbf{r}_s)}$

- For simplicity assume that $\Psi^*\Psi = I$. Then:

$$V = \Psi B \Psi^*$$

$$V\Psi = \Psi B$$

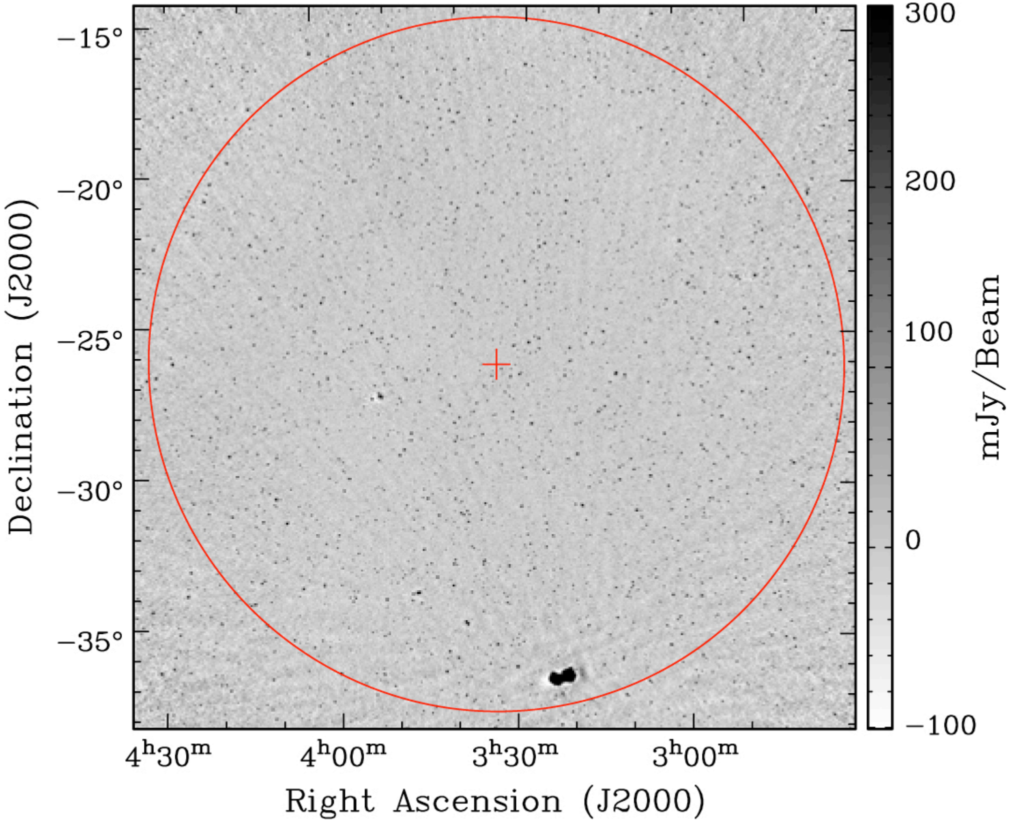
$$V\alpha^s = \lambda^s \alpha^s .$$

B is a diagonal covariance matrix, Ψ is a square matrix

Equivalent to eigenvalue decomposition!
Eigenvectors are given by $e^{k_p(\mathbf{r}_s)}$
Eigenvalues by $\mathcal{E}_s$

This holds even if the instrument isn't calibrated, ie sampling operator includes unknown complex gain terms $g_p(\mathbf{r}_s)$

