Dunlap Institute for Astronomy & Astrophysics
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PERIMETER INSTITUTE

Correlation Calibration

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Cosmology in the Alps
16 March 2026

arXiv:2602.06109

github.com/r-pascua/corrcal

Interferometry Overview

Interferometry Overview

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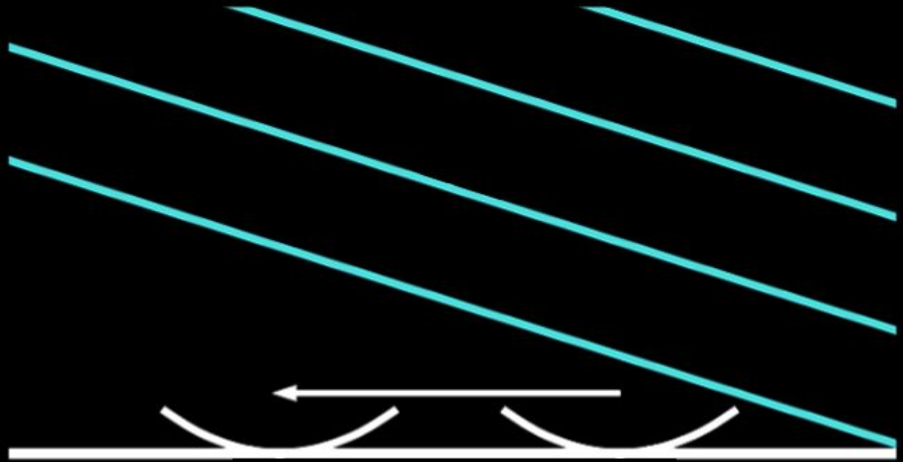
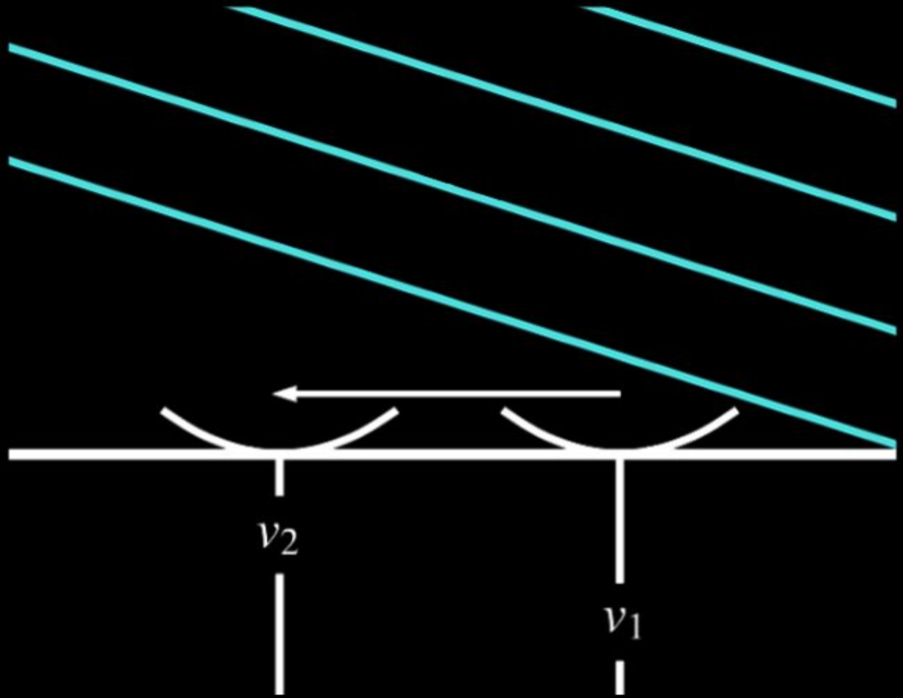


Interferometry Overview



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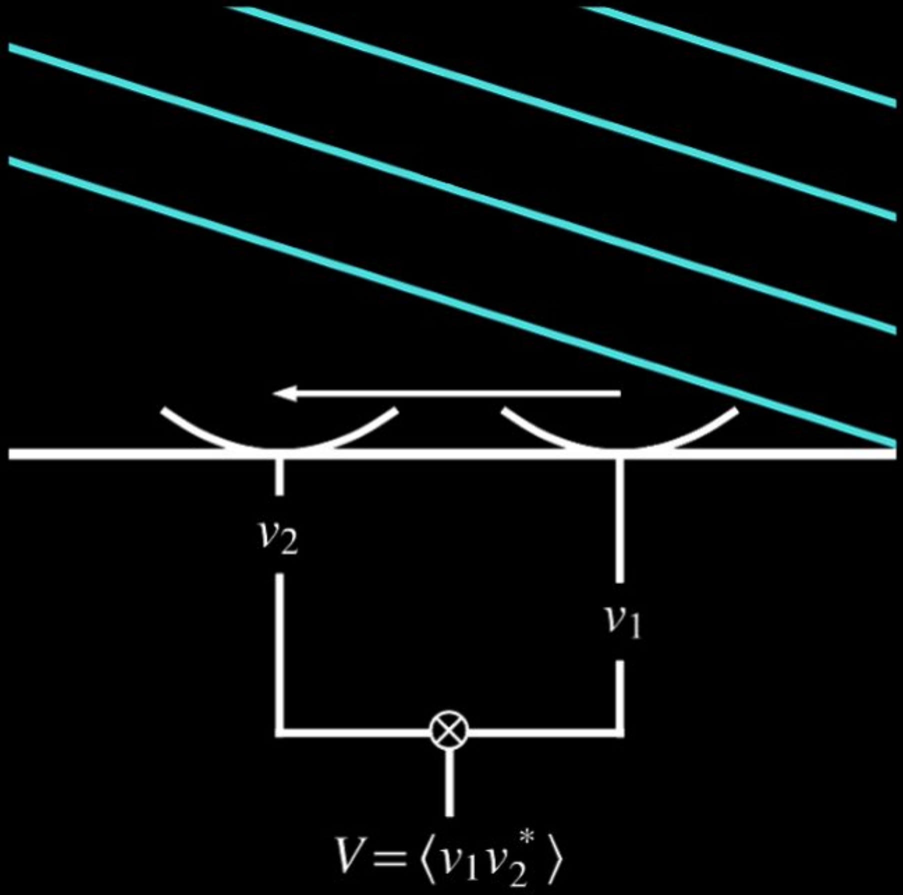

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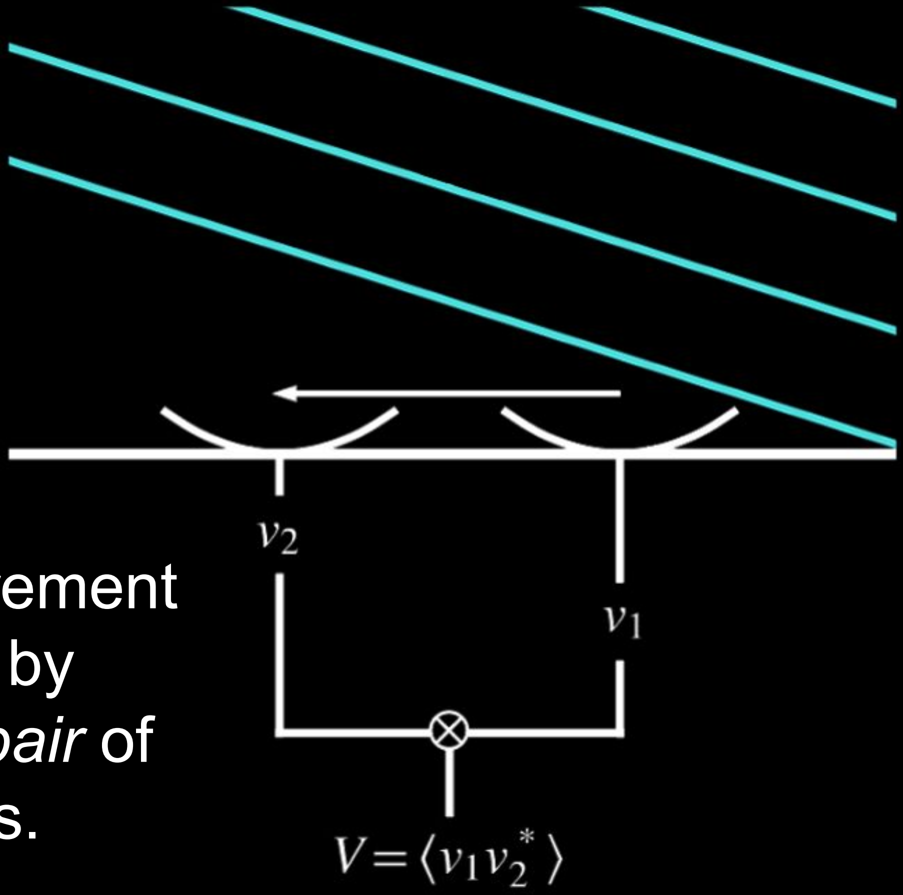
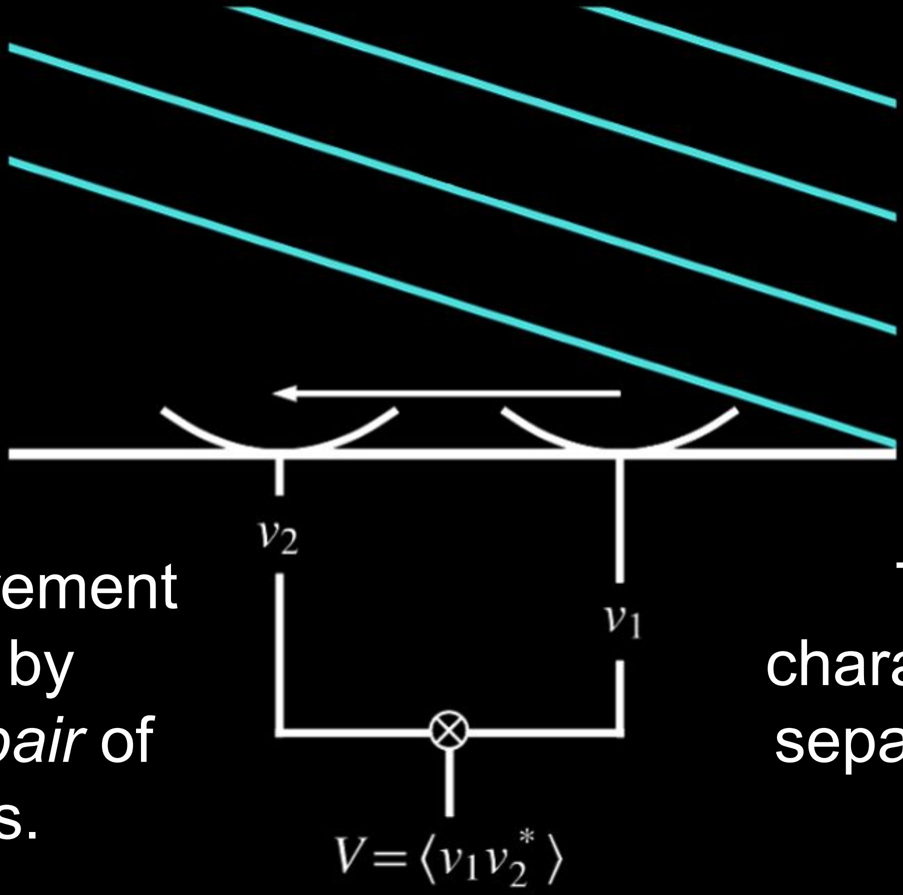


Each measurement is formed by choosing a *pair* of antennas.

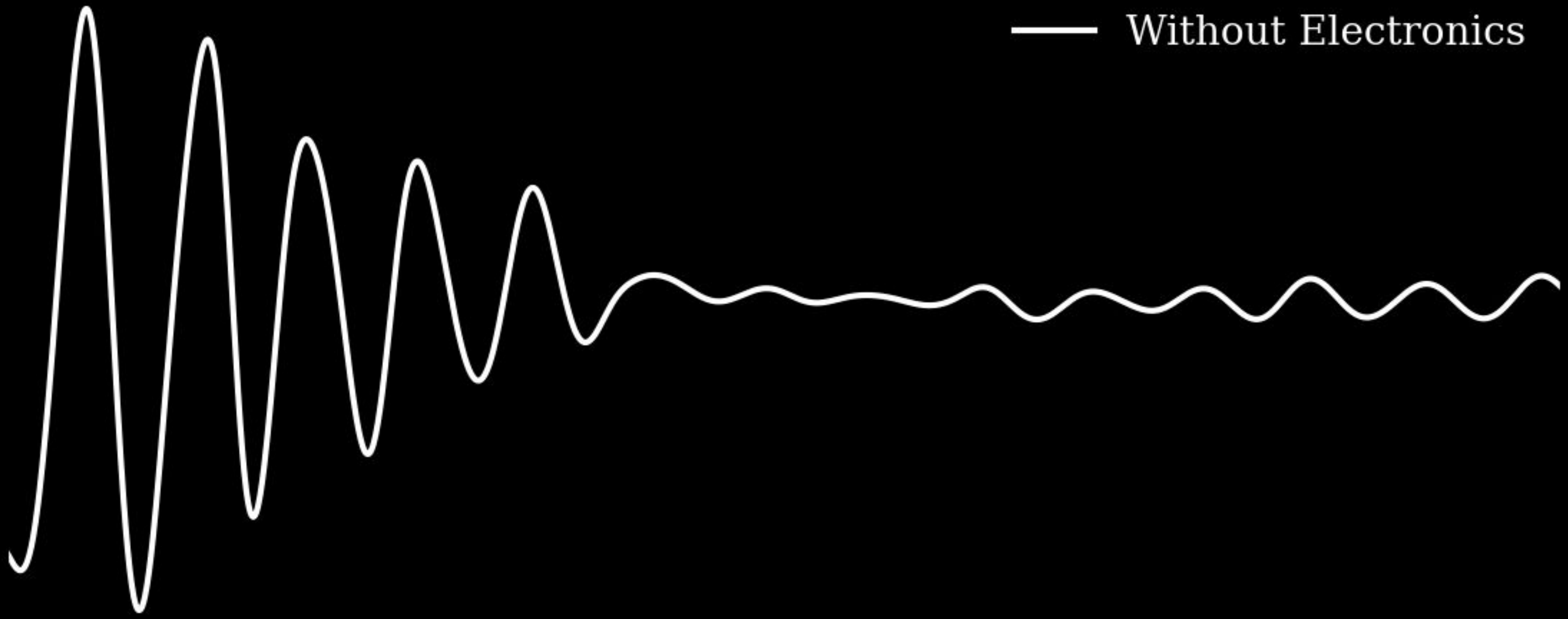
Interferometry Overview



Each measurement is formed by choosing a *pair* of antennas.

