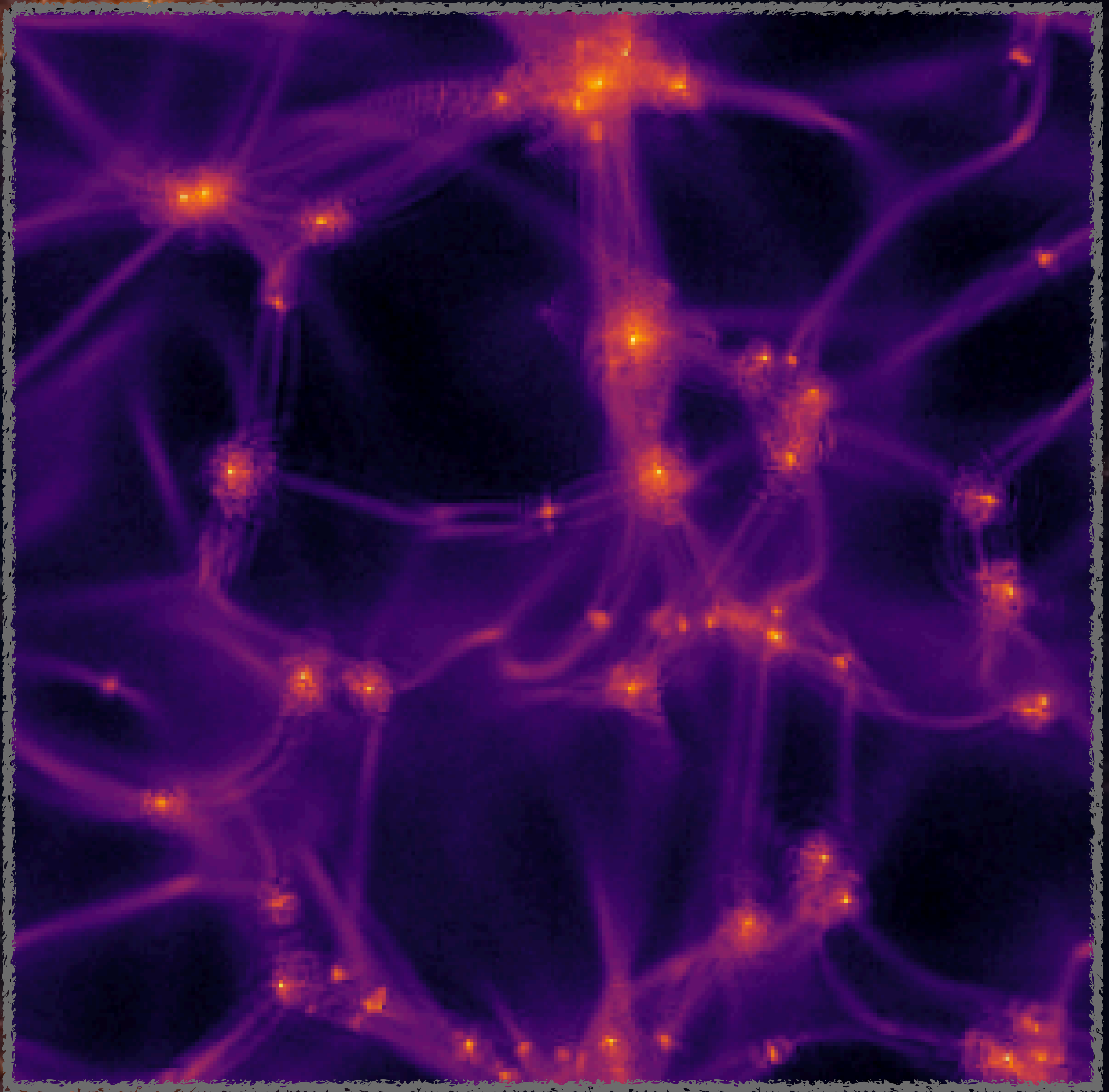
EPFL

Swiss National Science Foundation

Wave Dark Matter Simulations with Physics-Informed Neural Networks(PINNs)