A less trivial example

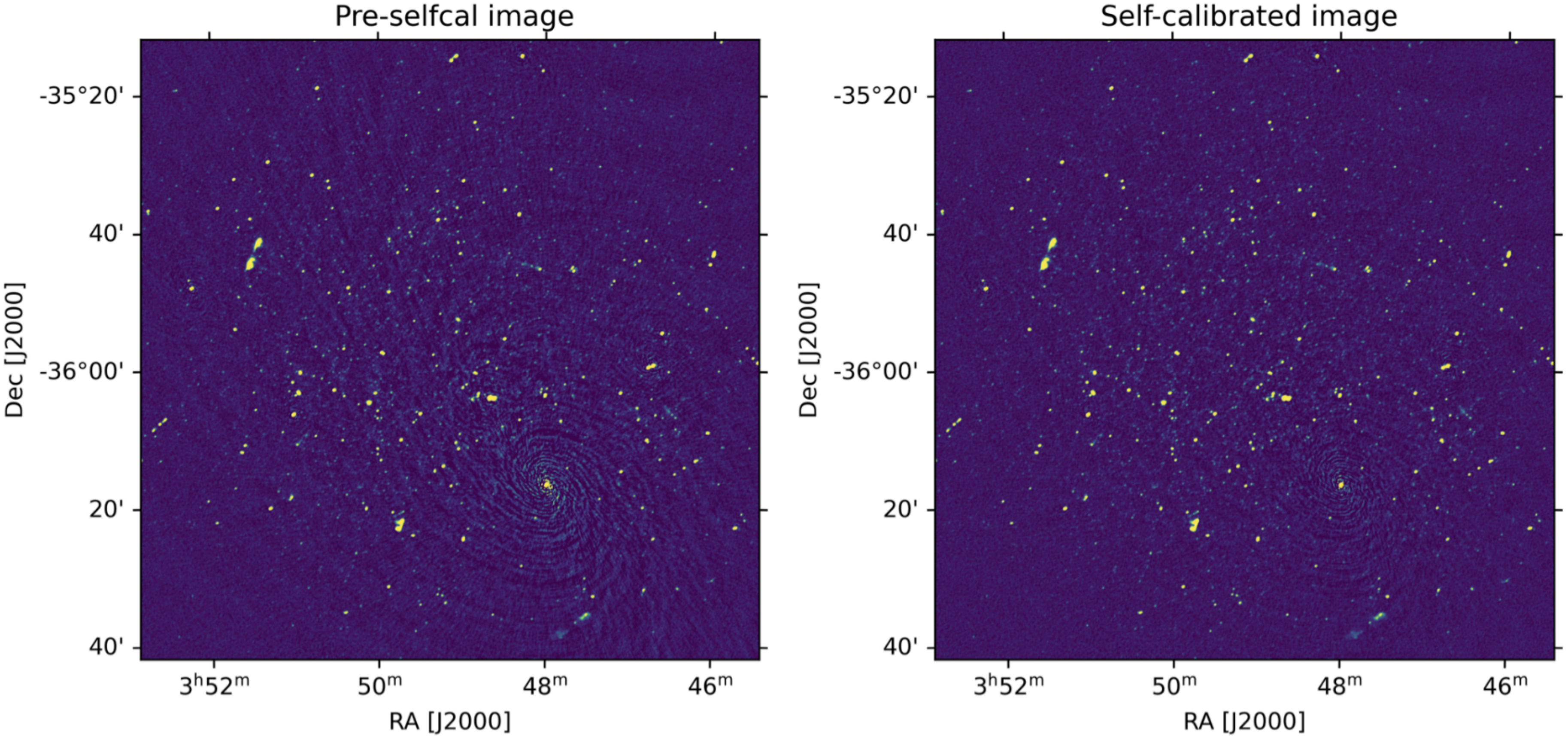


- In reality sky is not just point sources and # sources >> # antenna elements
- However, usually there are just a few bright sources in the FoV, and we still expect first eigenvector to "point" to direction of highest flux:

$$(\alpha^1)^* V \alpha^1 = \lambda^1.$$

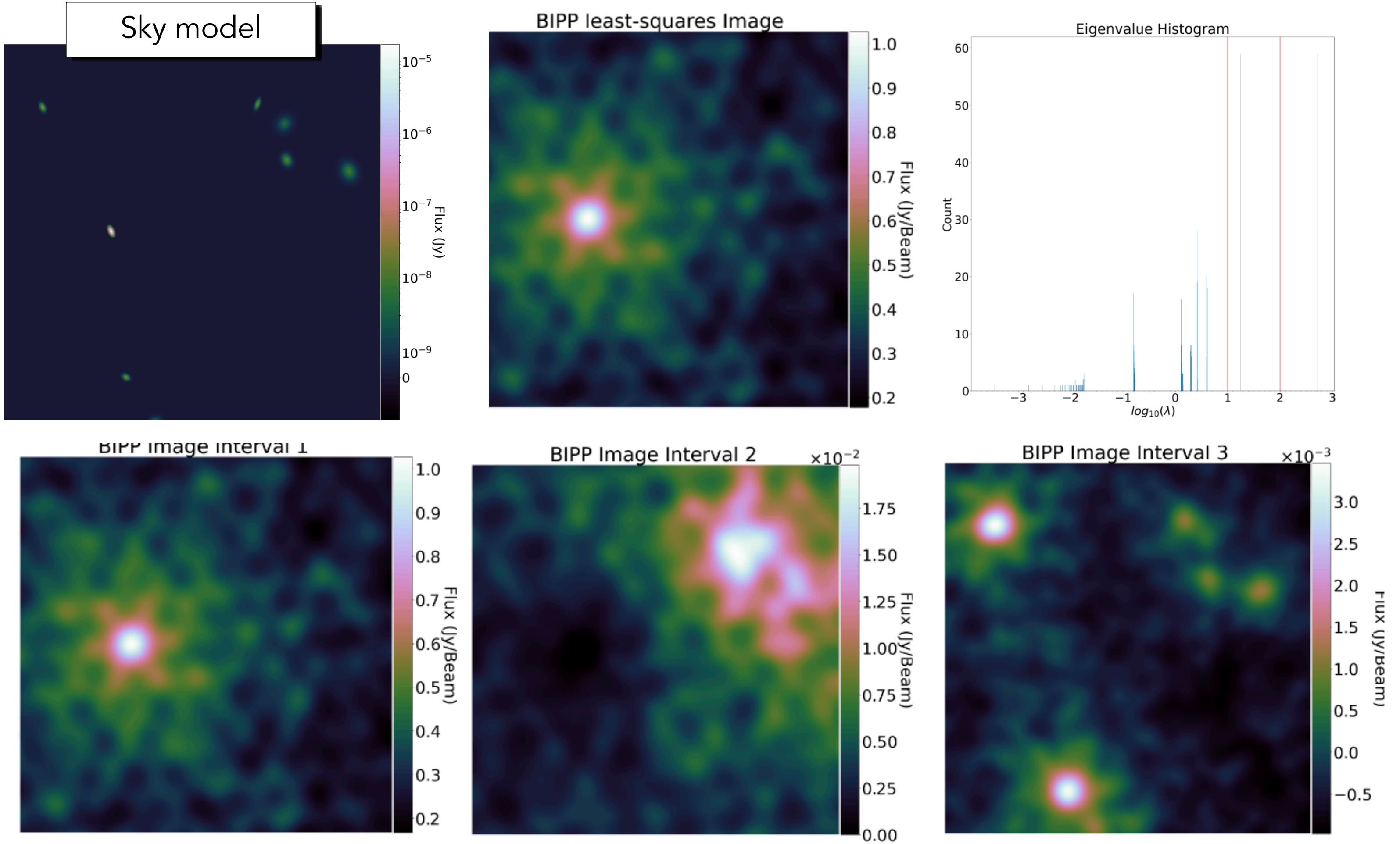
- Note that this acts like super resolution source finding
- Method breaks down with bright diffuse sources, multiple sources with same flux value