The data is characterized by the separation between antennas.

The Need for Calibration



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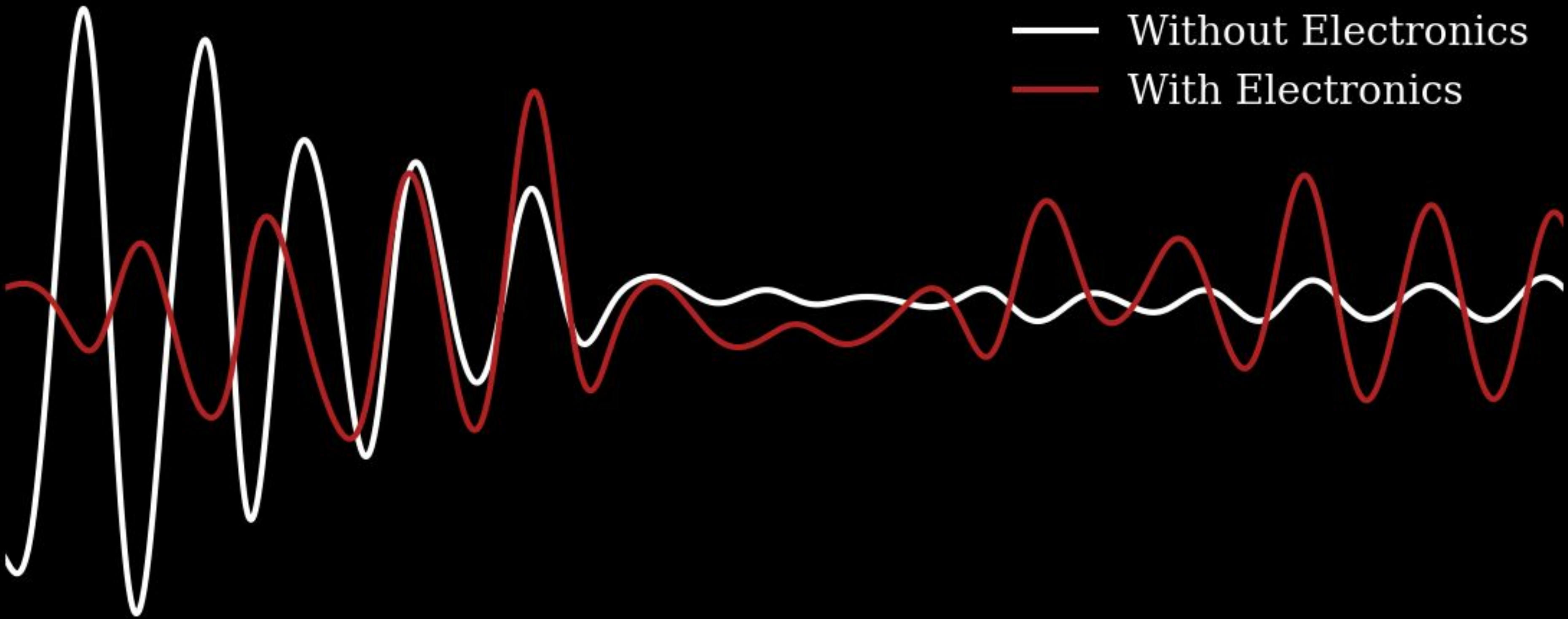
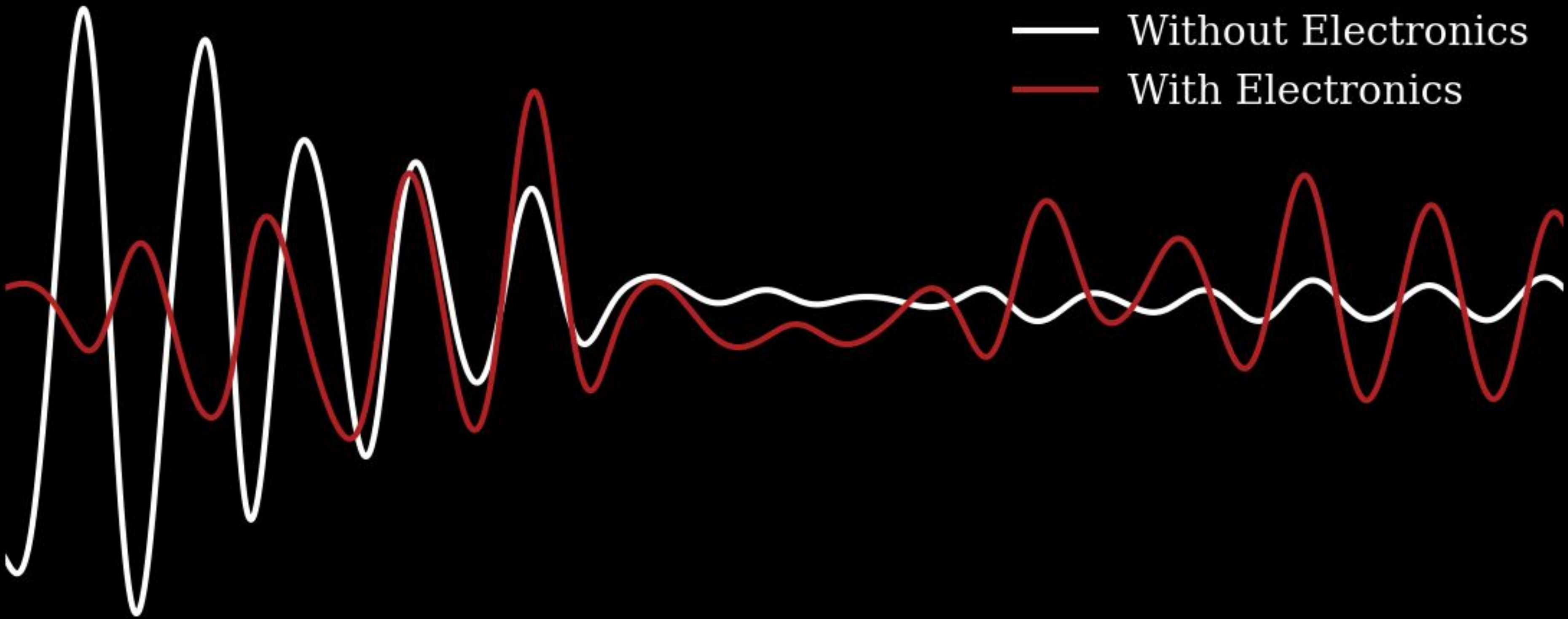


The Need for Calibration



Information about the sky is corrupted by the electronics!

Calibration Overview

$$V_{ij}^{\mathrm{meas}} = g_i g_j^* V_{ij}^{\mathrm{true}} + n_{ij}$$

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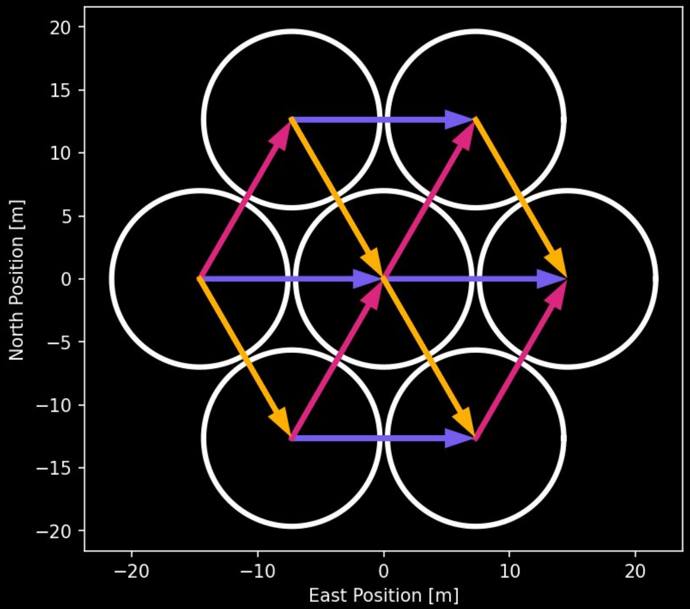
$$V_{ij}^{\text{meas}} = g_i g_j^* V_{ij}^{\text{true}} + n_{ij}$$

$$\chi^2 = \sum_{i,j} \frac{\left| V_{ij}^{\text{meas}} - g_i g_j^* V_{ij}^{\text{model}} \right|^2}{\sigma_{ij}^2}$$

Calibration Overview

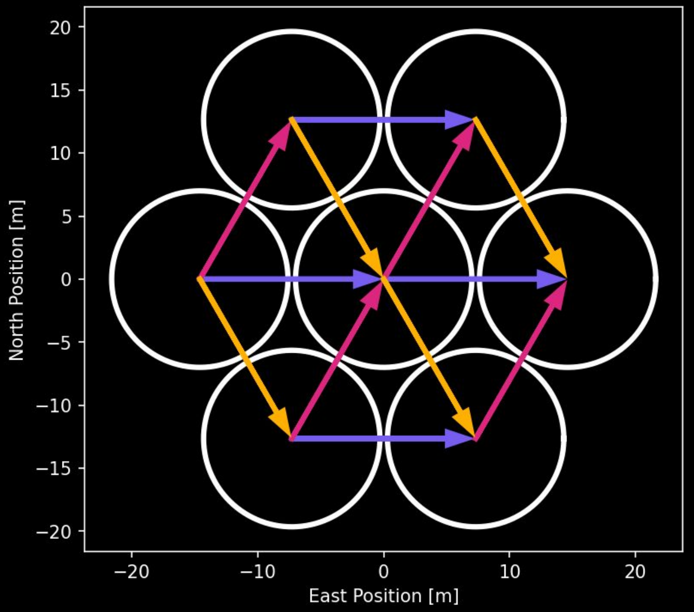
Redundant Calibration



Leverage internal consistency
to fit for model visibilities
alongside per-antenna gains

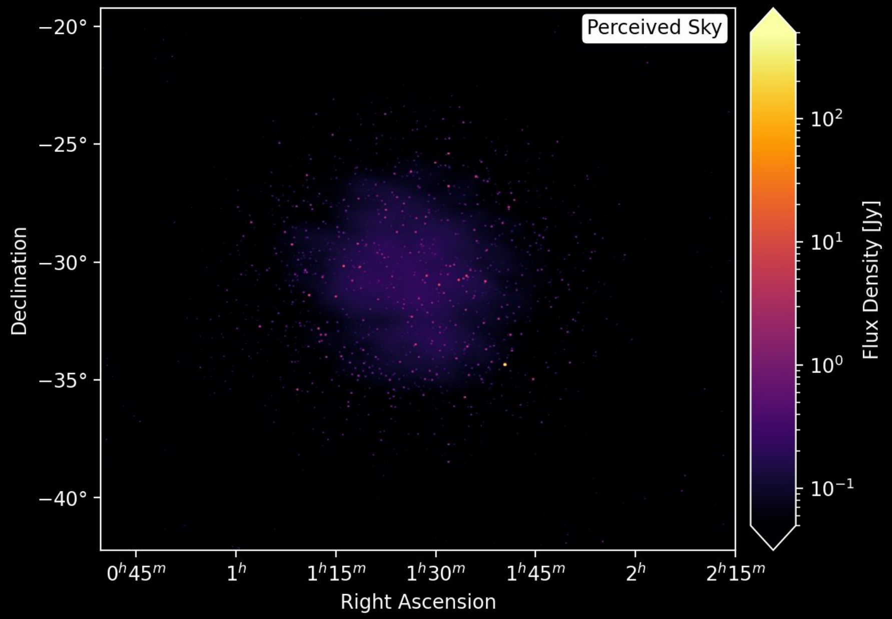
Calibration Overview

Redundant Calibration



Leverage internal consistency to fit for model visibilities alongside per-antenna gains