Ashutosh K. Mishra[1] (PhD Student)
Advisor: Emma Tolley
26 January 2026

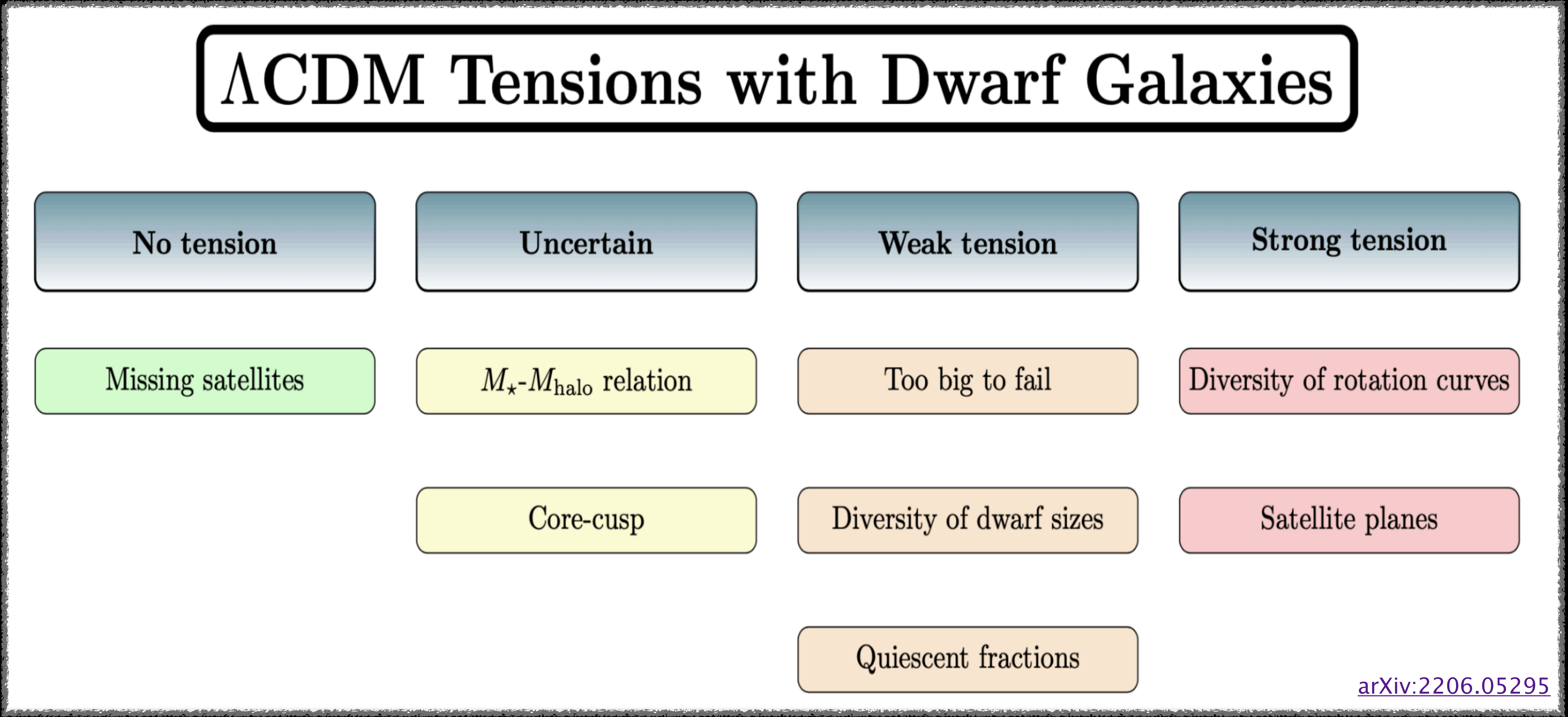
FDM Simulation



SKACH Winter Meeting

[1]Email: ashutosh.mishra@epfl.ch

Small Scale Challenges in CDM Model



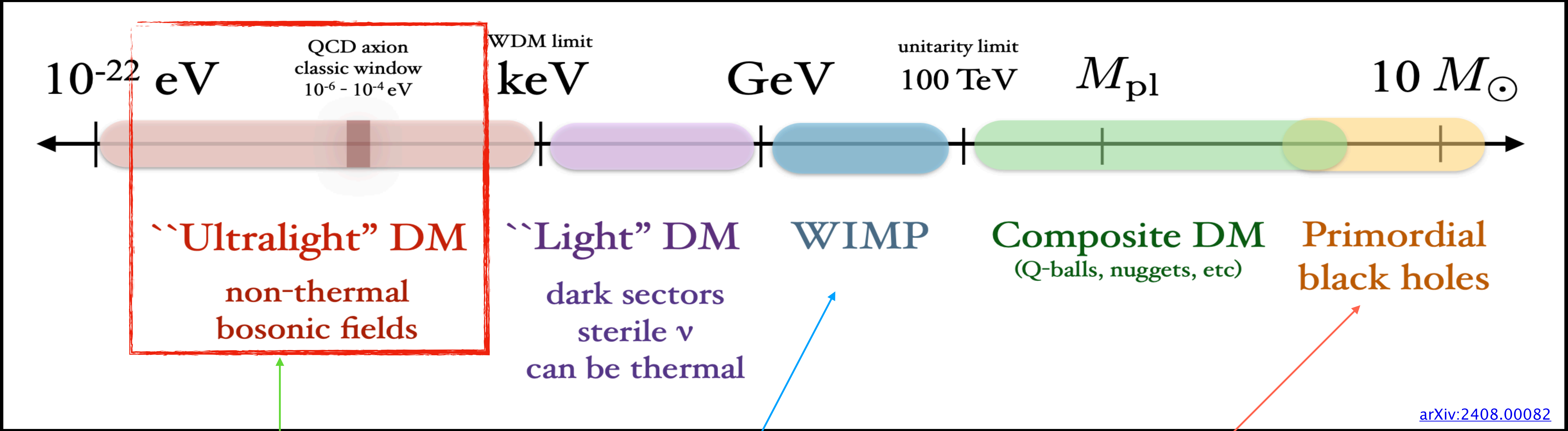
arXiv:2206.05295

Potential Problem: Absence of **Baryonic Processes** (Feedback, Formation) and/or Nature of **DM**!

Alternative Dark Matter Models

Warm Dark Matter (WDM): favored mass range in tension with Lyα observation & abundance of high-z galaxies

Self-interacting Dark Matter (SIDM): Needs fine-tuned cross-sections & struggles to explain full range of observations



Still a viable window Under observational constraints!

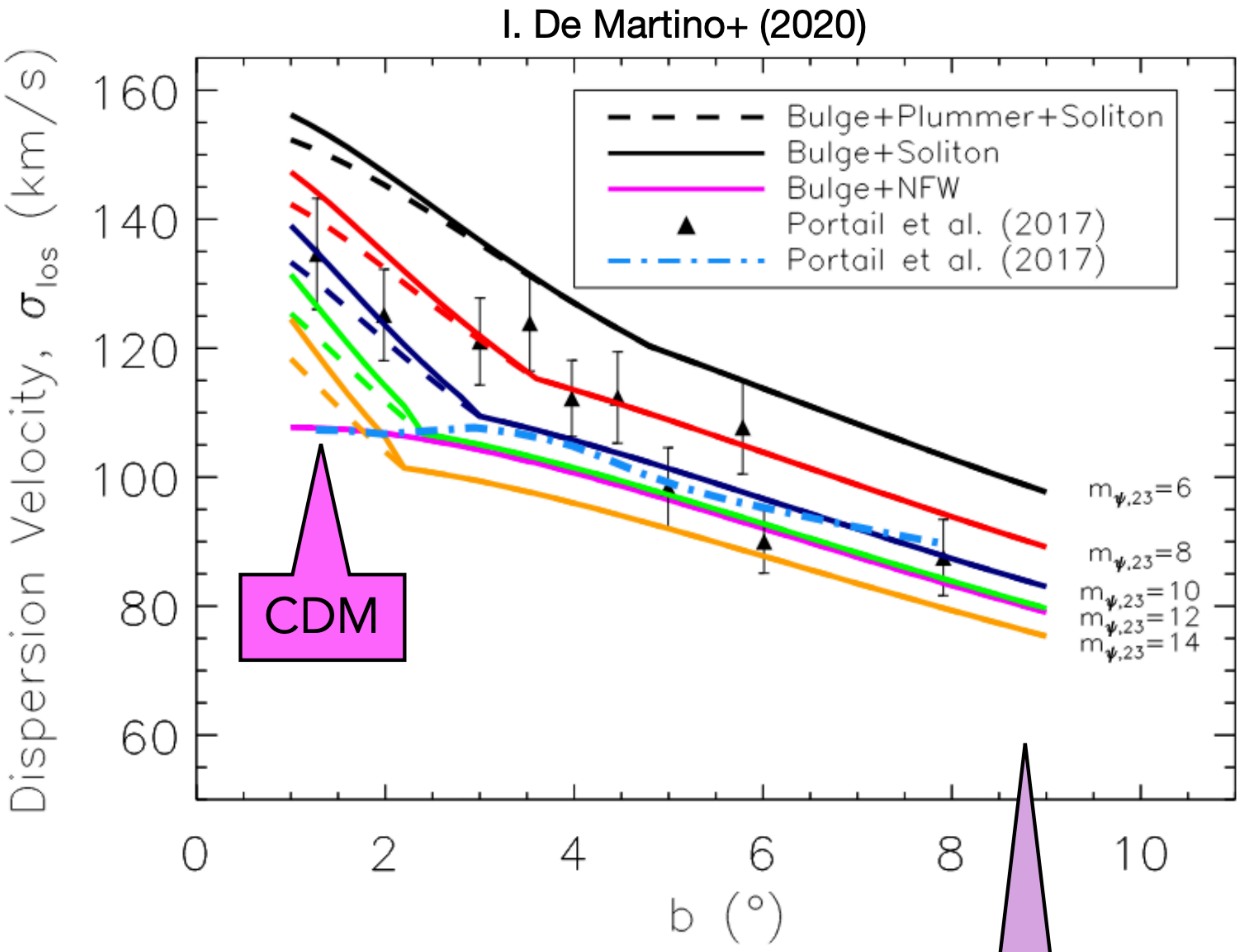
No Empirical Detection so far!

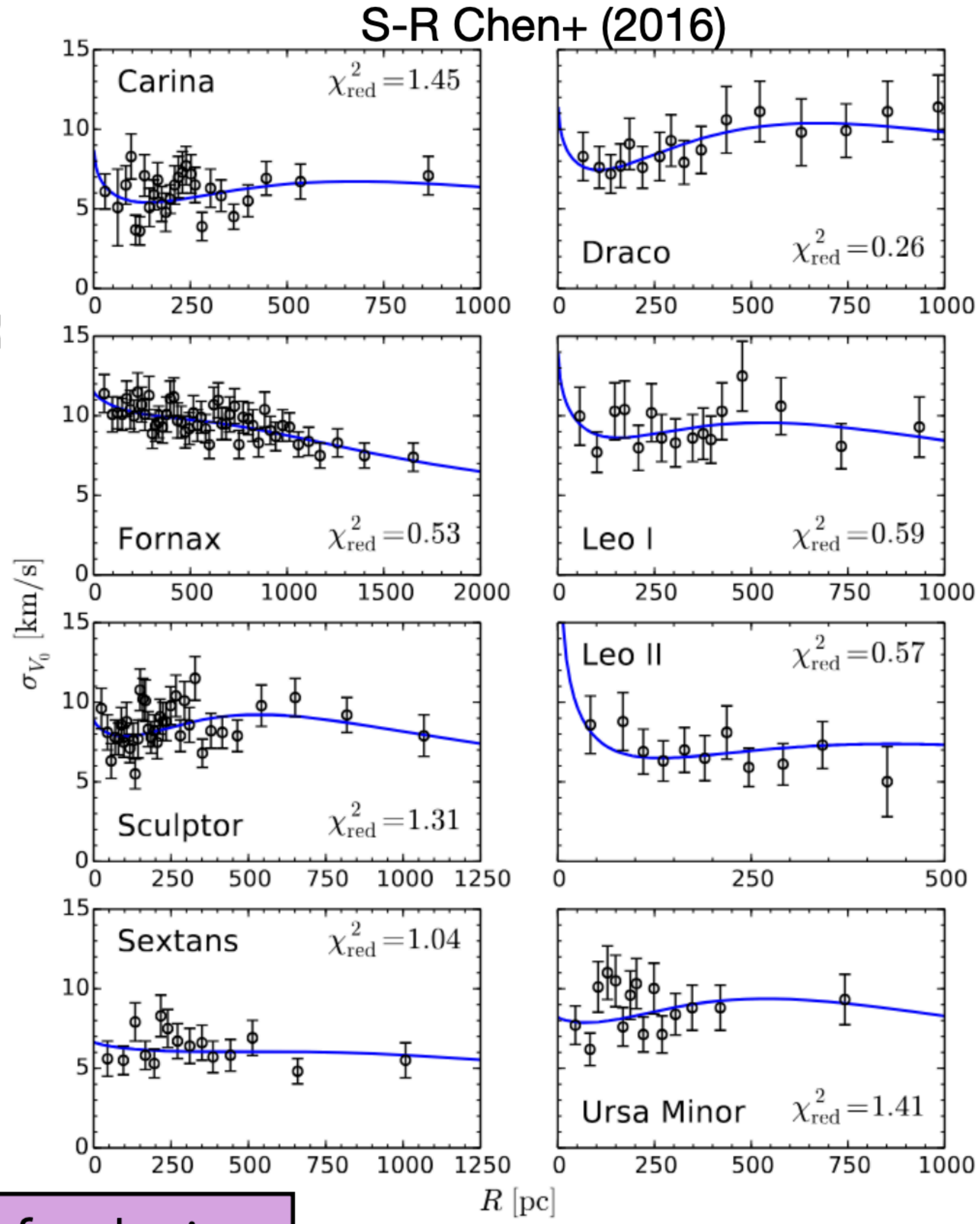
Strong constraints with microlensing observations!