Peeling bright sources



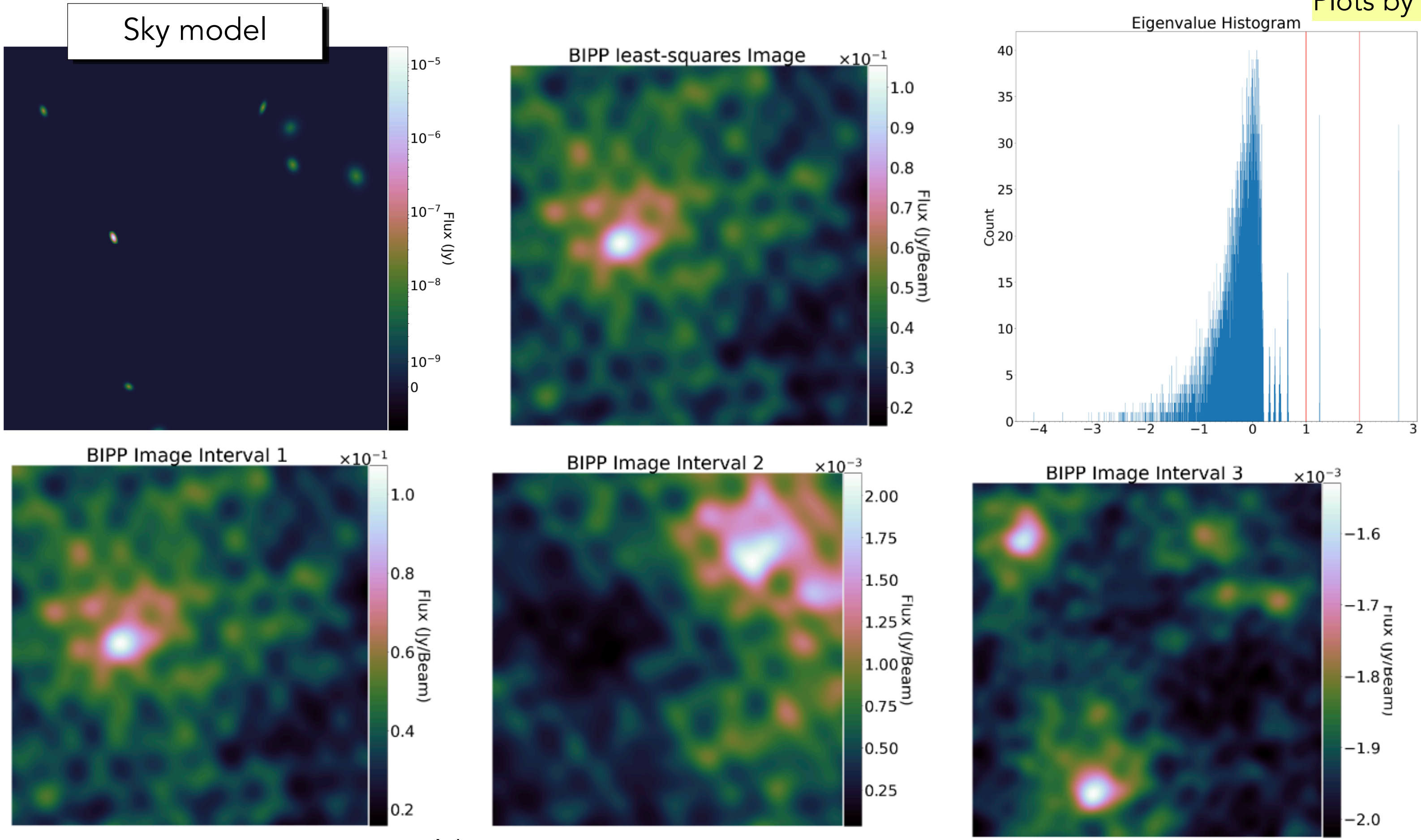
A simulated example

Plots by Shreyam Krishna

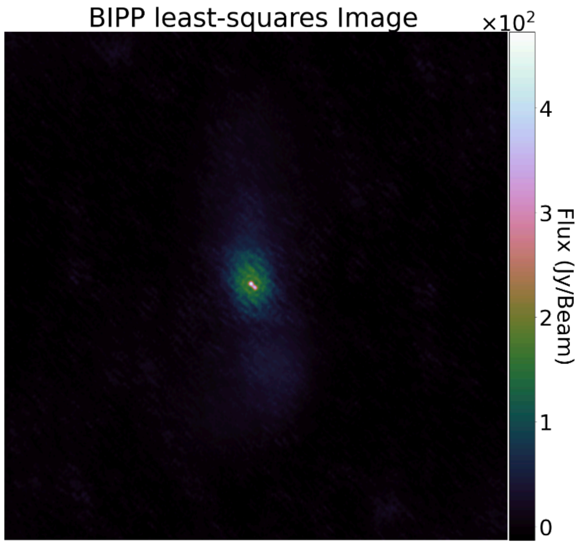


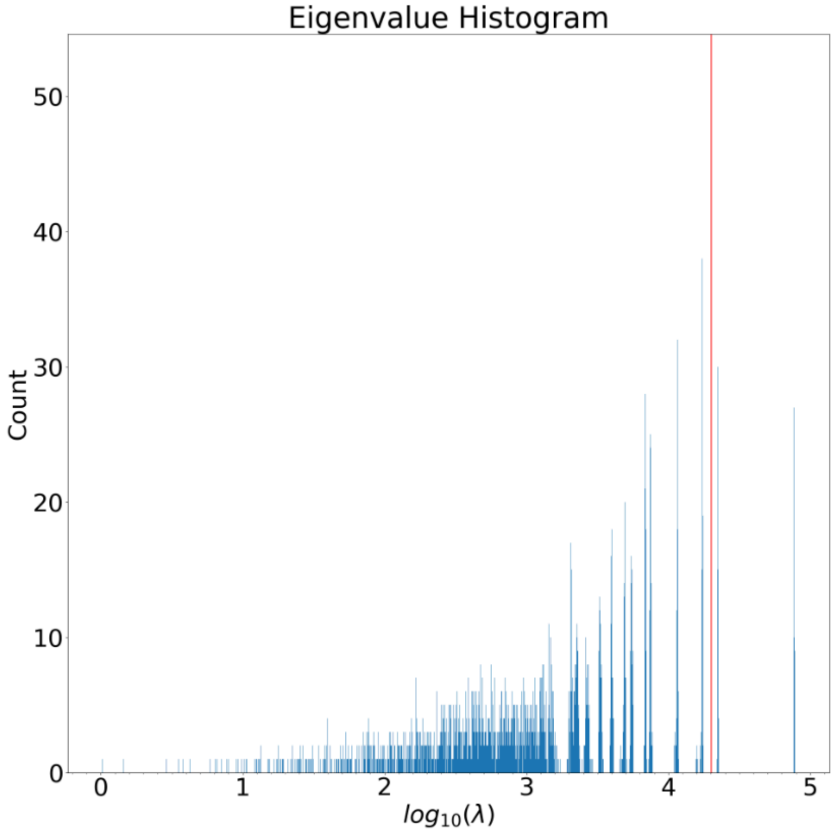
A simulated example with noise & gain errors

Plots by Shreyam Krishna



An example on real MWA data





Plots by Shreyam Krishna

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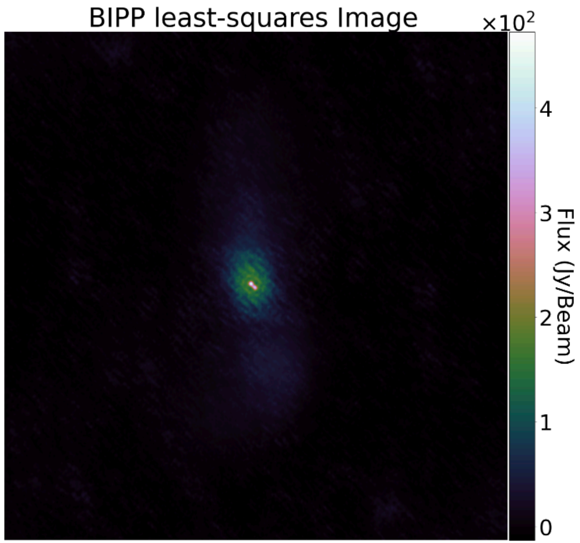