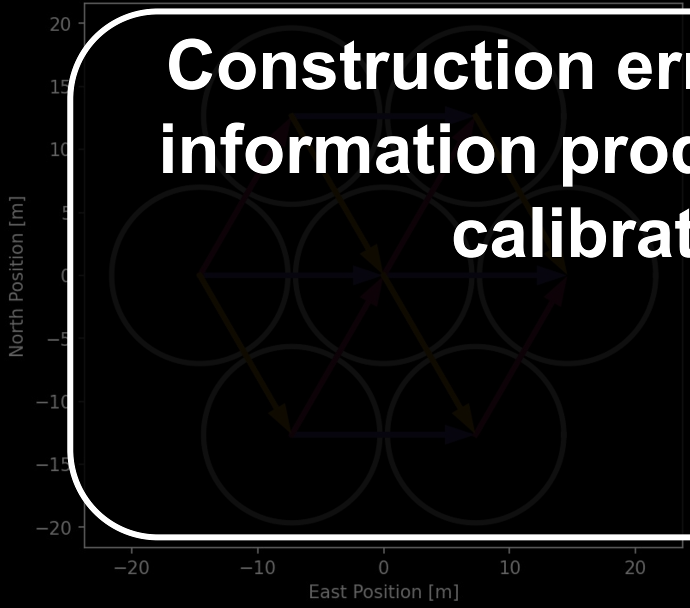
Sky-based Calibration



Use sky and instrument information to pre-compute model visibilities

Calibration Overview

Redundant Calibration



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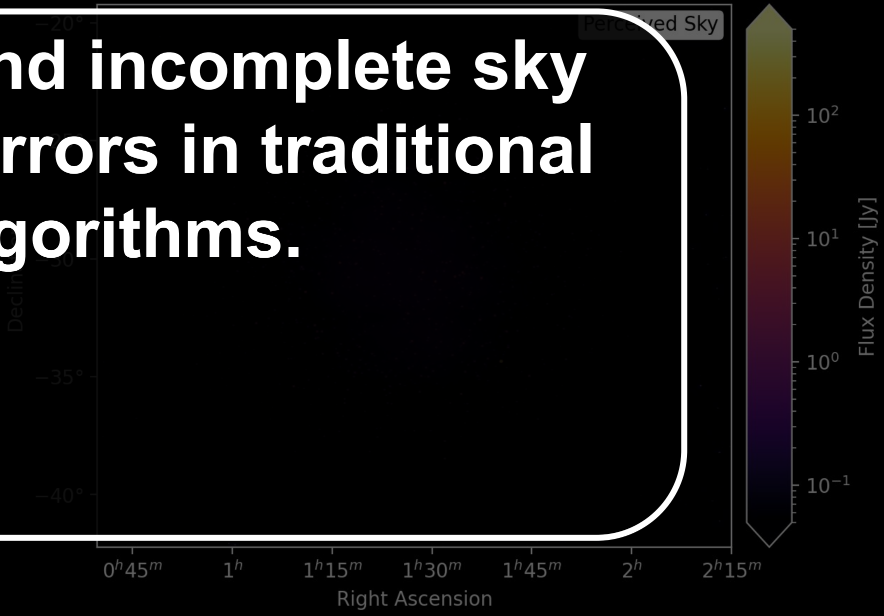
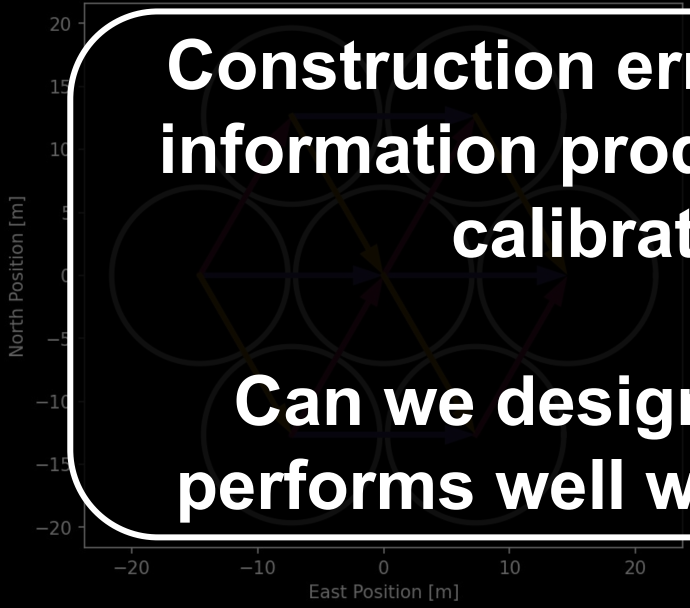


Use sky and instrument information to pre-compute model visibilities

Construction errors and incomplete sky information produce errors in traditional calibration algorithms.

Calibration Overview

Redundant Calibration



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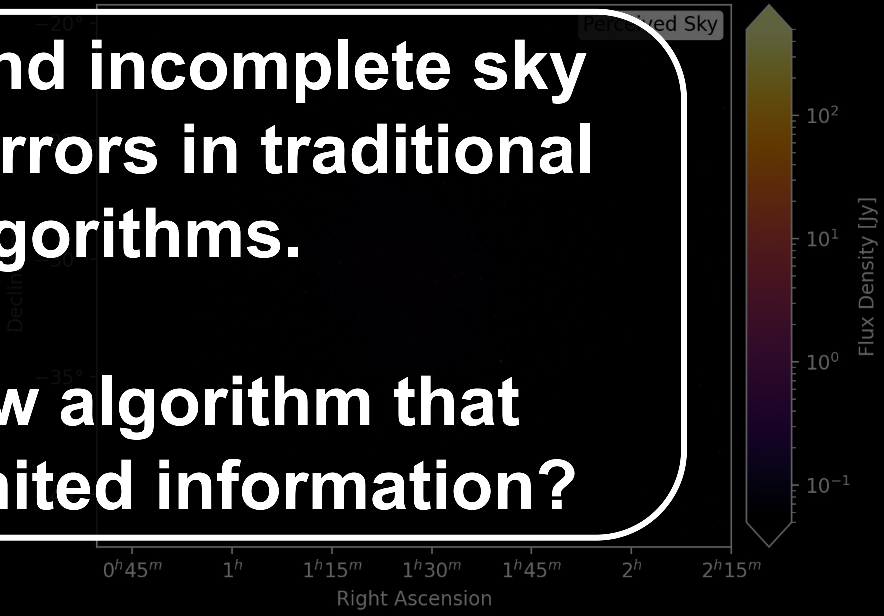

Construction errors and incomplete sky information produce errors in traditional calibration algorithms.

Can we design a new algorithm that performs well with limited information?

Leverage internal consistency to fit for model visibilities alongside per-antenna gains

Use sky and instrument information to pre-compute model visibilities

Correlation Calibration mitigates modeling limitations through *covariance* optimization, rather than *expectation value* optimization.

Correlation Calibration

$$-\log \mathcal{L} = \log \det \mathbf{C} + \boldsymbol{d}^T \mathbf{C}^{-1} \boldsymbol{d}$$

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Maximize the likelihood that the data is described by the model

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Treat the data as if it were Gaussian distributed and normalize properly

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Maximize the likelihood that the data is described by the model

Treat the data as if it were Gaussian distributed and normalize properly

Match covariance in the data to a model of the covariance

Correlation Calibration

$$-\log \mathcal{L} = \log \det \mathbf{C} + \boldsymbol{d}^T \mathbf{C}^{-1} \boldsymbol{d}$$

For an array with ~500 antennas, the covariance has roughly 100,000 x 100,000 elements.

How could we possibly compute this likelihood?

Correlation Calibration

$$-\log \mathcal{L} = \log \det \mathbf{C} + \boldsymbol{d}^T \mathbf{C}^{-1} \boldsymbol{d}$$

$$\mathbf{C} = \mathbf{N} + \mathbf{G}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^T\mathbf{G}^T + \mathbf{G}\boldsymbol{\Delta}\boldsymbol{\Delta}^T\mathbf{G}^T$$

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Noise
(diagonal)

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Noise
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Electronic effects
(solve for these; diagonal)

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Covariance from the sky
(only need ~5 bright sources)

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Noise
(diagonal)

Electronic effects
(solve for these; diagonal)

Covariance from the sky
(only need ~5 bright sources)

Covariance from redundancy
(a few numbers per group)

Correlation Calibration

$$-\log \mathcal{L} = \log \det \mathbf{C} + \boldsymbol{d}^T \mathbf{C}^{-1} \boldsymbol{d}$$

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$$\mathbf{C}^{-1} = \mathbf{N}^{-1} - \bar{\boldsymbol{\Sigma}}\bar{\boldsymbol{\Sigma}}^T - \bar{\boldsymbol{\Delta}}\bar{\boldsymbol{\Delta}}^T$$

Correlation Calibration

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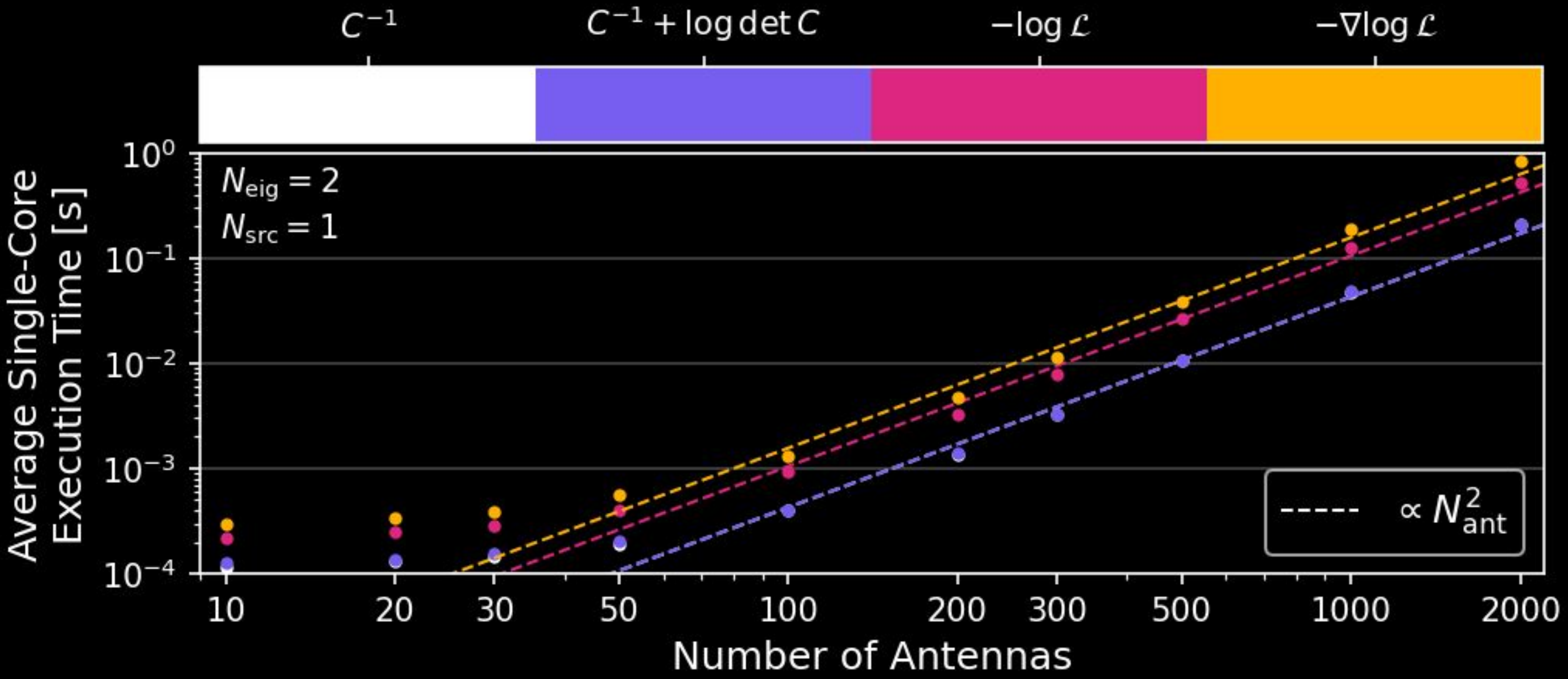
$$\mathbf{C} = \mathbf{N} + \boldsymbol{\Sigma}\boldsymbol{\Sigma}^T + \boldsymbol{\Delta}\boldsymbol{\Delta}^T$$

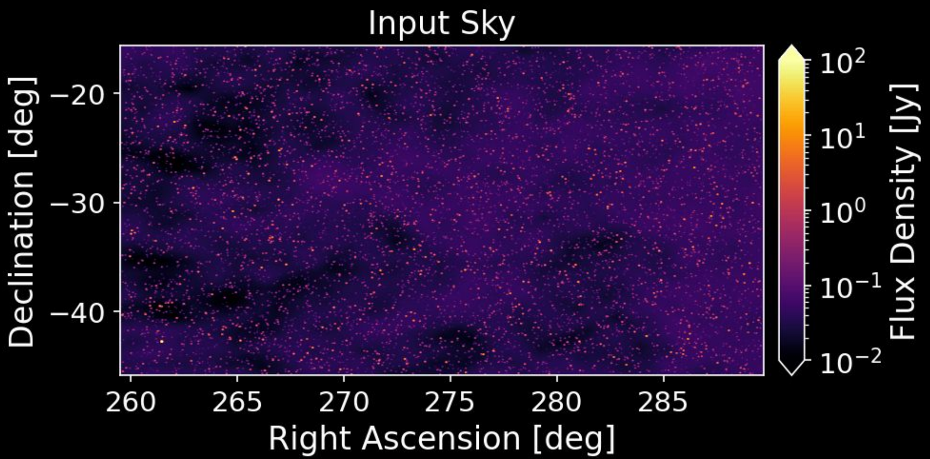
Liberally apply the Woodbury Identity and Cholesky decompositions

$$\mathbf{C}^{-1} = \mathbf{N}^{-1} - \bar{\boldsymbol{\Sigma}}\bar{\boldsymbol{\Sigma}}^T - \bar{\boldsymbol{\Delta}}\bar{\boldsymbol{\Delta}}^T$$