Fuzzy Dark Matter

(F(C)DM, BECDM, ULDM, ELBDM, (ultra-light) axion (-like) DM (ULA, ALP))

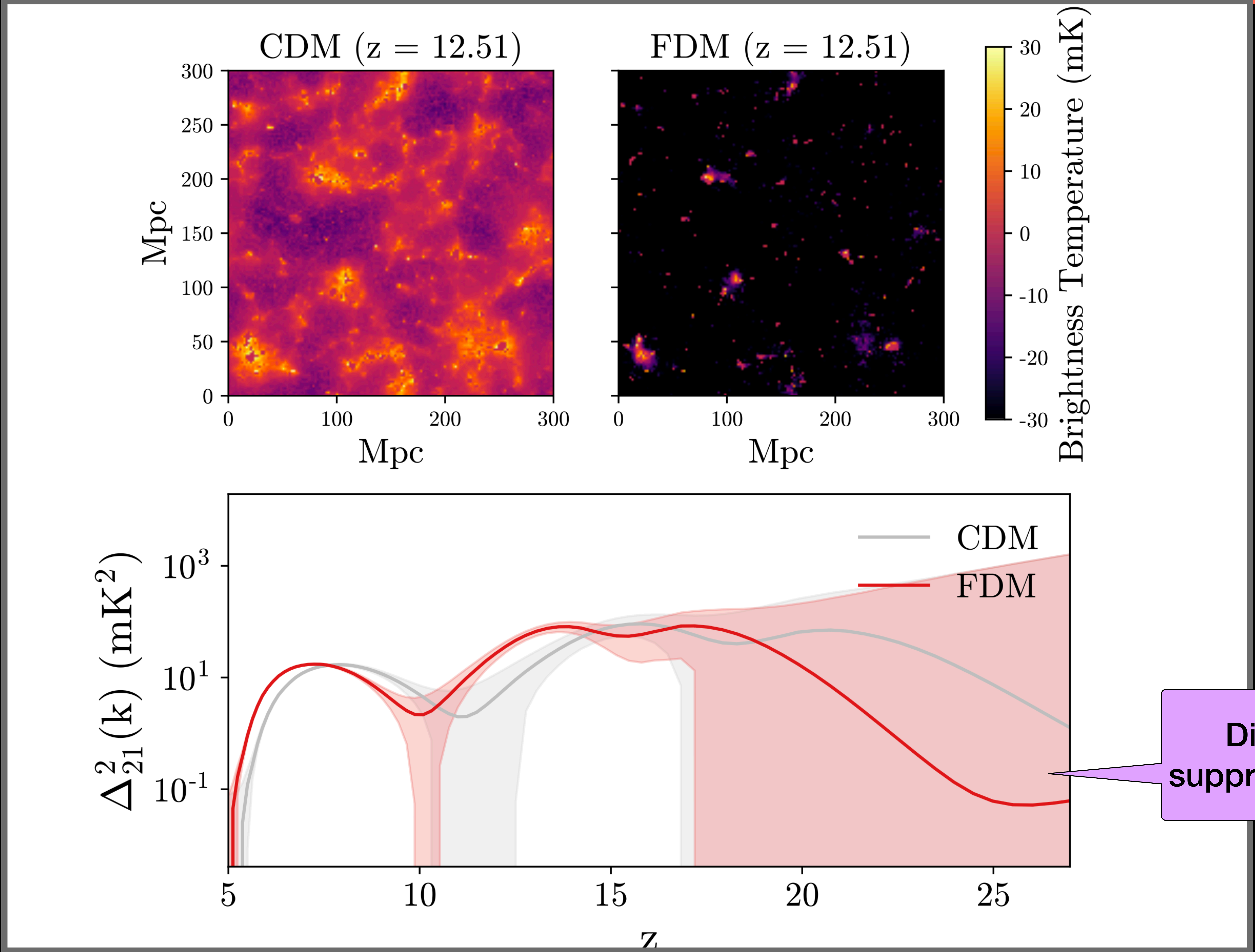
- ✦ ***Extremely light scalar*** **particle ($m \sim 10^{-20}$ - 10^{-22} eV)**
- ✦ **Non-thermally produced (thus not ultra-hot)**
- ✦ **Clumps to form Bose-Einstein Condensate (BEC)!**
- ✦ **Quantum effects counteract gravity at small scales**
- ✦ **Tiny mass**
  - — **large de-Broglie wavelength ($\sim 1/m$)**
  - — **macroscopic quantum effects at kpc scales**





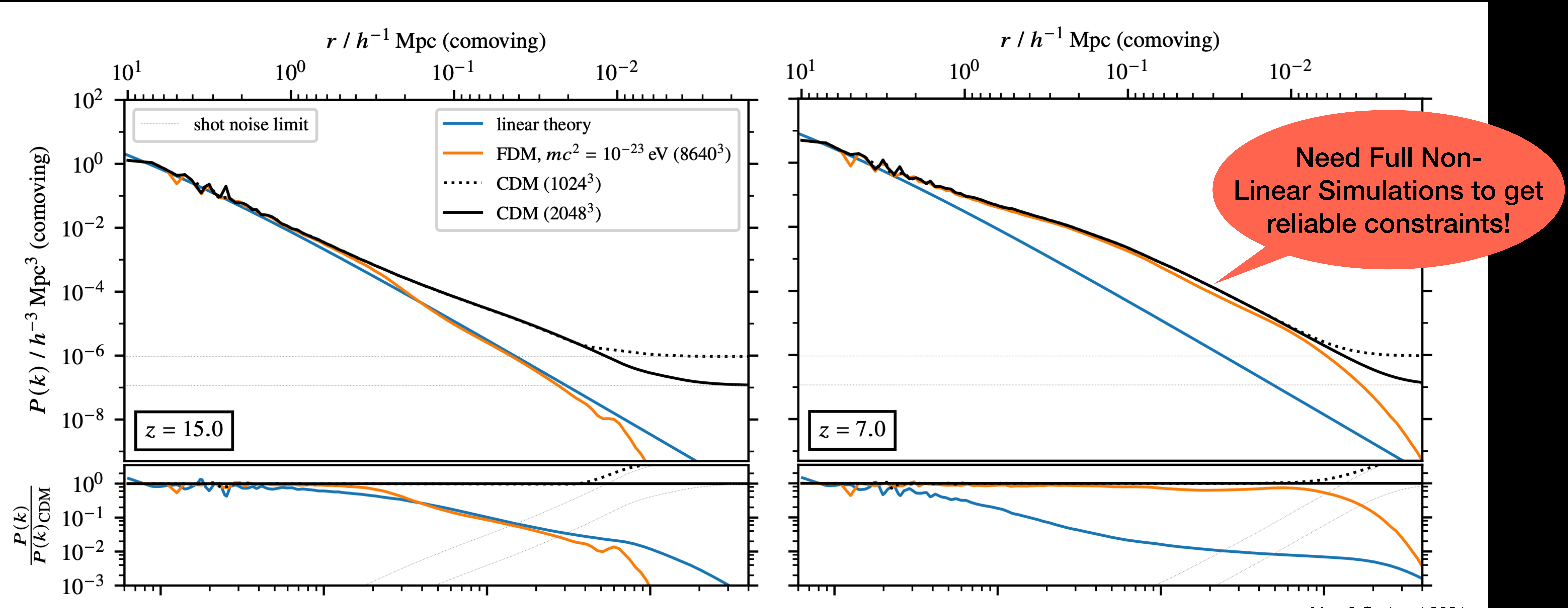
FDM can explain halo profiles of our galaxy & many dwarf galaxies

FDM potential signature in 21cm power spectrum



Distinct power suppression at high z!