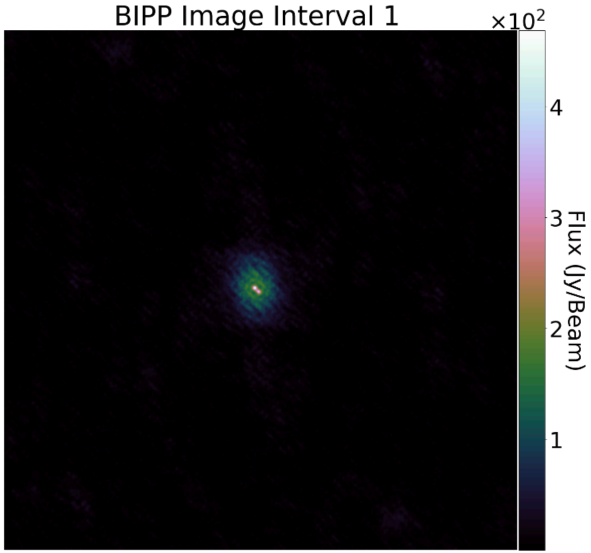

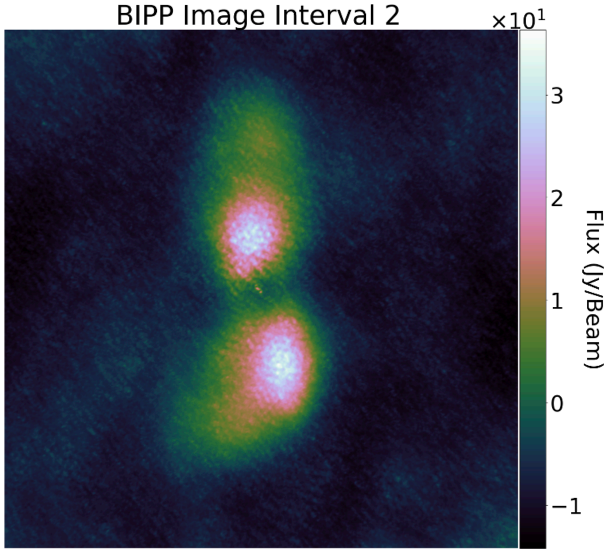

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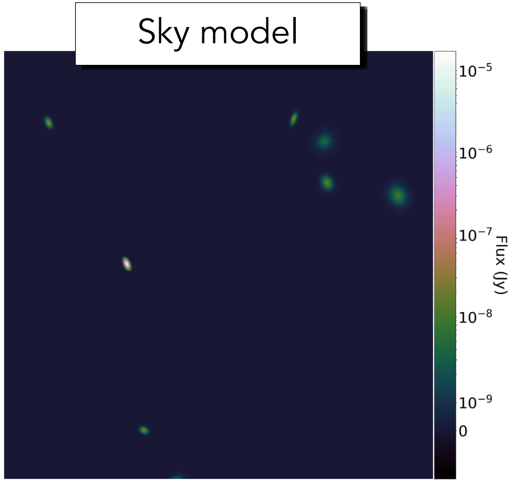
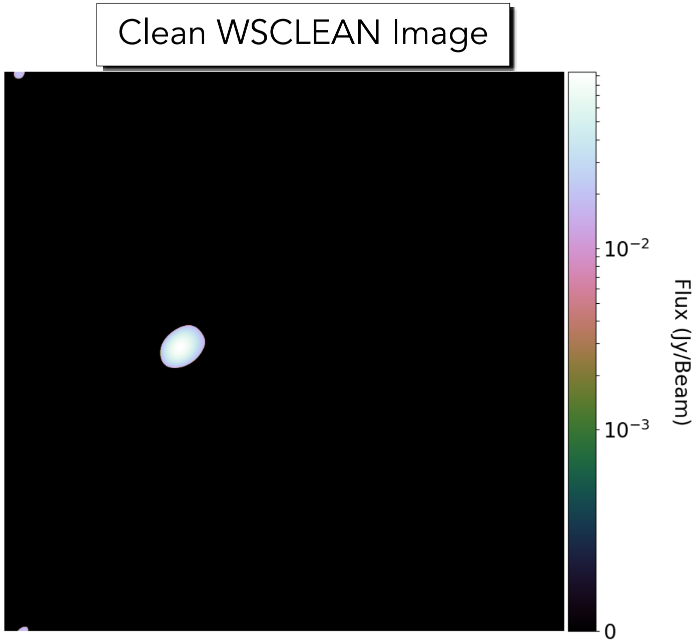
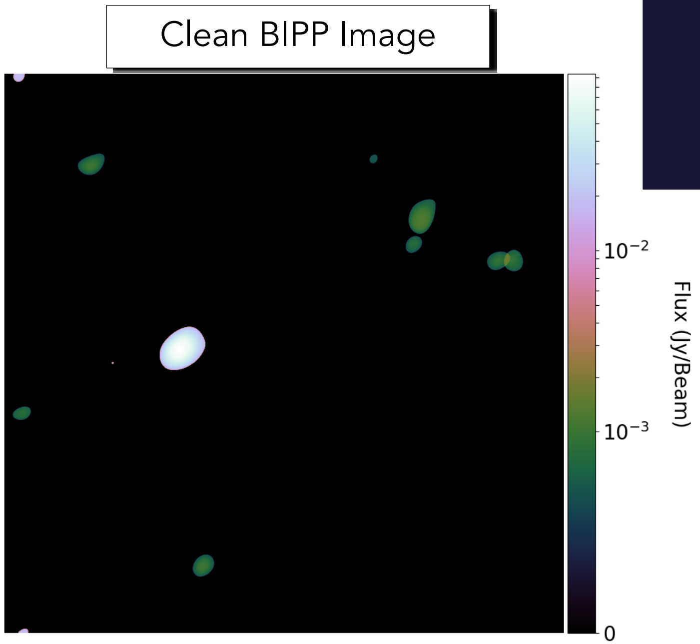
Better SNR?



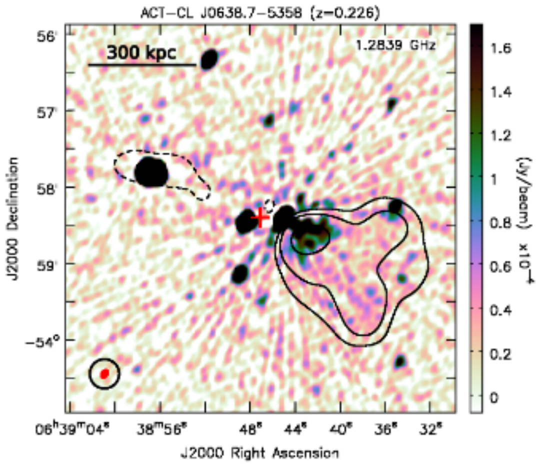




(Levels cleaned separately)

Next steps

- Would like to check the impact of this peeling on a few scientific use cases
    - Using it in the MeerKAT calibration pipeline (R. Kincaid)
    - Seeing if peeling can improve measurements of diffuse flux (R. Poitevineau)
    - If peeling can improve EoR foreground removal, or make fields contaminated by bright A-team sources useful for EoR measurements (S. Krishna)
- Also working on our own NUFFT library, separating eigenvalue decomposition from imaging



If you are interested in using this for your own work the code is publicly available on GitHub: https://github.com/epfl-radio-astro/bipp