Computational Complexity



Validation Simulations



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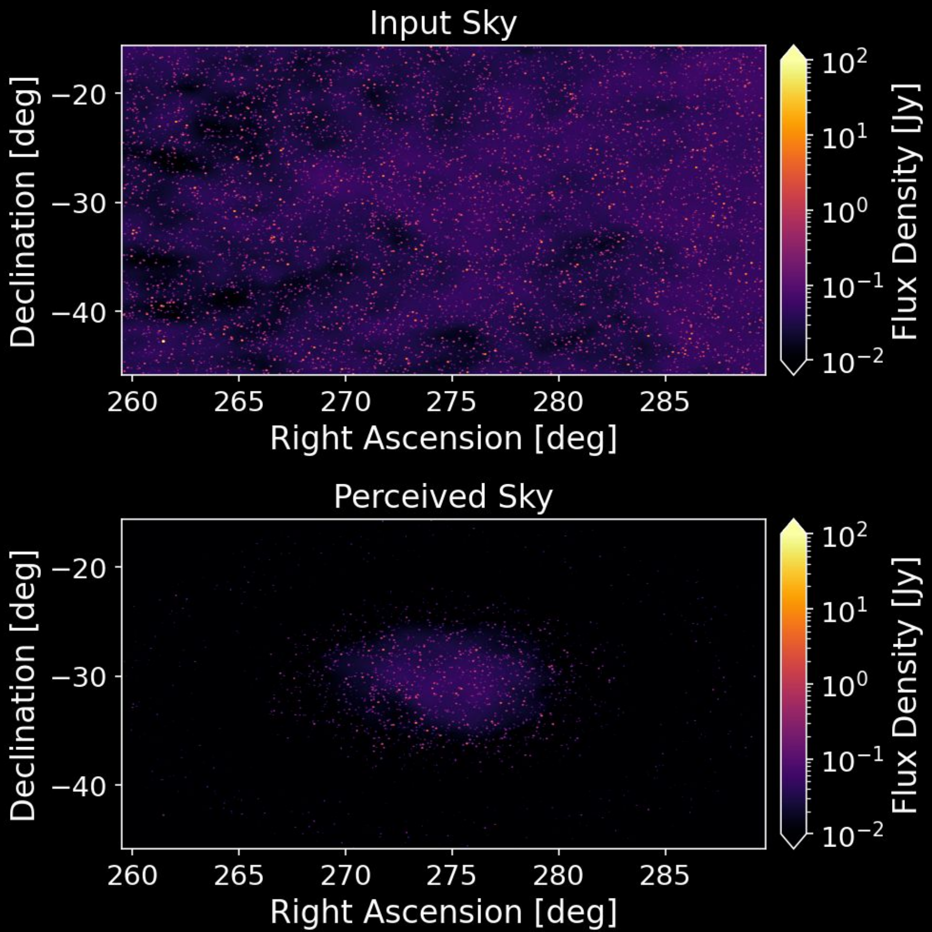

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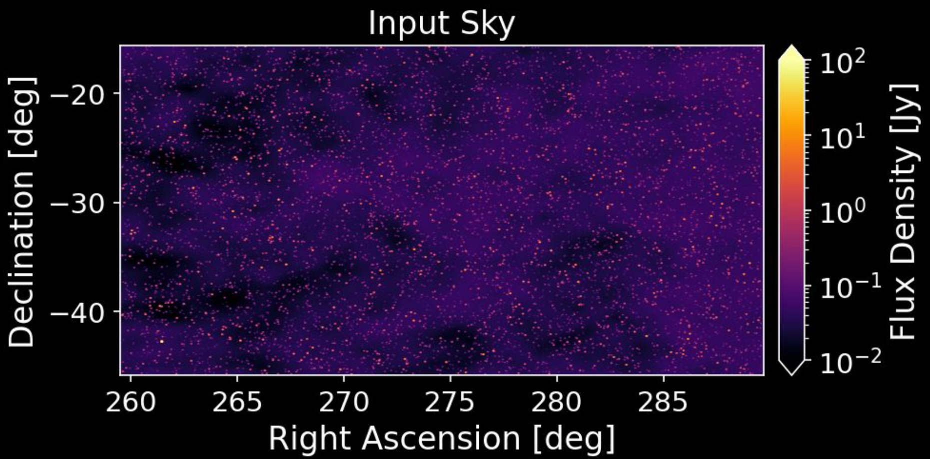

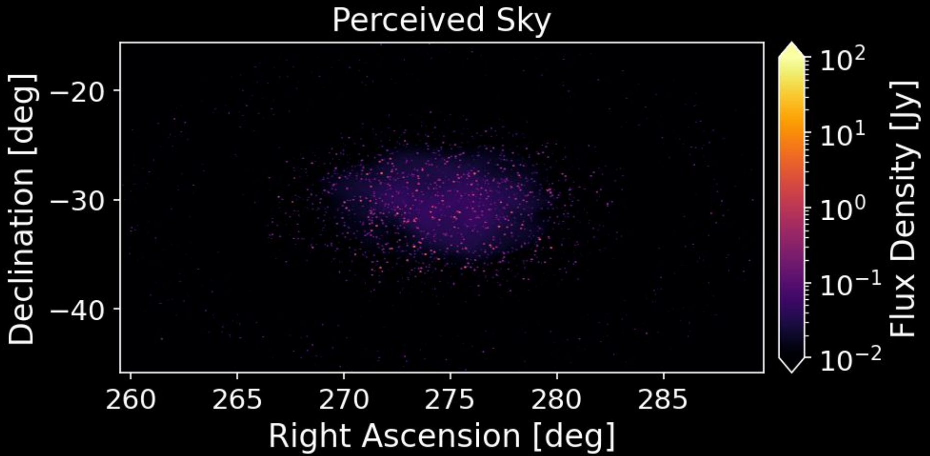

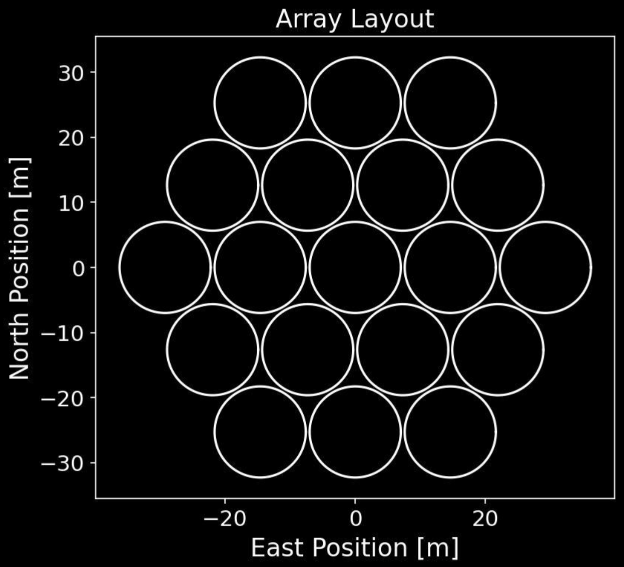

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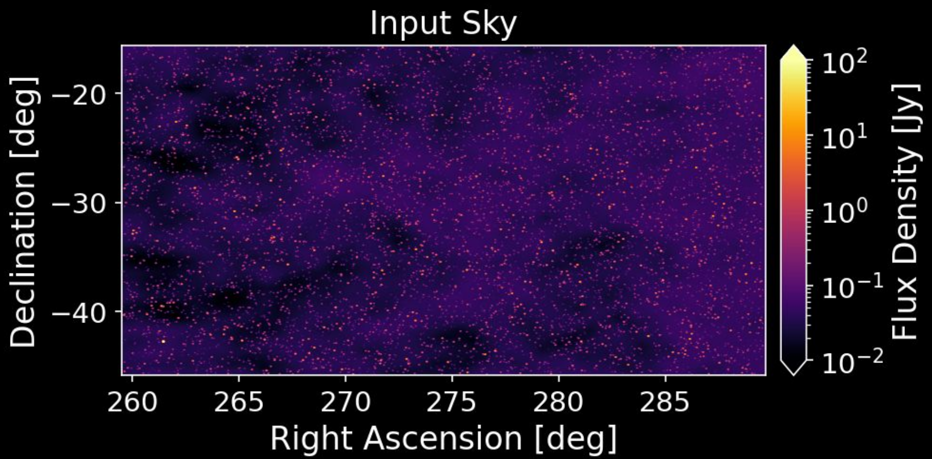
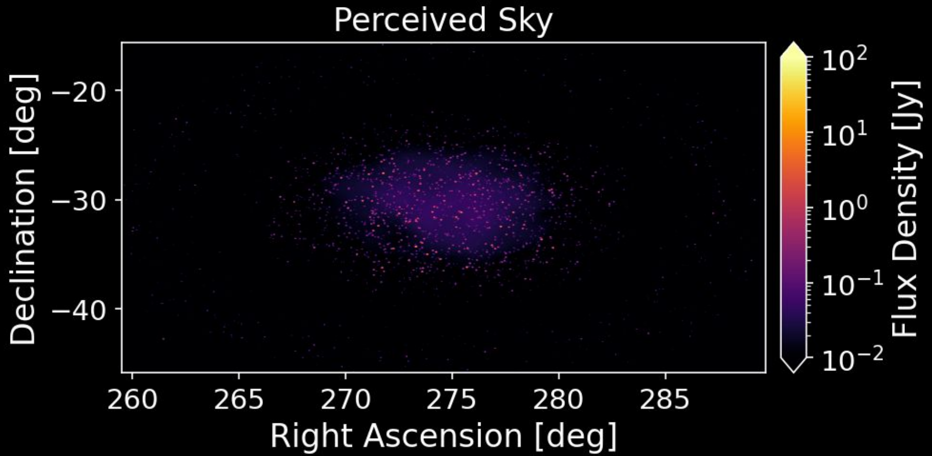
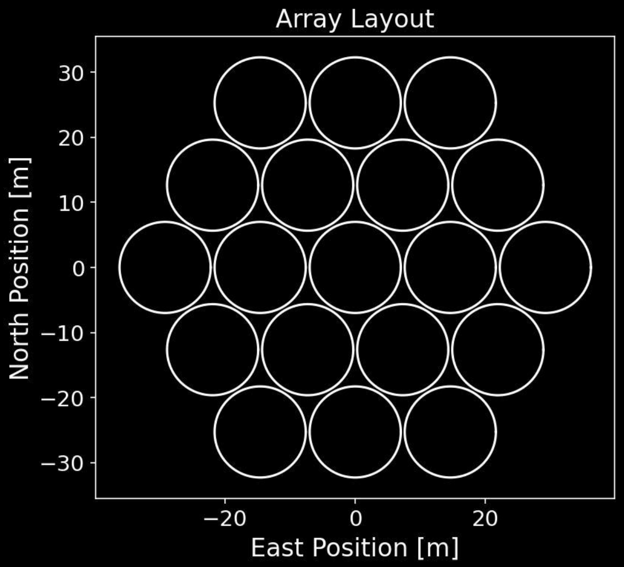






Parameter	Distribution
True Gain Amplitude, $g_a^{\rm true}$	$\mathcal{N}(1, 0.1^2)$
True Gain Phase, $\phi_a^{\rm true}$	$\mathrm{Uniform}(0, 2\pi)$
Initial Gain Amplitude Error, $\frac{g_a^{\rm init}}{g_a^{\rm true}}$	$\mathcal{N}(1, 0.05^2)$
Initial Gain Phase Error, $\phi_a^{\rm init} - \phi_a^{\rm true}$	$\mathcal{N}(0, 0.02^2)$

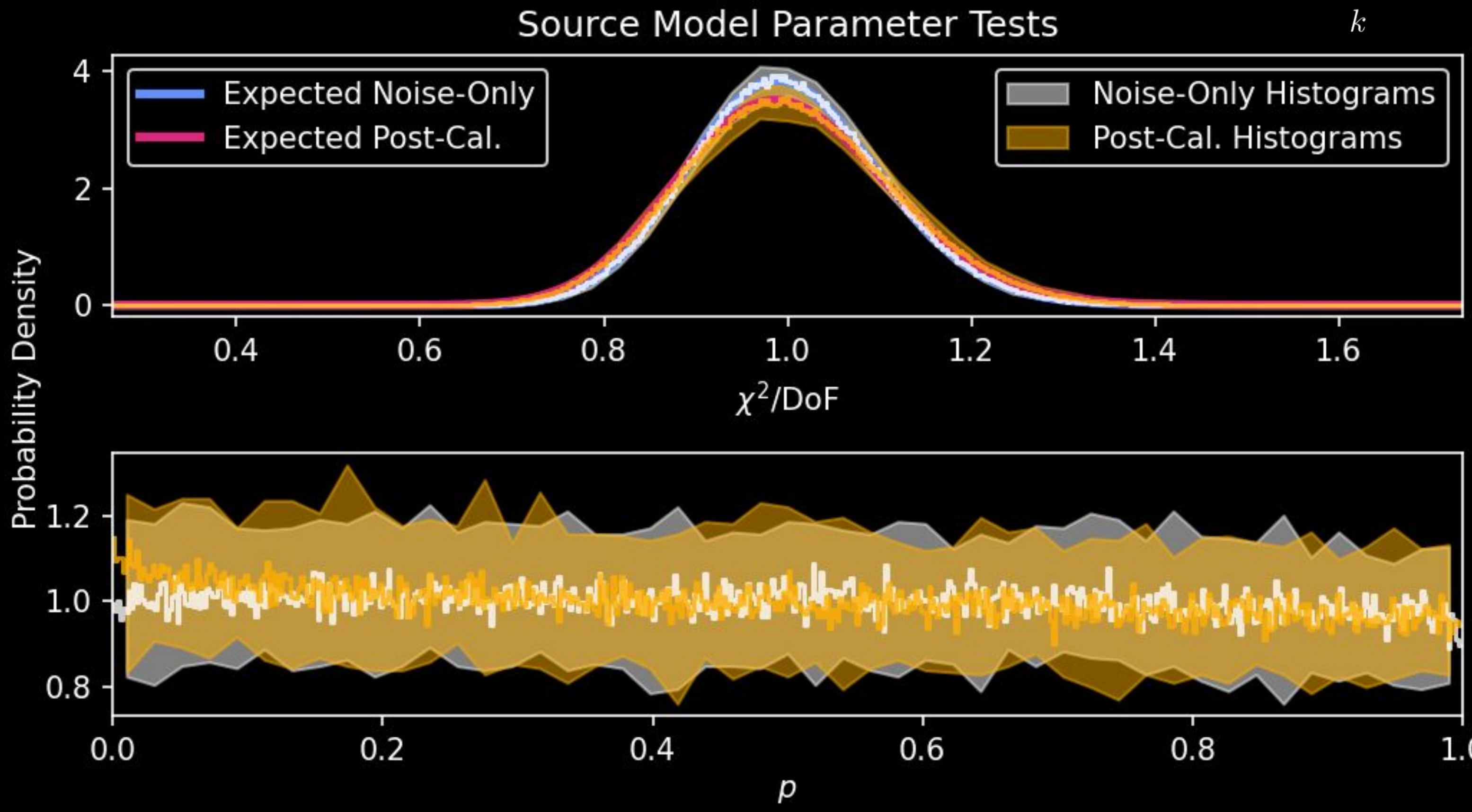
Source Model Tests

Test Hyperparameter	Options
Average Flux Error [%]	(0, 1, 2, 5, 10, 20)
Number of Calibration Sources	(1, 5, 10, 20)

Calibration performance as a function of source model accuracy was tested with the outer product of the above “source model error” options.