FDM mass constraints based on P_{Lin} 🤔



May & Springel 2021

FDM Limit ruled out !? (From Ultra-dwarf galaxies)

★Result:
$m > 8 \times 10^{-18} eV \quad (95\,\%\, C.L)$

★Bound is conservative:
perturbative heating underestimates full SP result -> true limit likely stronger!!

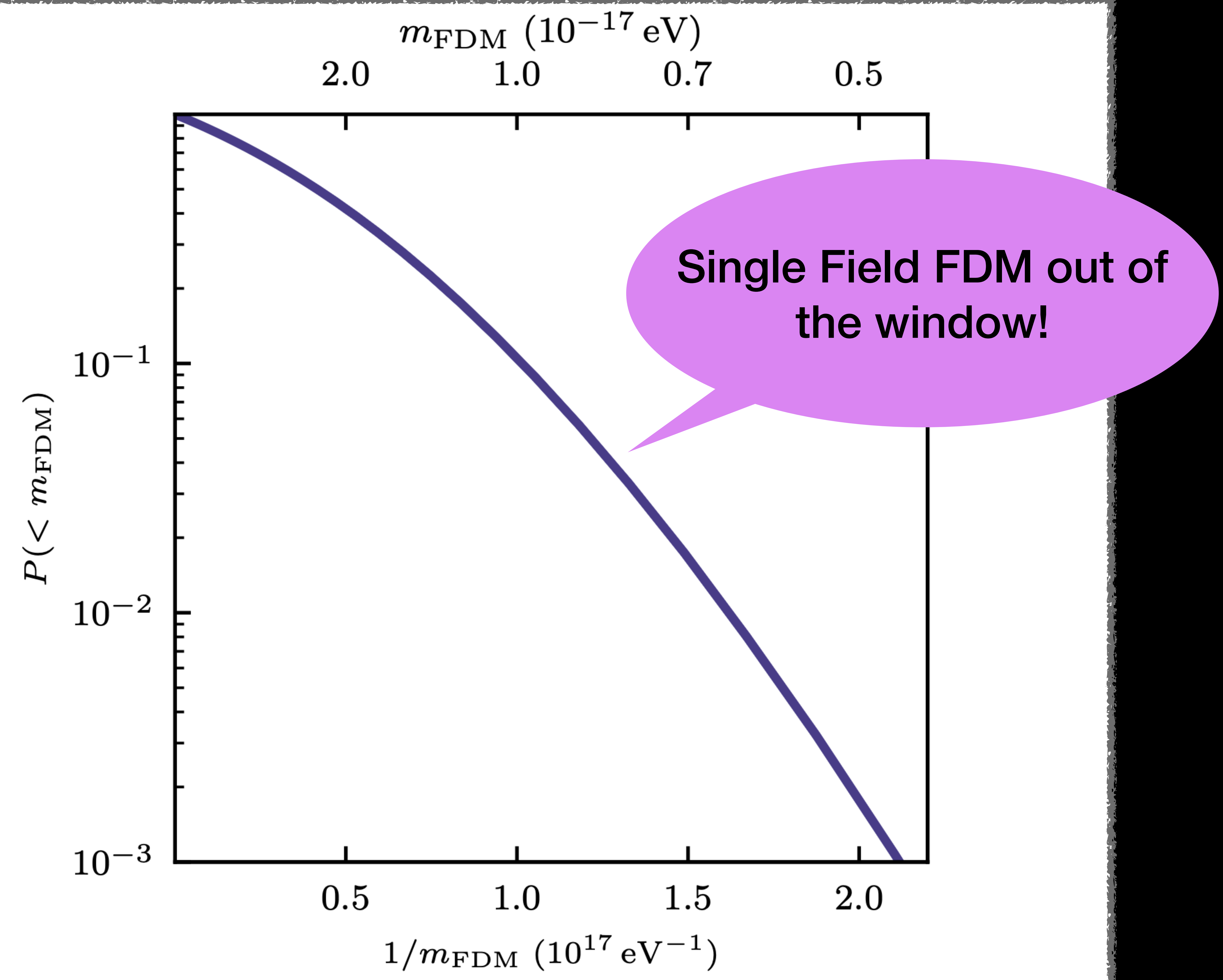
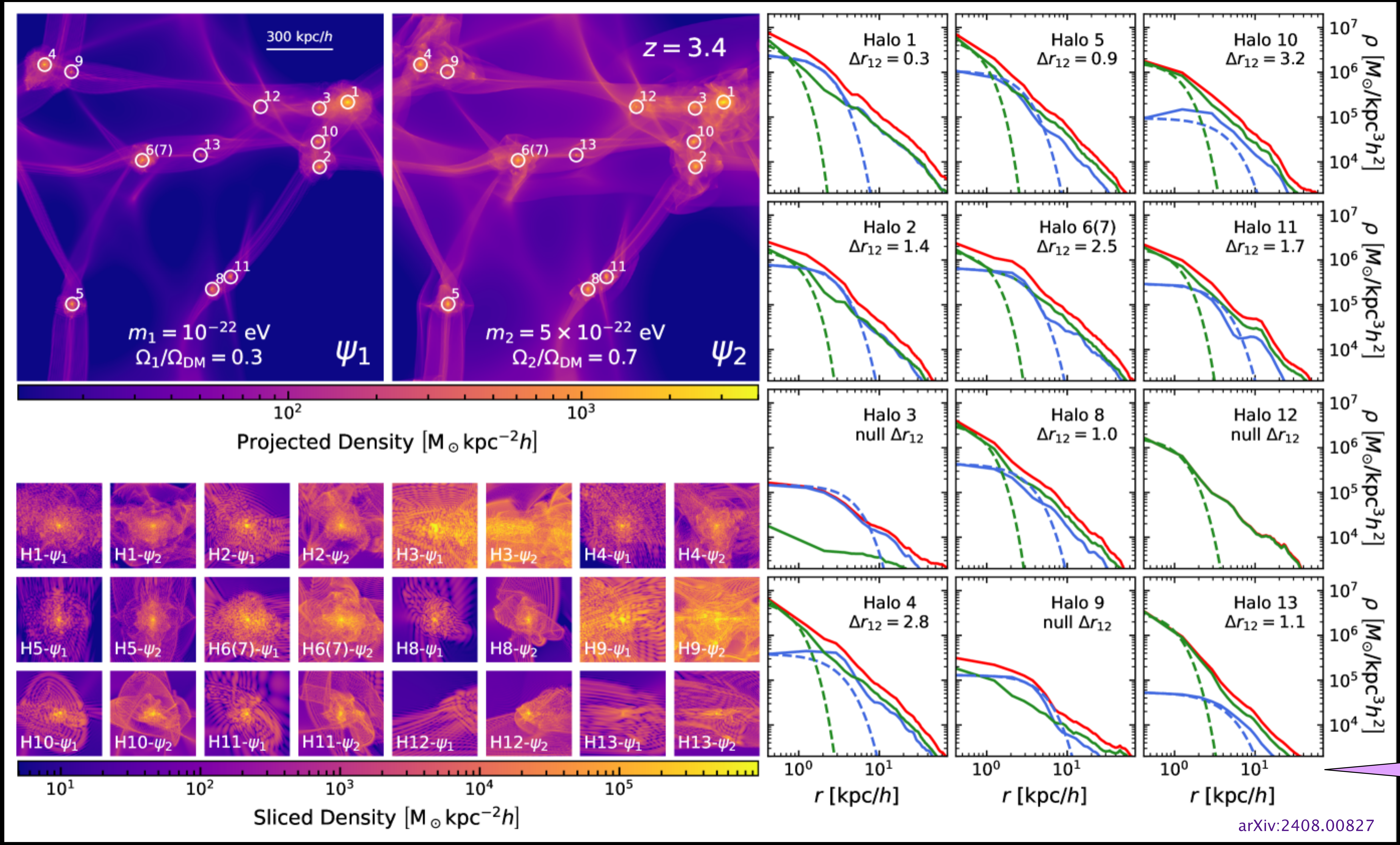


FIG. 6. Cumulative posterior on DM mass m from Ursa Major III, using $r_{1/2} = (4.0 \pm 1.3)$ pc in 3D, and $\sigma_v = (1.9 \pm 1.4)$ km s^{-1} [14]. We find that $m > 8 \times 10^{-18}$ eV at 95 % confidence.

May et al. 2025 (arXiv:2509.02781)

Diversity of Halos in Two Field FDM cosmology 🧐



2-Field can get rid of small scale tensions!

Governing Equations

A. Wave Formalism (Schrödinger-Poisson Equations)

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi$$

$$\nabla^2 V = 4\pi G m(|\psi|^2 - |\psi_0|^2)$$

Mean Field Interpretation: Single Macroscopic WF of BEC

B. Madelung Formalism (Fluid Dynamics Representation)

$$\partial_t\rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

$$\partial_t\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{m}\vec{\nabla}\left(V - \underbrace{\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}}_{=\,Q}\right)$$

$$\nabla^2 V = 4\pi G m(\rho - \rho_0)$$

$$\psi = \sqrt{\frac{\rho}{m}}e^{iS}$$

$$\rho = m|\psi|^2$$

$$v = \frac{\hbar}{m}\nabla S$$

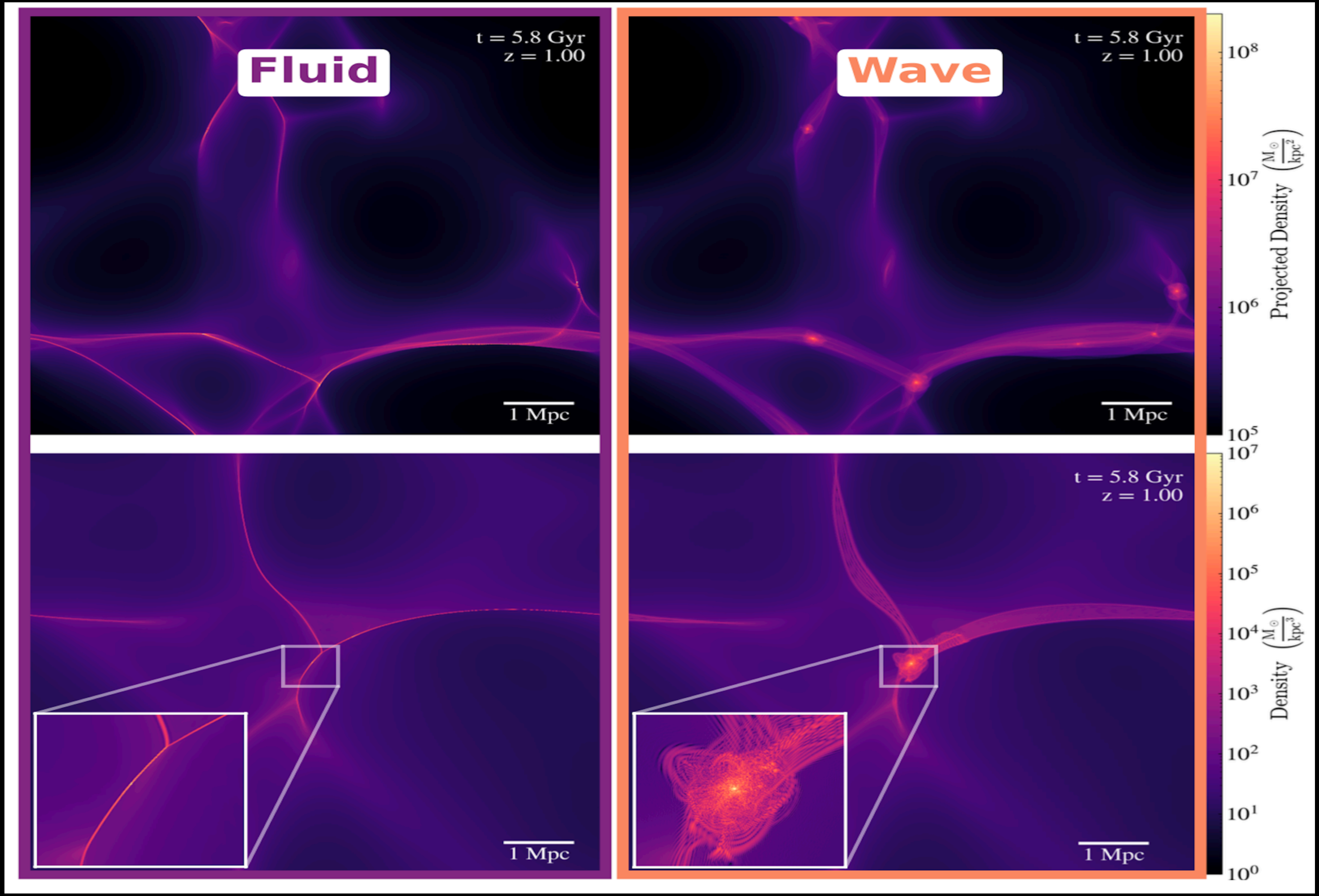
"Quantum Pressure"

Q ill-defined at $\rho = 0$!

Fuzzy Dark Matter Simulations

Fluid Solver unable to capture interference effects!

Stick to SP-Equations for evolution!



A. Kunkel et.al. 2024

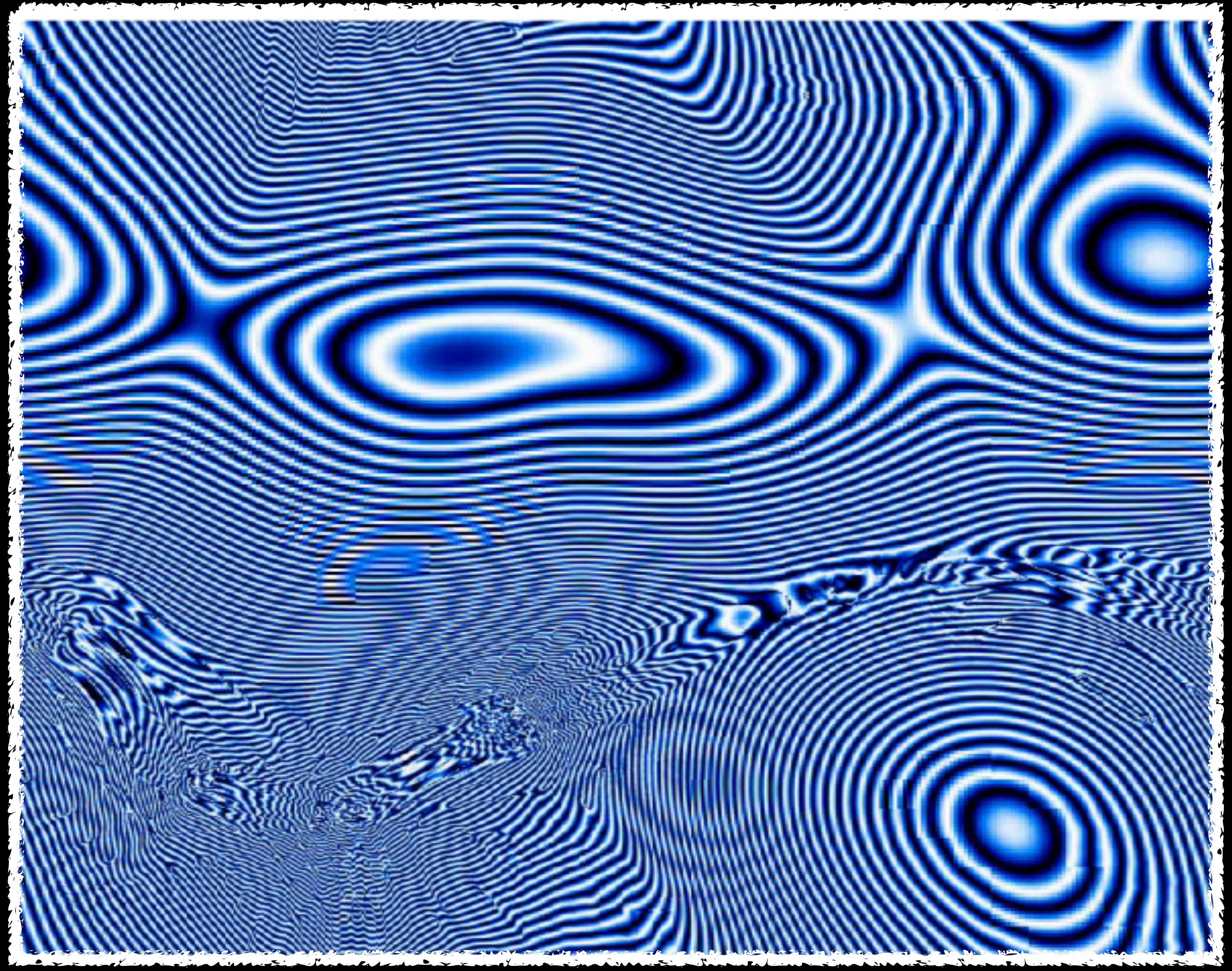
Challenges in Simulating Fuzzy Dark Matter