Source Model Tests

$$\chi^2 = \sum_k \frac{|V_k^{\mathrm{cal}} - V_k^{\mathrm{true}}|^2}{\sigma_k^2}$$



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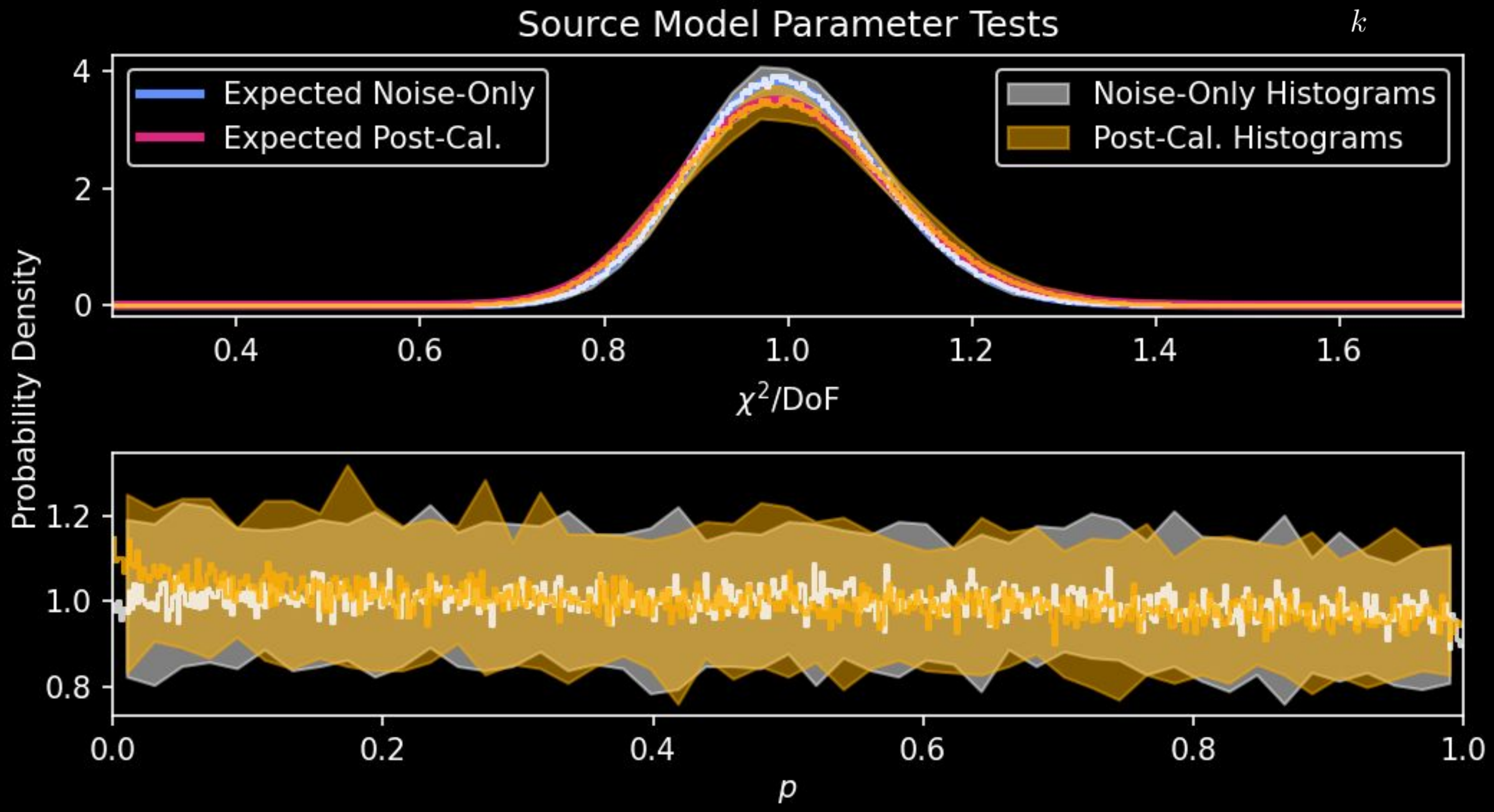
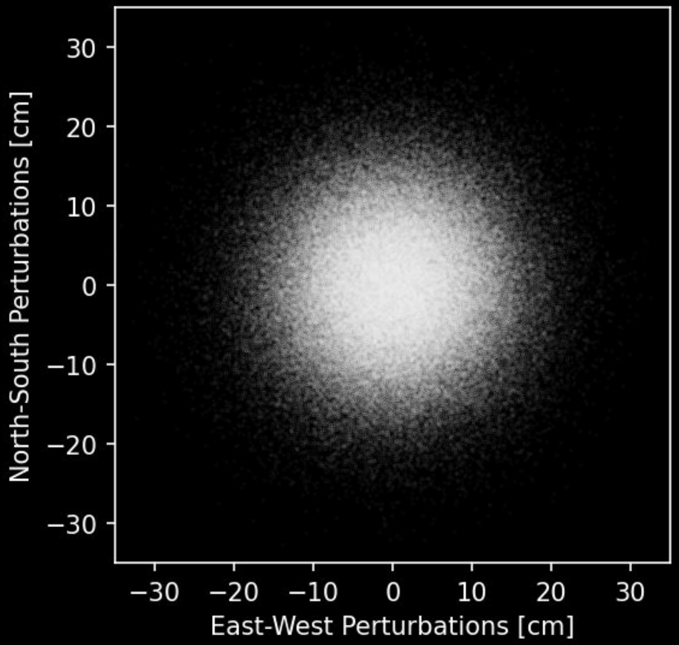


CorrCal is robust to uncertainties in true source fluxes.

Nonredundancy Tests



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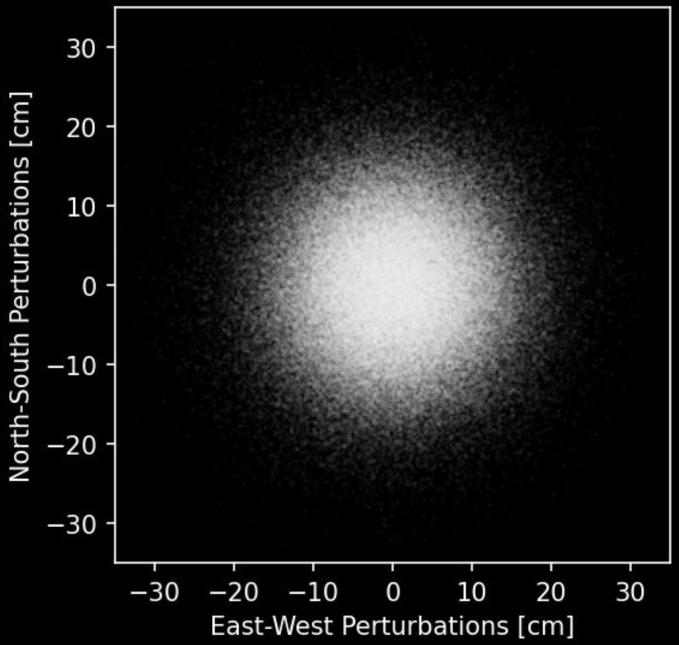