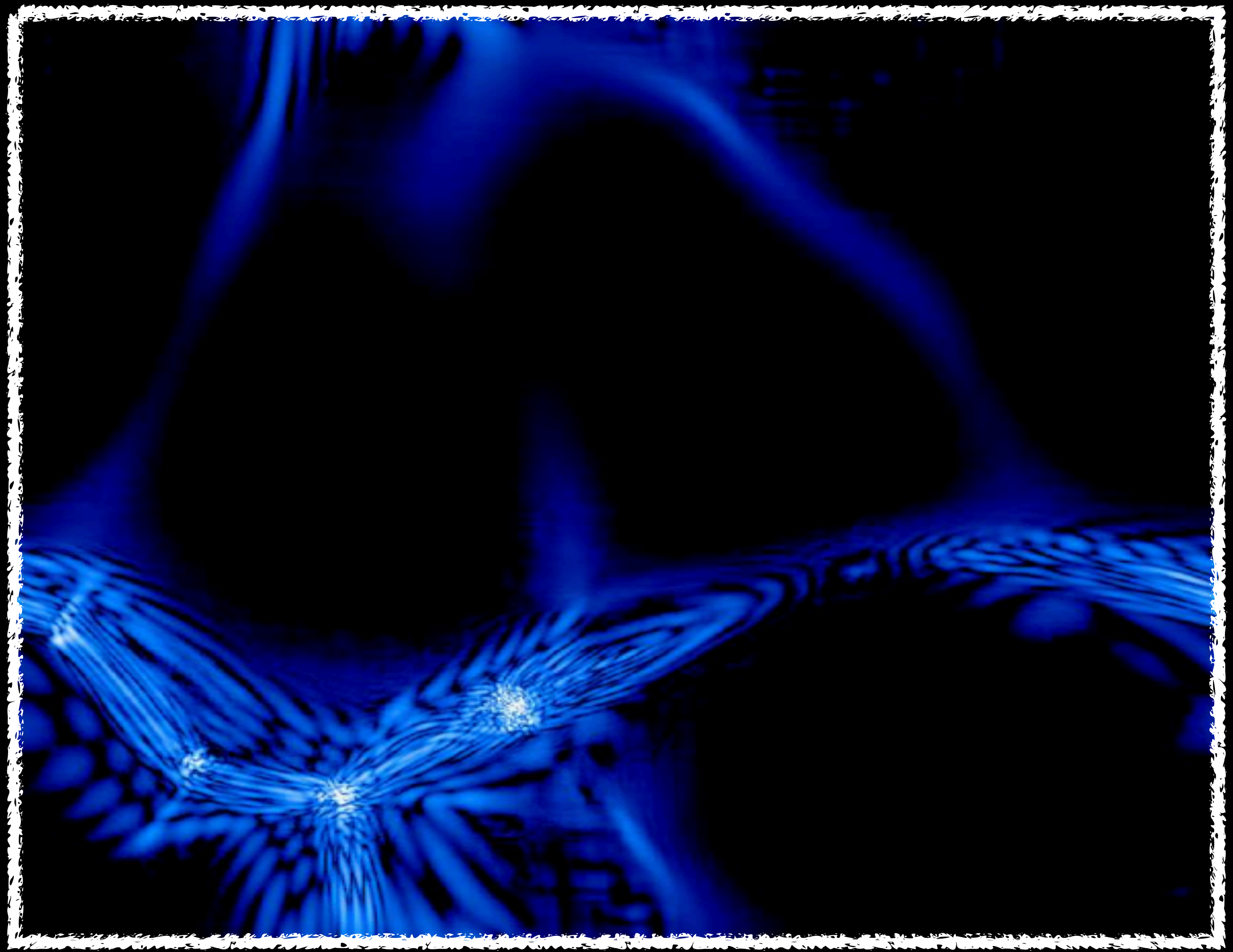
Both Mpc-scale and kpc-scales need to be
Resolved for accurate evolution

Time step scaling: $\Delta t \sim \Delta x^2$

Hydrodynamical codes are used in N-body
Simulation (but Fluid Formulation
For FDM evolution?)

So far sims. restricted to small box sizes of 10Mpc/h





Schive, Chieuh, & Broadhurst (2014)

Challenges in Simulating Fuzzy Dark Matter

Schroedinger-Poisson Equations

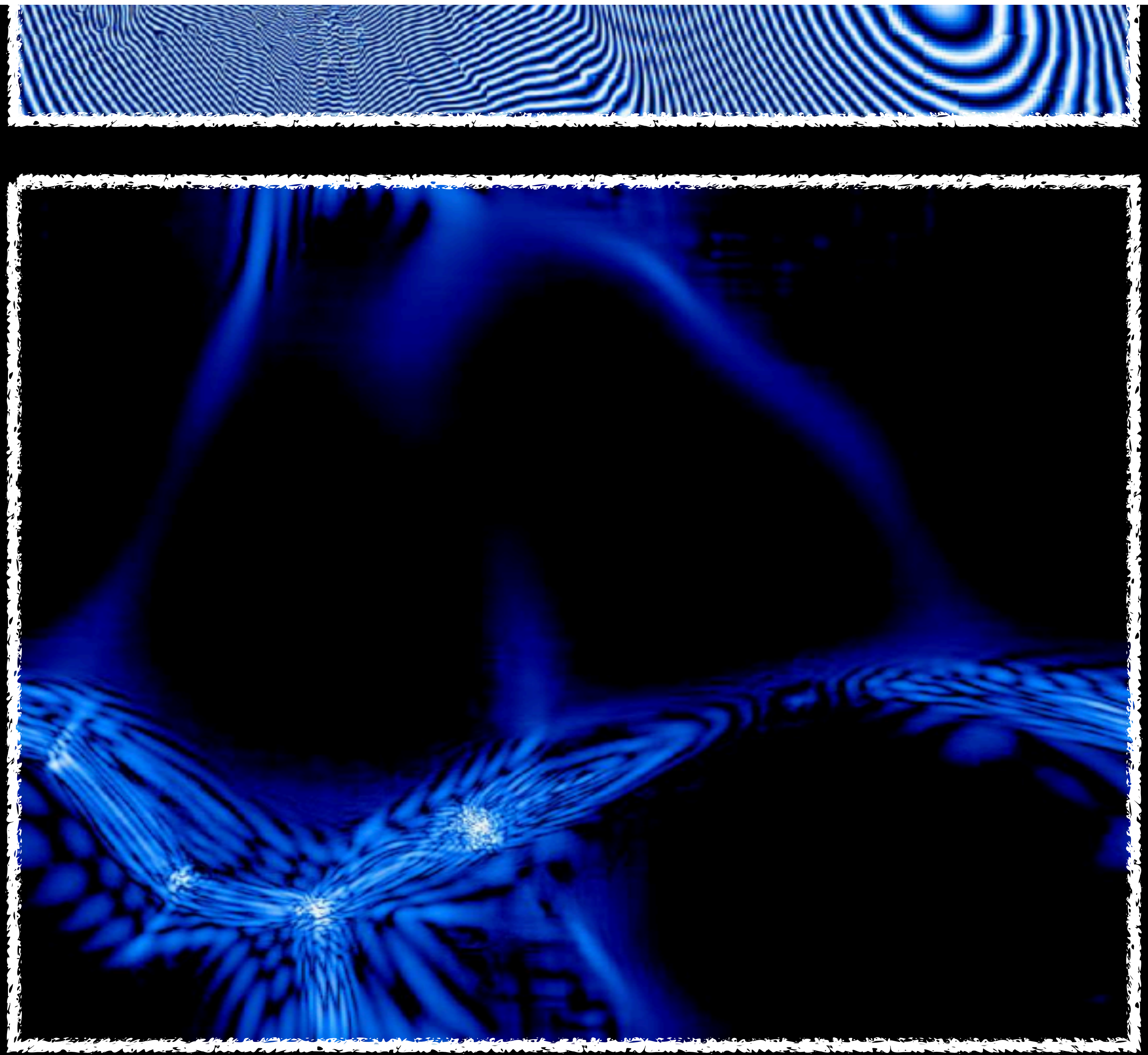
$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi,$$

$$\nabla^2 V = 4\pi G(\rho - \bar{\rho}),$$

Can we solve this with machine learning?

Hydrodynamical codes are used in N-body Simulation (but Fluid Formulation For FDM evolution?)

So far sims. restricted to small box sizes of 10Mpc/h



Schive, Chieuh, & Broadhurst (2014)

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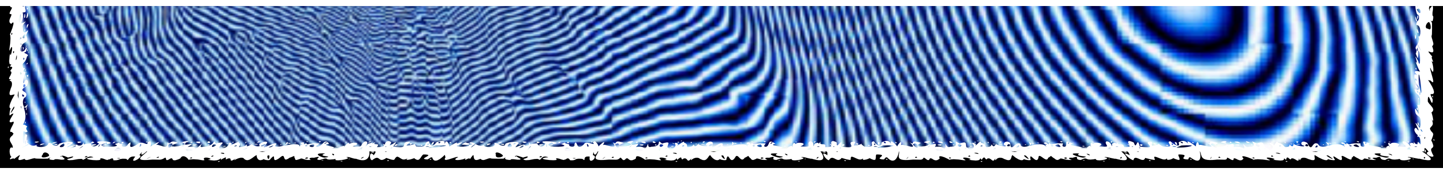


Neural networks are universal function approximators

Theorem (*Cybenko, 1989*)

Let σ be any continuous sigmoidal function. Then, the finite sums of the form

$$g(x) = \sum_{j=1}^{N} w_j^2 \sigma((w_j^1)^T x + b_j^1)$$

are dense in $C(I_d)$.

Schive, Chieuh, & Broadhurst (2014)

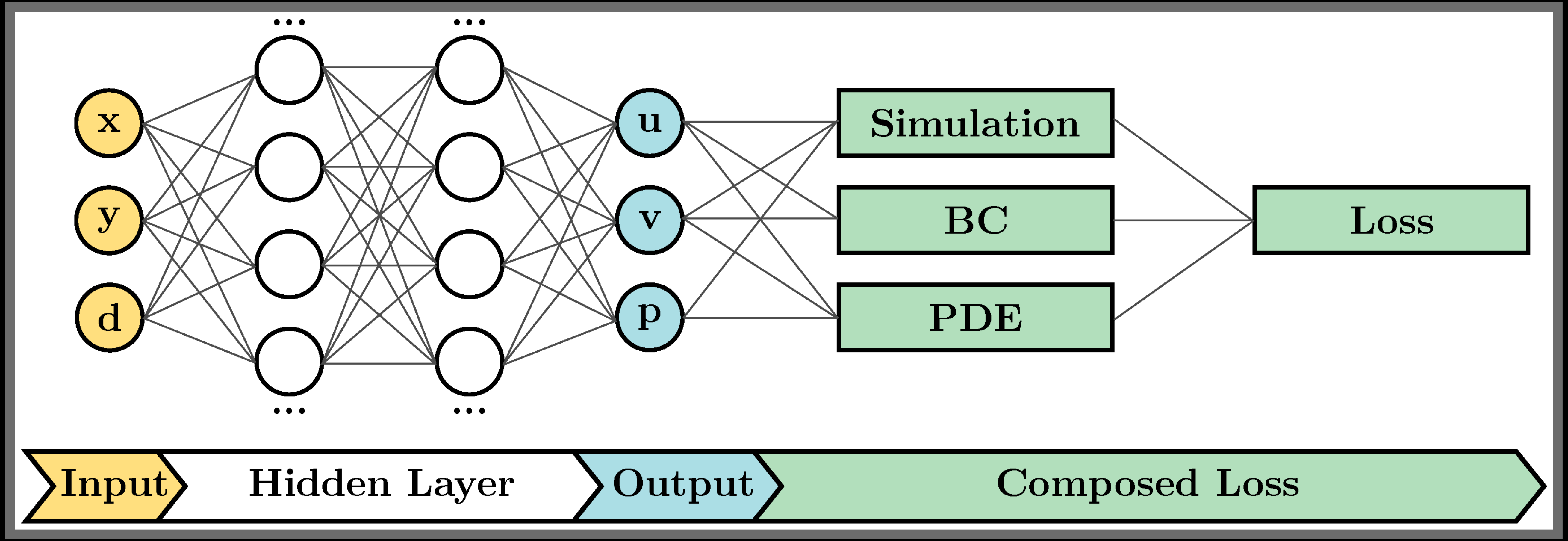
Physics Informed Neural Networks

General Framework:

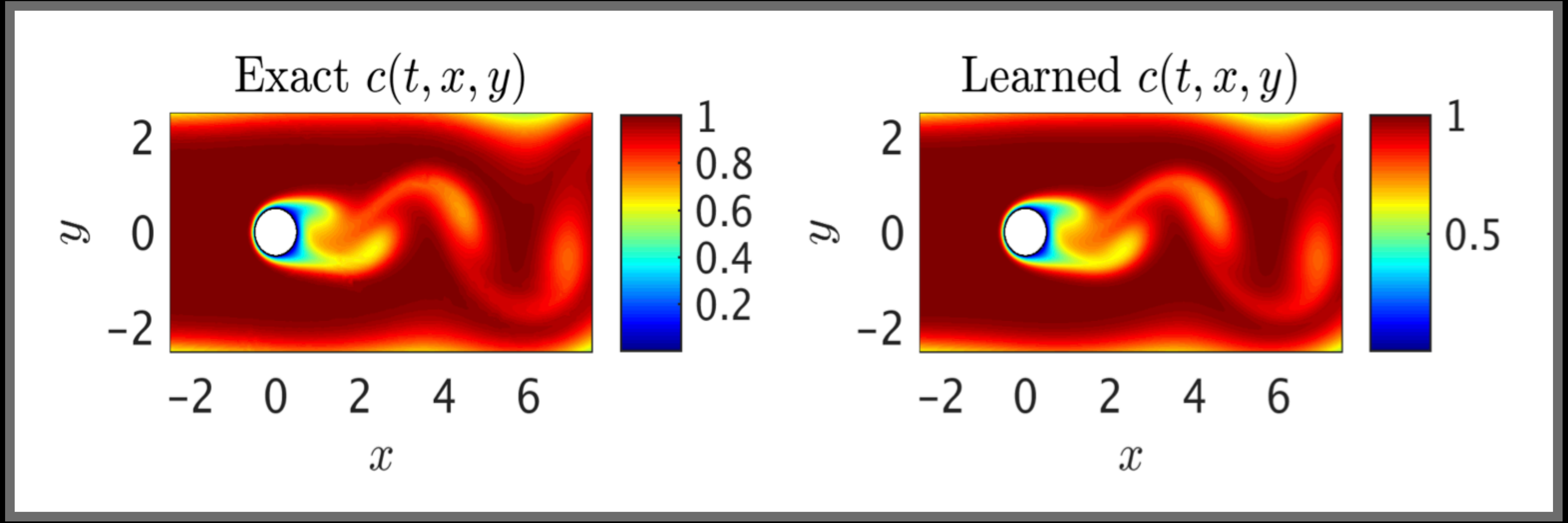
$$\mathscr{D}[NN(X,\theta);\lambda] = f(X), \quad X \in \Omega$$
$$\mathscr{B}[NN(X,\theta);] = g(X) \quad X \in \partial\Omega$$

Custom Loss Function: with PDE and boundary conditions as additional constraints

Pretty Successful in Fluid and Climate Simulations!