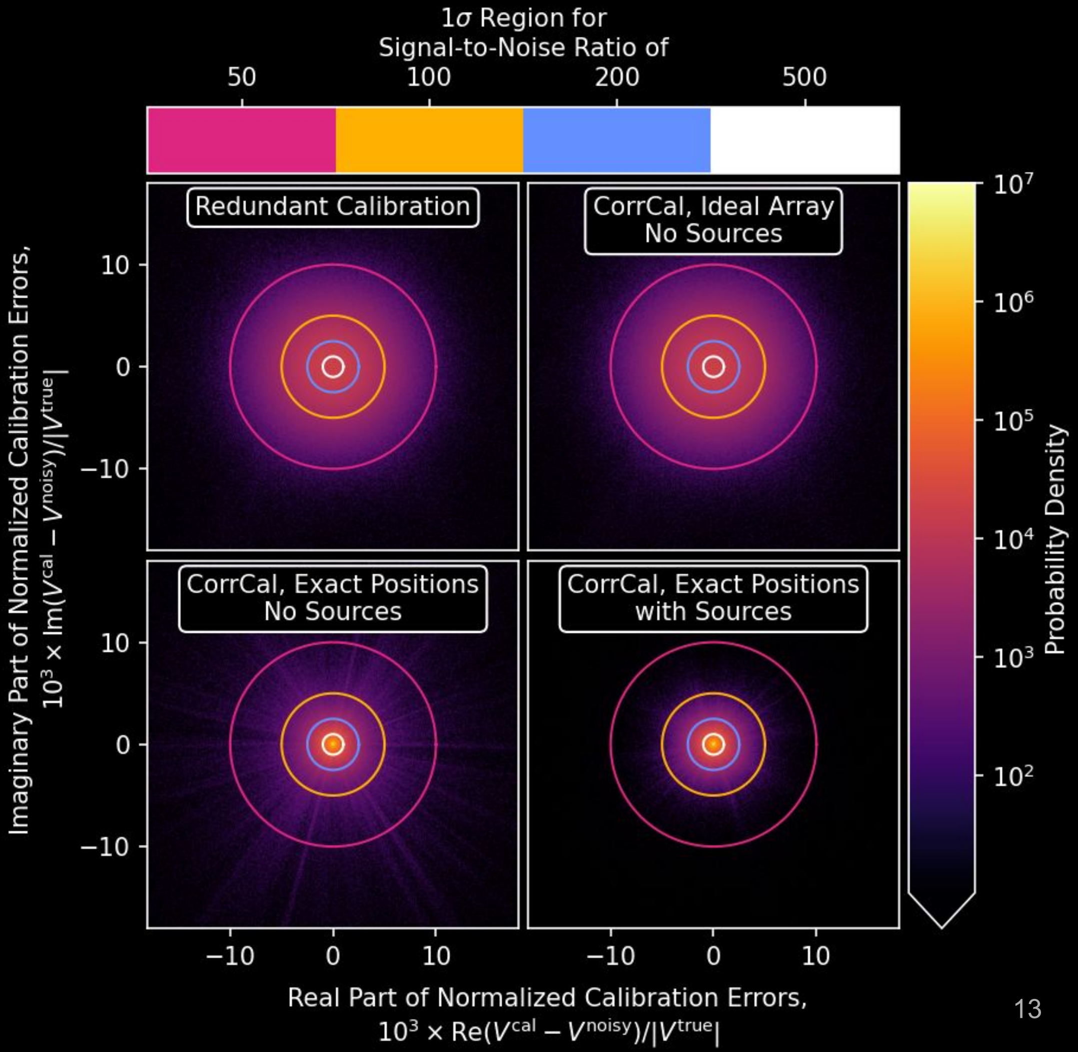

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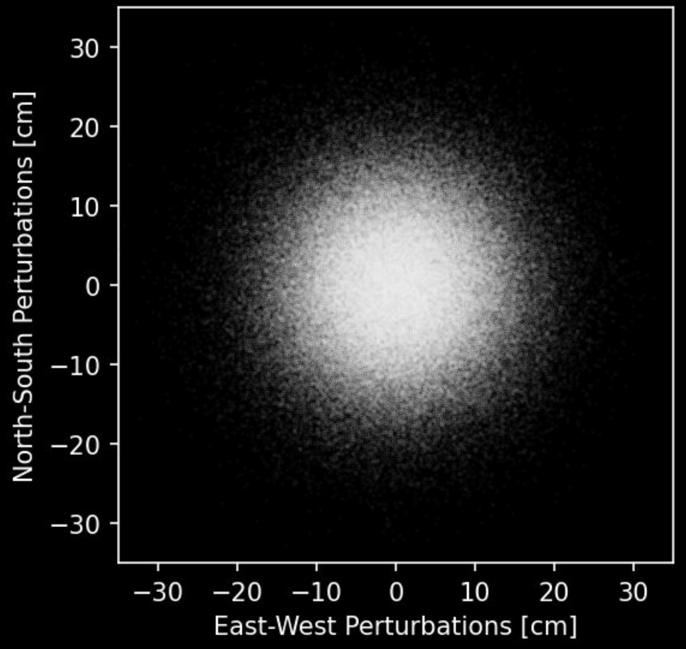
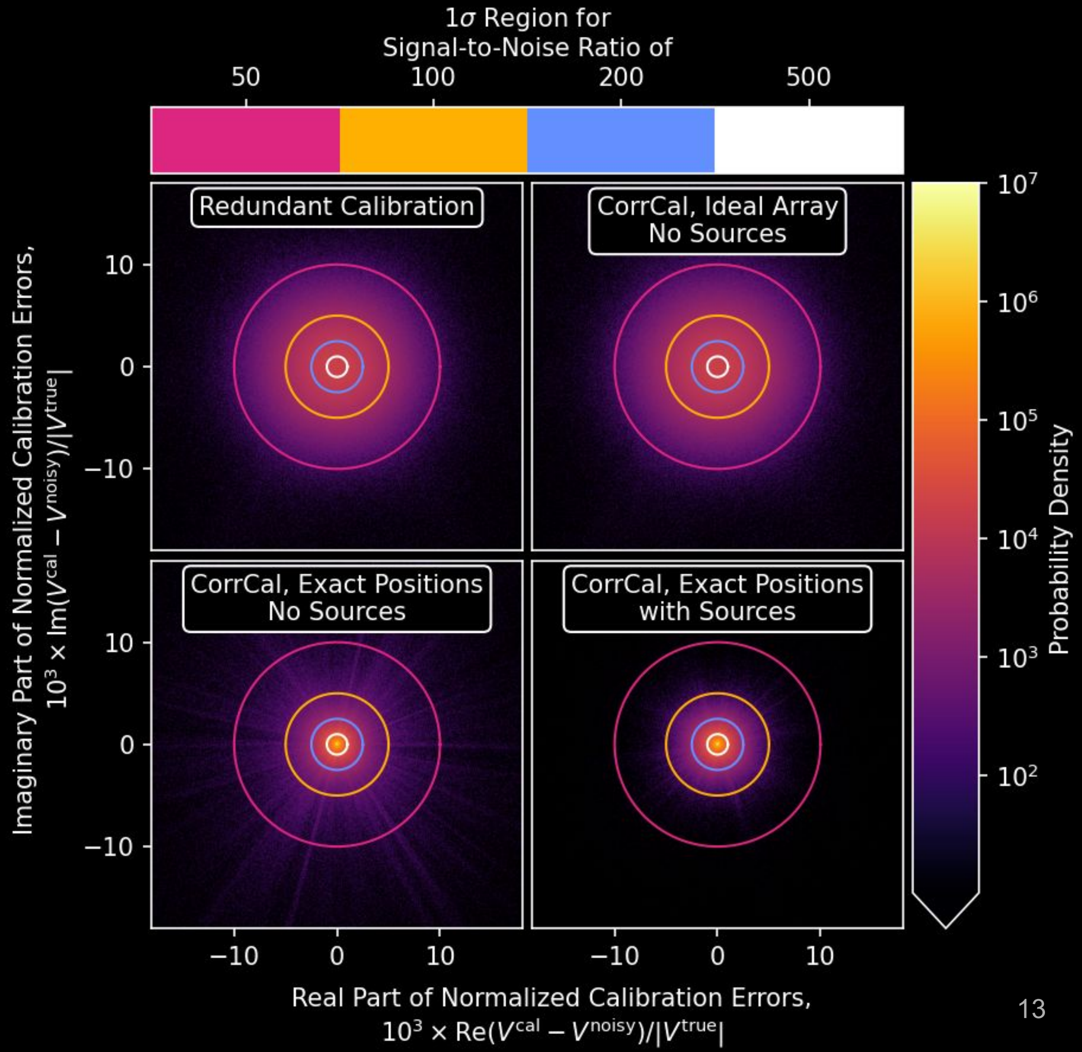
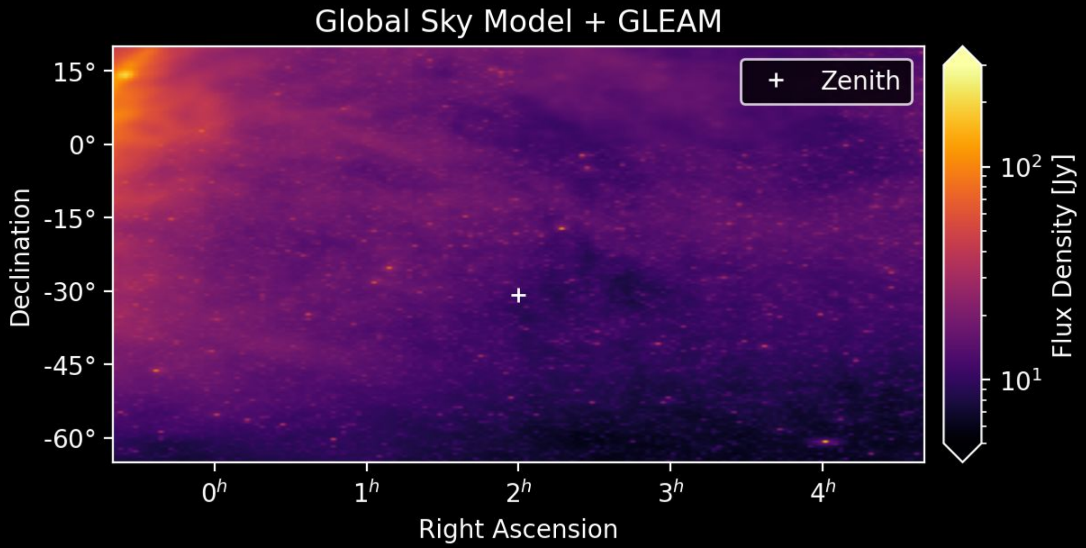




Calibration errors from nonredundancy are extremely small.

Non-Gaussian Sky Tests



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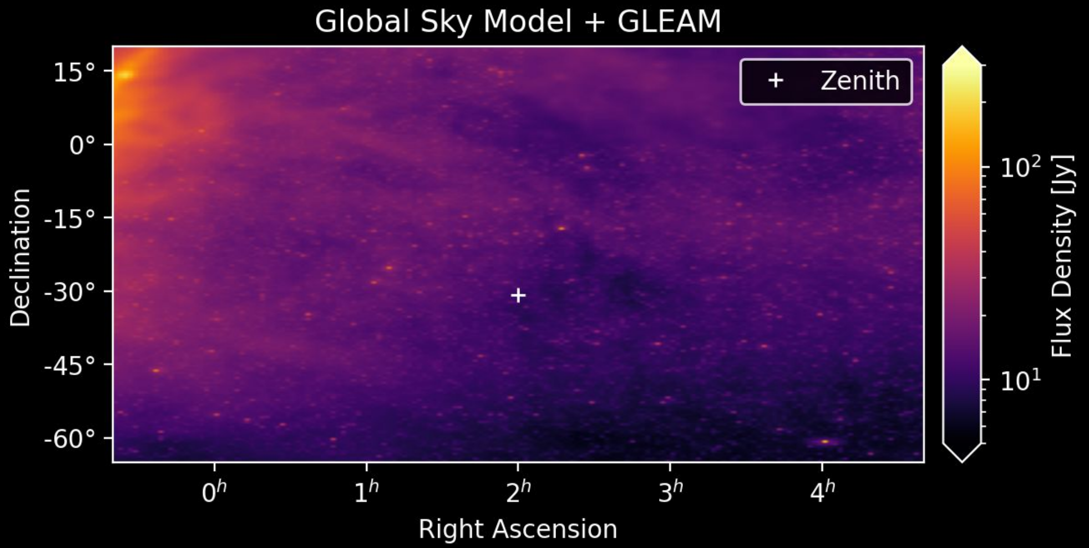

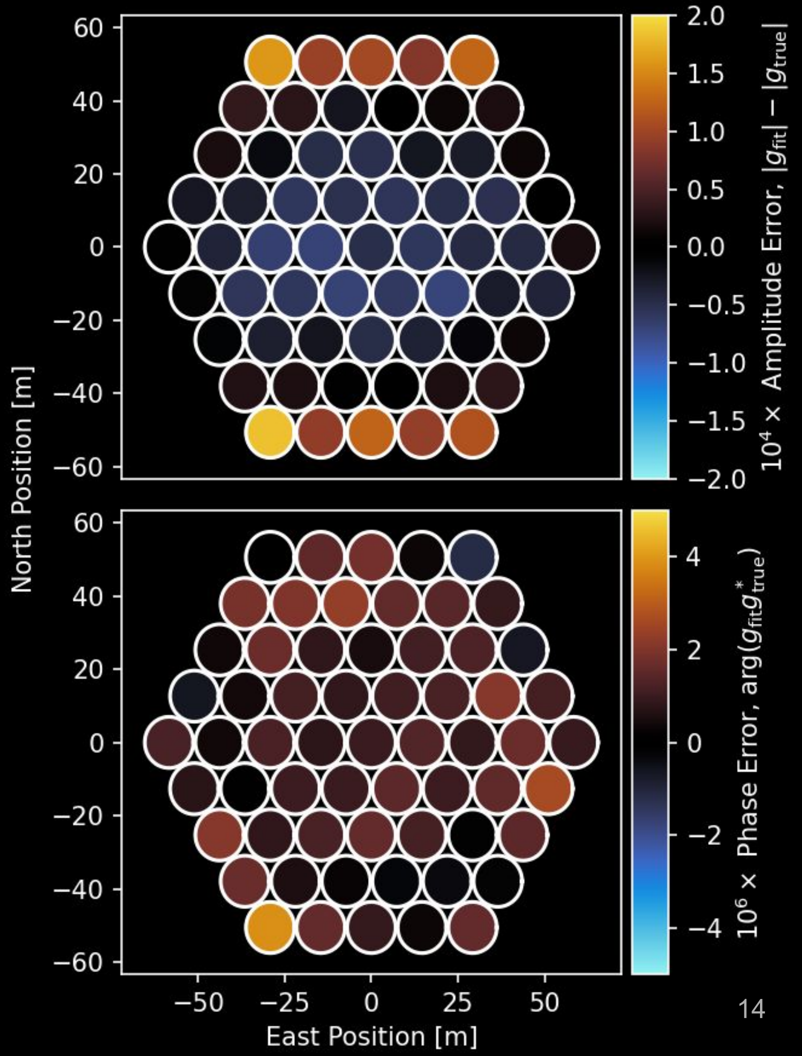

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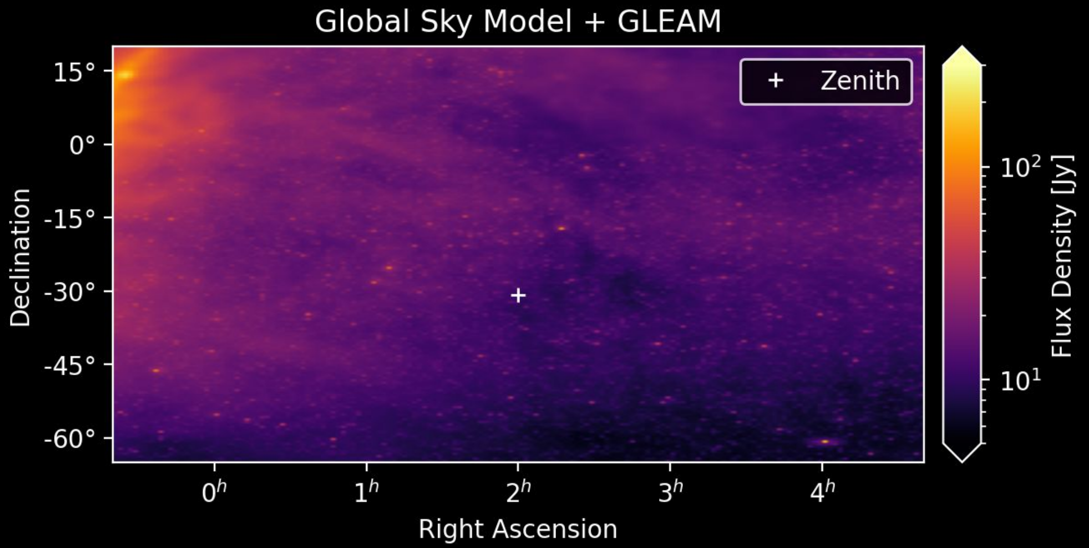

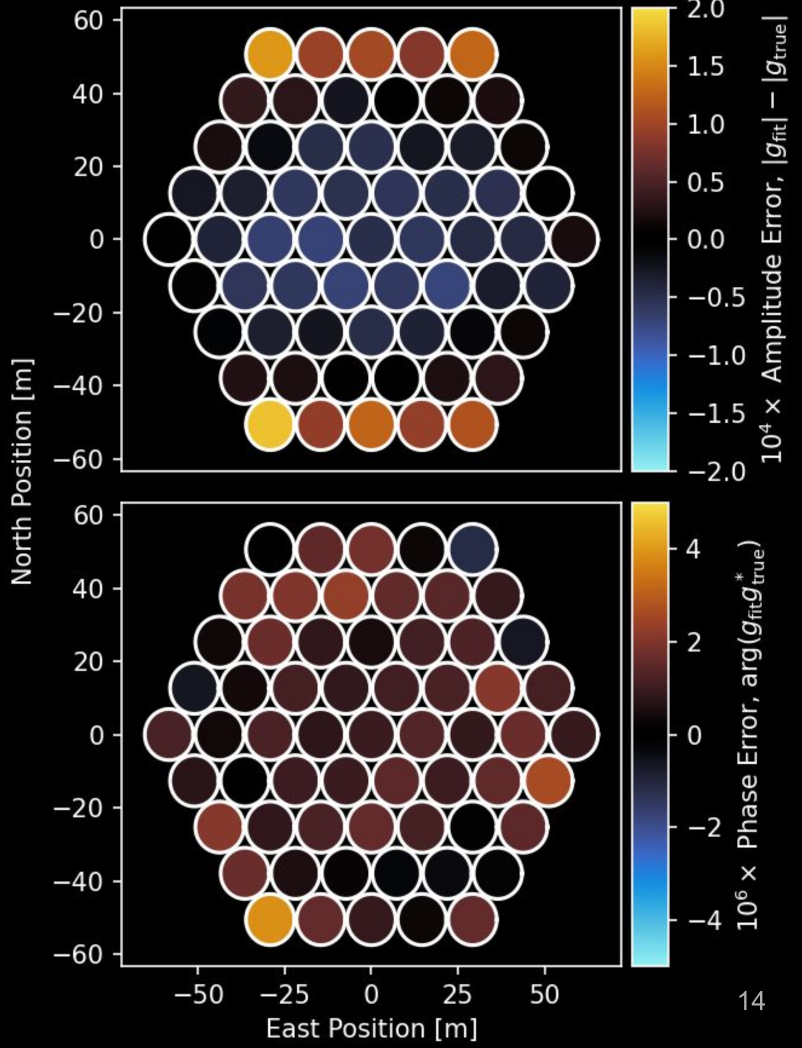

Non-Gaussianities in the sky do not significantly affect the quality of calibration solutions.

RedCal Degeneracies

Redundant Calibration chi-squared

$$\chi^2 = \sum_r \sum_{(i,j)\in r} \frac{\left|d_{ij} - g_i g_j^* V_r\right|^2}{\sigma_{ij}^2}$$

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Overall Phase (unphysical)

$$g_j \rightarrow e^{i\phi} g_j$$

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Flux Scale

$$g_i \rightarrow A g_i$$

$$V_r \rightarrow A^{-2} V_r$$

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$$g_i \longrightarrow A g_i$$
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Phase Gradient (pointing error)

$$g_j \longrightarrow e^{i\nabla\Phi\cdot\boldsymbol{x}_j} g_j$$
$$V_r \longrightarrow e^{-i\nabla\Phi\cdot\boldsymbol{b}_r} V_r$$

RedCal Degeneracies

Redundant Calibration chi-squared

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The redcal degeneracies are “weakly broken” by CorrCal

Flux Scale

$$g_i \longrightarrow A g_i$$

$$V_r \longrightarrow A^{-2} V_r$$

Phase Gradient (pointing error)

$$g_j \longrightarrow e^{i\nabla\Phi\cdot \boldsymbol{x}_j} g_j$$

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CorrCal "Degeneracies"

Flux Scale

$$g_i \longrightarrow Ag_i$$

$$V_r \longrightarrow A^{-2}V_r$$

Flux scale is set by the power included in the calibration model

CorrCal "Degeneracies"

Flux Scale

$$g_i \rightarrow A g_i$$

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Flux scale is set by the power included in the calibration model

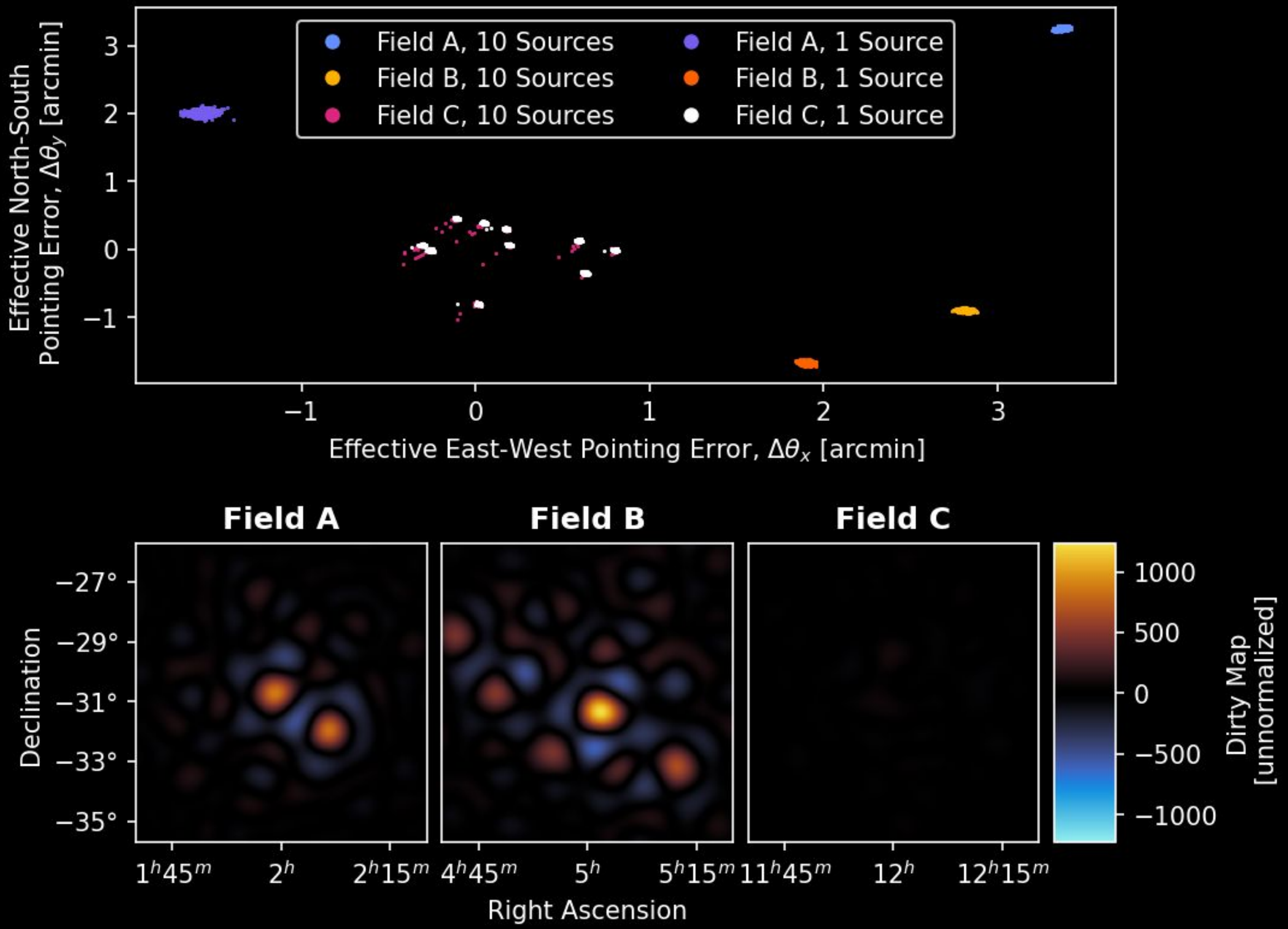
Phase Gradient (pointing error)

$$g_j \rightarrow e^{i\nabla\Phi\cdot\boldsymbol{x}_j} g_j$$

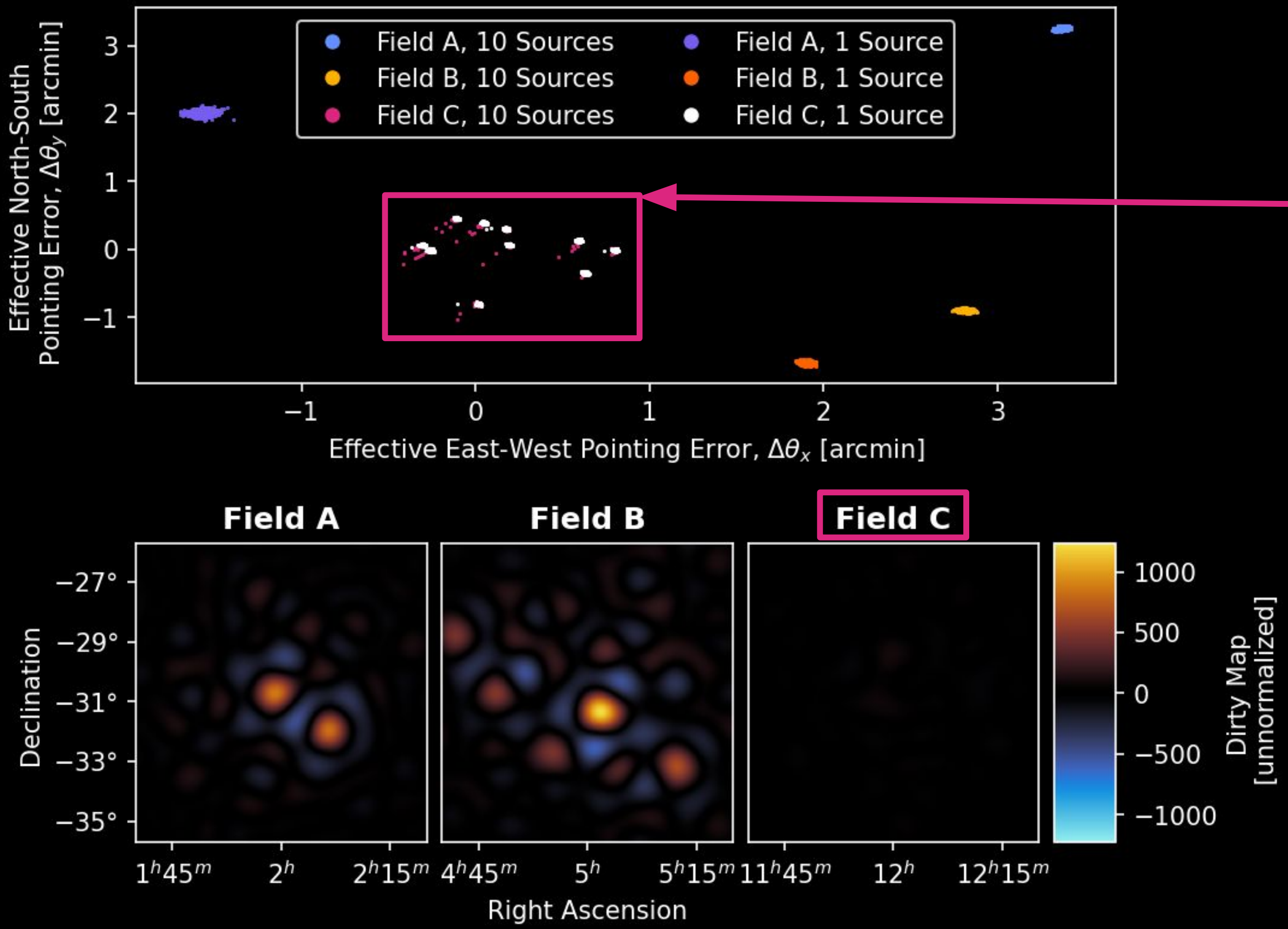
$$V_r \rightarrow e^{-i\nabla\Phi\cdot\boldsymbol{b}_r} V_r$$

Phase gradient is fixed to some (nonzero!) value by point sources used for calibration

Phase Gradients

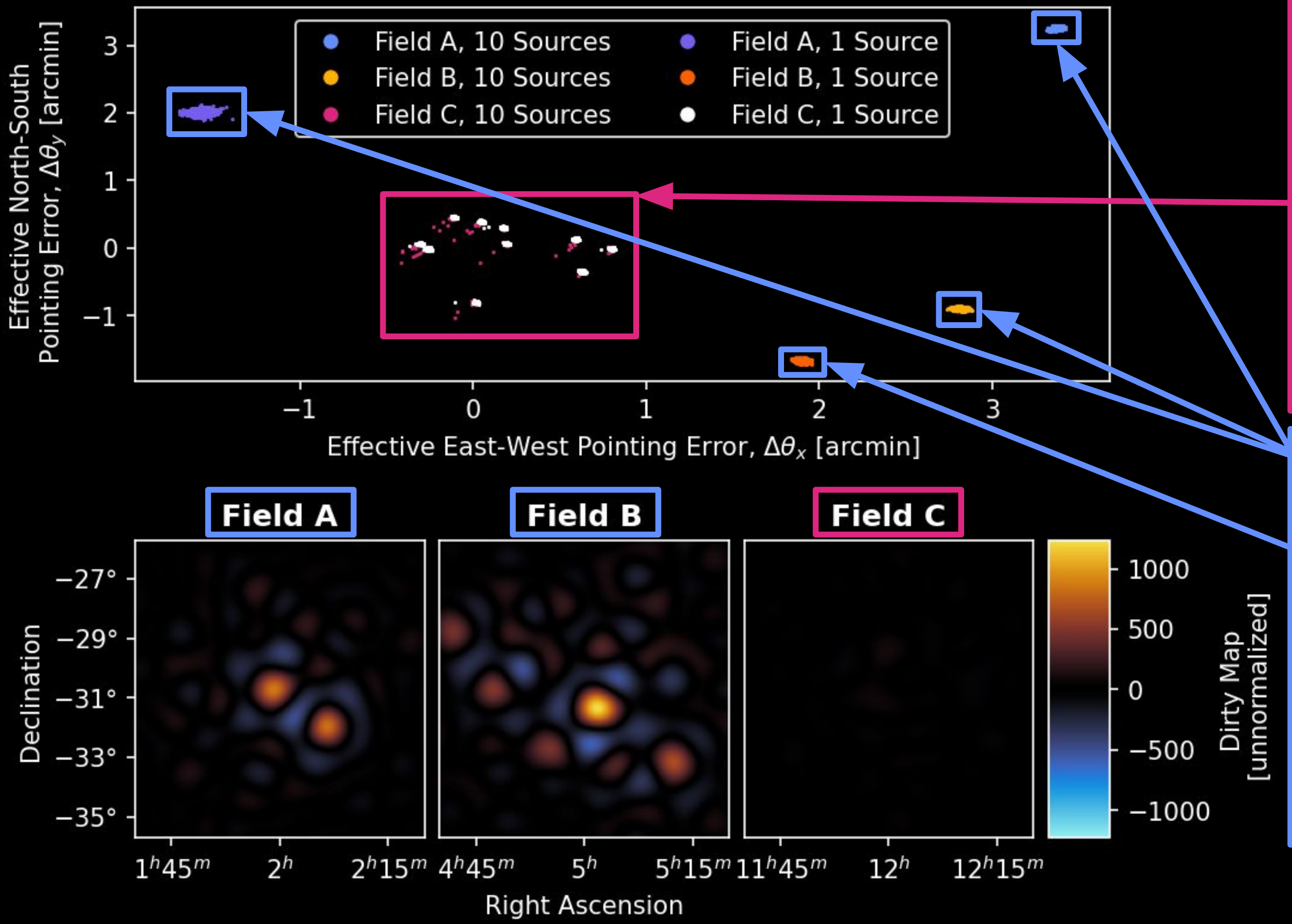


Phase Gradients



Without point sources available for calibration, the CorrCal likelihood cannot constrain the visibility phases.

Phase Gradients



Without point sources available for calibration, the CorrCal likelihood cannot constrain the visibility phases.

When point sources are available, calibrating with an incomplete catalog leaves a residual phase gradient.

Conclusions & Future Work

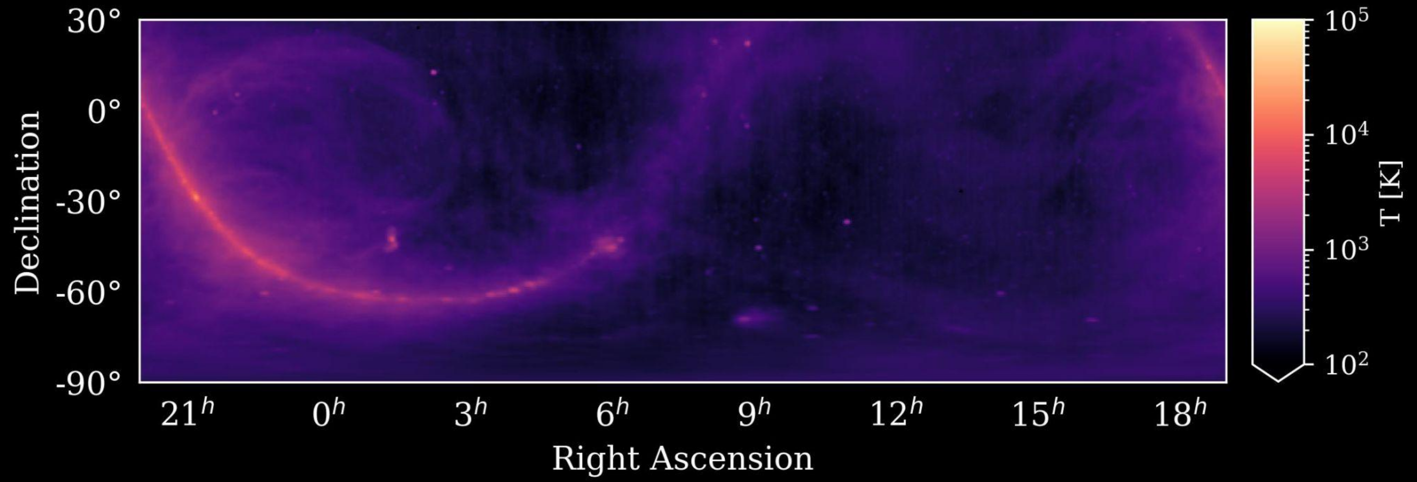
- The extreme dynamic range between foregrounds and the cosmological 21 cm signal places stringent constraints on calibration performance.
- CorrCal alleviates modeling limitations in traditional calibration through a covariance optimization approach to calibration.
- CorrCal is computationally efficient and robust to a variety of modeling errors.
    - However, incomplete point source models produce nonzero (but sub-synthesized beam) phase gradient errors.
- An extension to CorrCal that leverages spectral correlations is currently in progress.
- CorrCal will soon be tested on real data from CHORD and HERA.
- Further tests with more systematics (e.g., mutual coupling) are planned.

arXiv:2602.06109 github.com/r-pascua/corrcal

Backup Slides

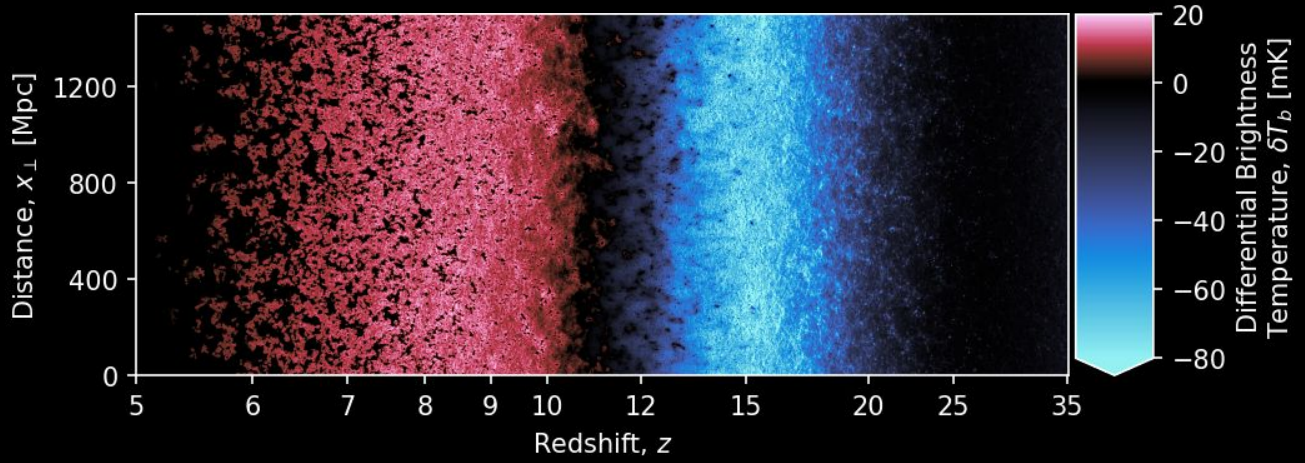
The Importance of Calibration

Foregrounds



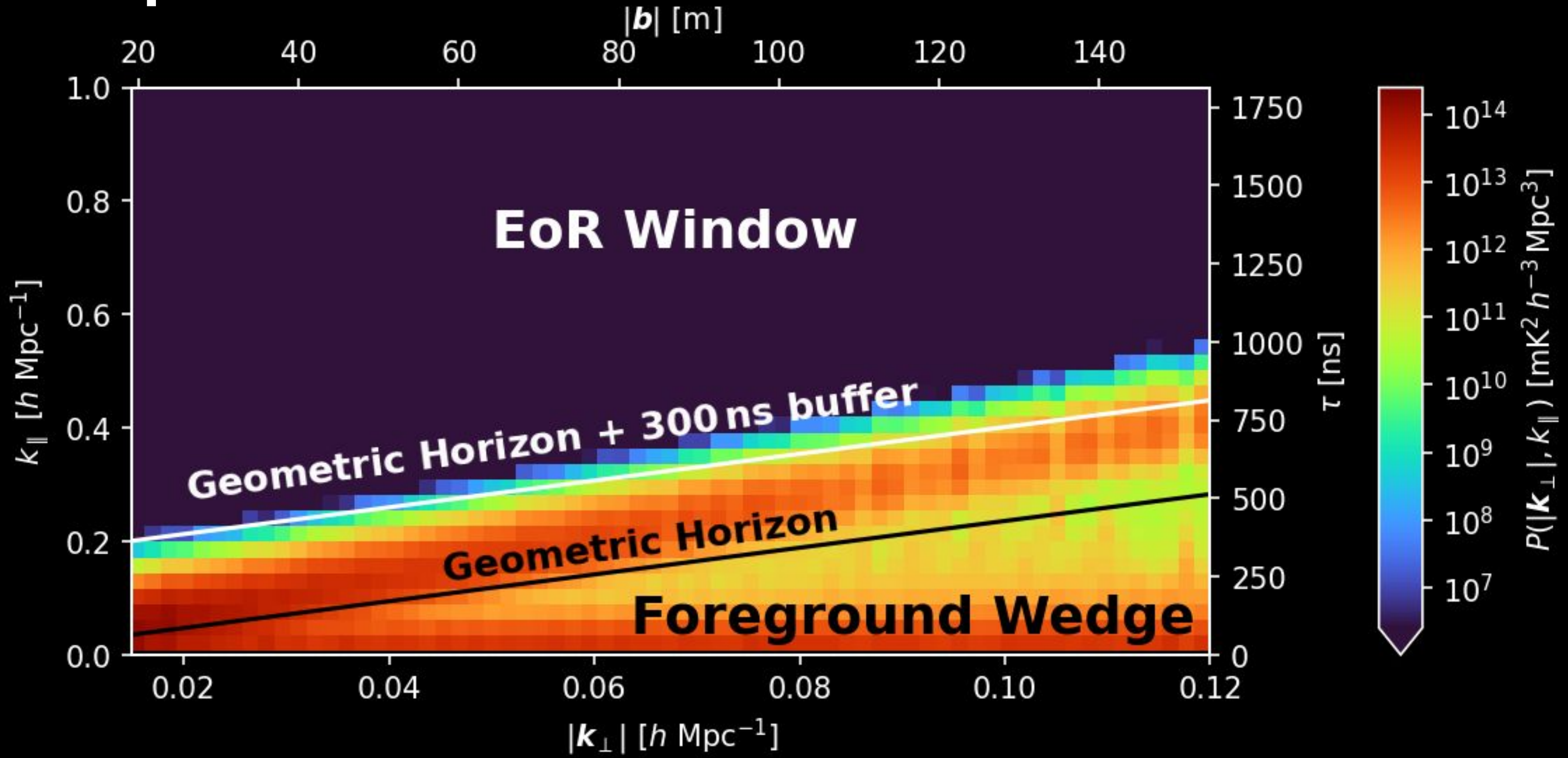
≥ 100 K

21-cm Signal



≲ 100 mK

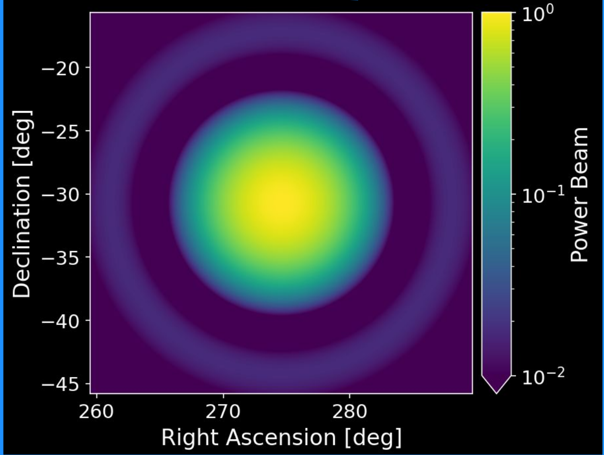
The Importance of Calibration

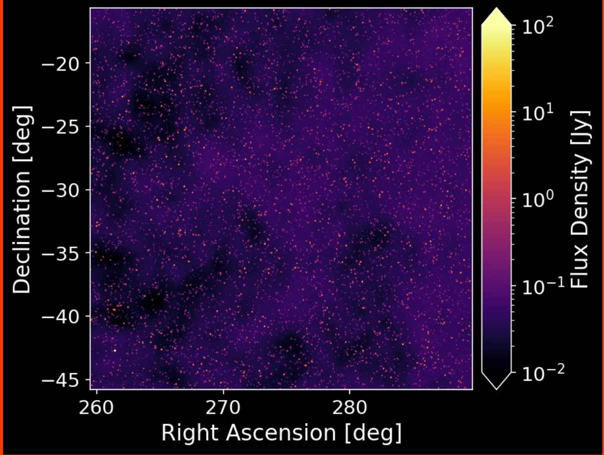


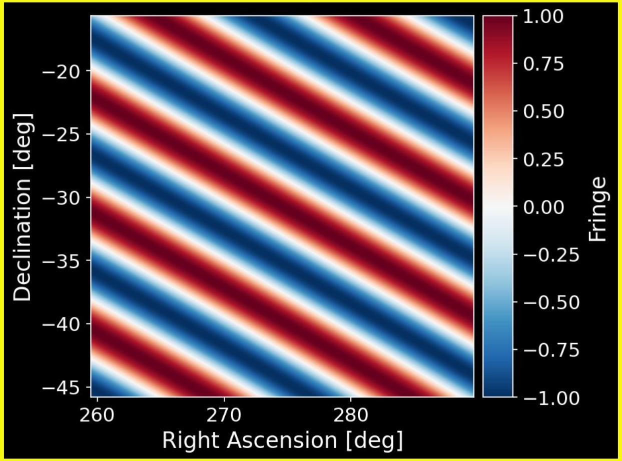
Spectrally structured calibration errors scatter foreground power into the EoR Window.

Interferometry Overview

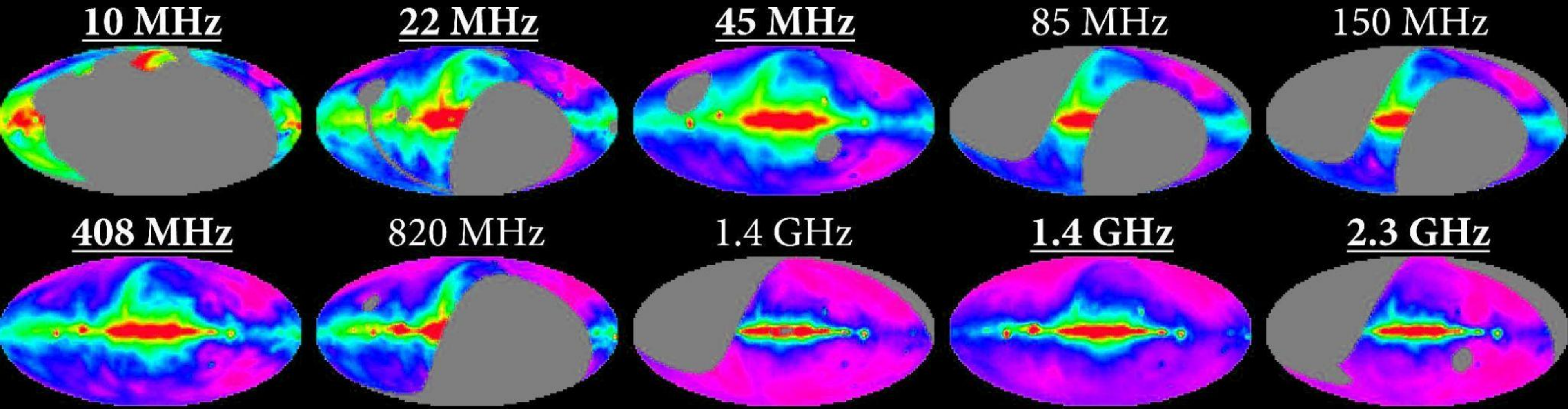
$$V(\nu, \mathbf{b}) = \int A_\nu(\hat{\mathbf{n}}) I_\nu(\hat{\mathbf{n}}) e^{-i2\pi\nu \mathbf{b}\cdot\hat{\mathbf{n}}/c} d\Omega$$







Trouble in the Sky



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