Adapted from F. Pioch et.al.2023



Raissi, Yazdani, Karinadakis 2020

Outline of the approaches

01 **Coordinate based approach**

- Directly solve the Schrödinger–Poisson (SP) equations using Physics-Informed Neural Networks (PINNs).

02 **Generative Modeling**

- Learn the mapping from Cold Dark Matter (CDM) to Fuzzy Dark Matter (FDM) cosmological simulations at a fixed redshift.

03 **Physics-Informed Generative Modeling**

- Interpolate and simulate FDM cosmological boxes across multiple redshifts using physics-constrained generative models.

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03 **Physics-Informed Generative Modeling**

- Interpolate and simulate FDM cosmological boxes across multiple redshifts using physics-constrained generative models.

Schrödinger-Poisson Equations used

$\lambda = \frac{\hbar}{m} \implies$

$$i\frac{\partial}{\partial t}\Psi(\boldsymbol{x}, t) = \left(-\frac{\lambda}{2}\nabla^2 + \frac{1}{\lambda}V[\Psi(\boldsymbol{x}, t)]\right)\Psi(\boldsymbol{x}, t)$$

$$\nabla^2 V[\Psi(\boldsymbol{x}, t)] = (|\Psi(\boldsymbol{x}, t)|^2 - 1)$$

$\frac{1}{\lambda}$: the strength of potential

$\lambda \to 0$, Gravitational Potential Term is dominant in the SP Equations!

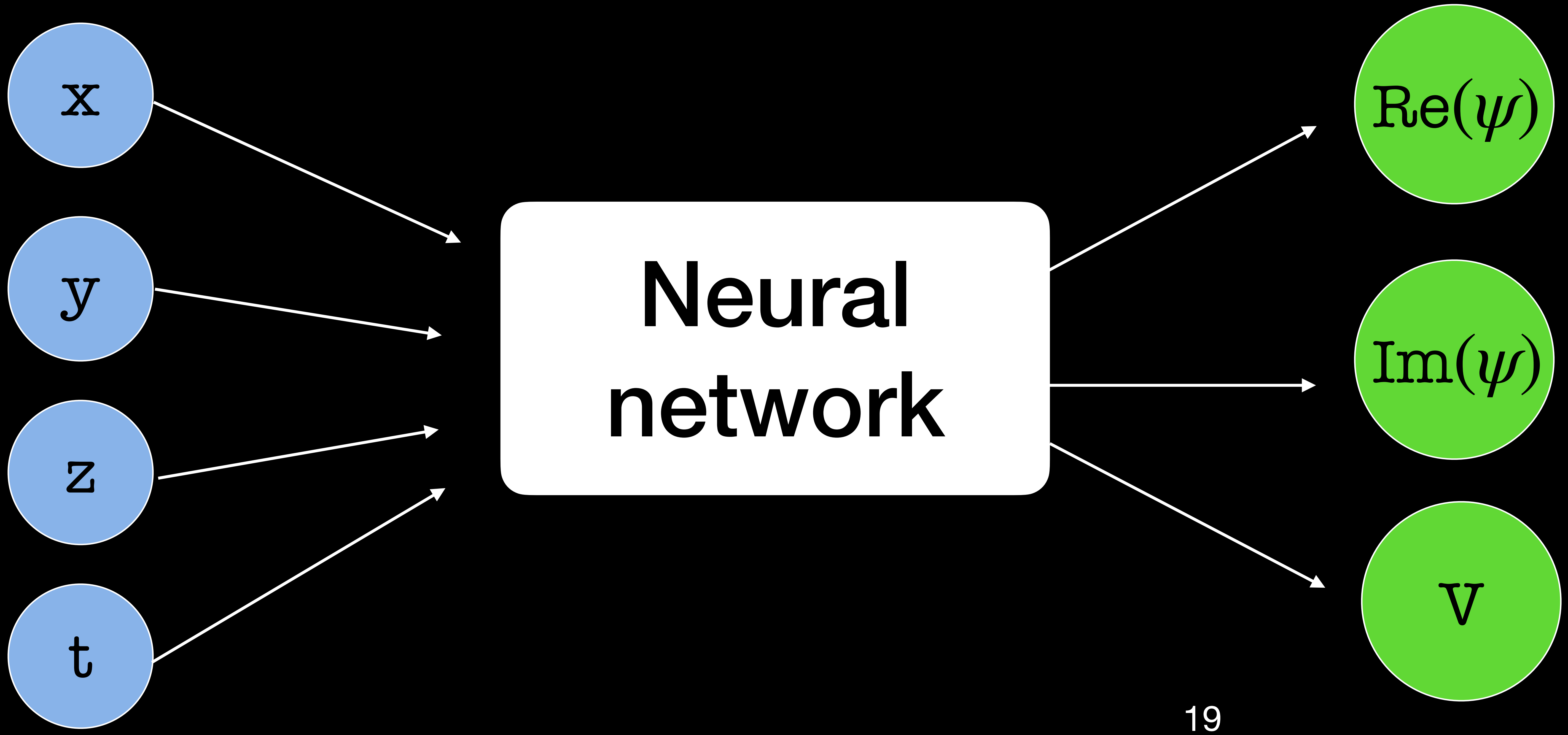
$\lambda \to \infty$, Gravitational Potential Term vanishes, Free Schrodinger Equation representing diffusion!

$\lambda = 1$ throughout this work!

Schrödinger-Poisson Informed Neural Networks (SPINN)

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi$$

$$\nabla^2 V = 4\pi Gm(|\psi|^2 - |\psi_0|^2)$$



Schrödinger-Poisson Informed Neural Networks (SPINN)

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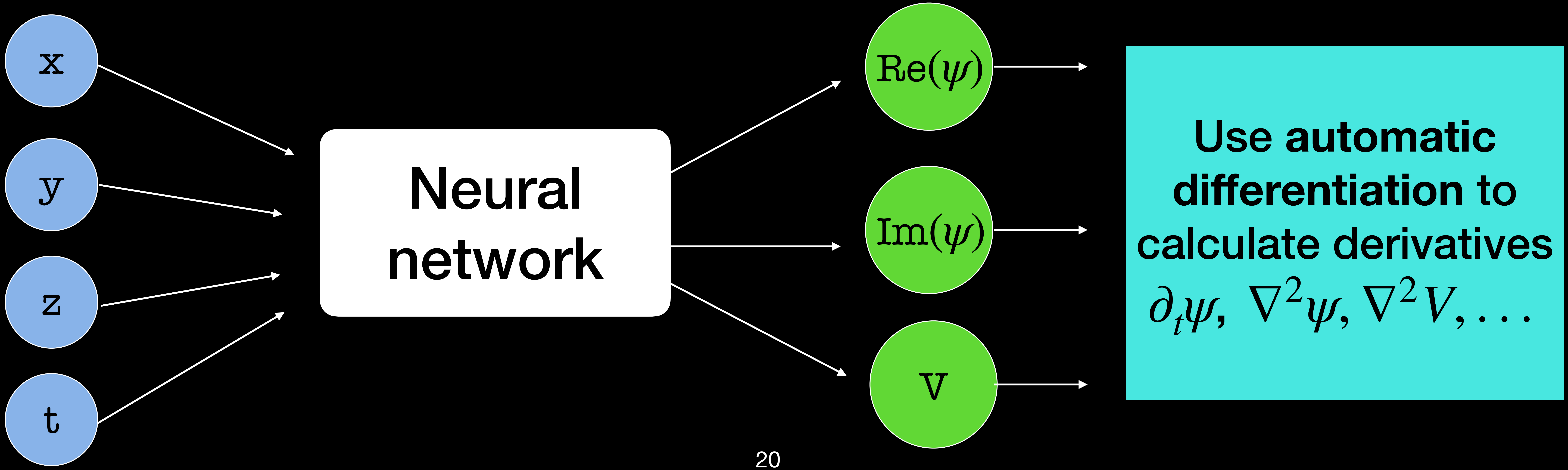


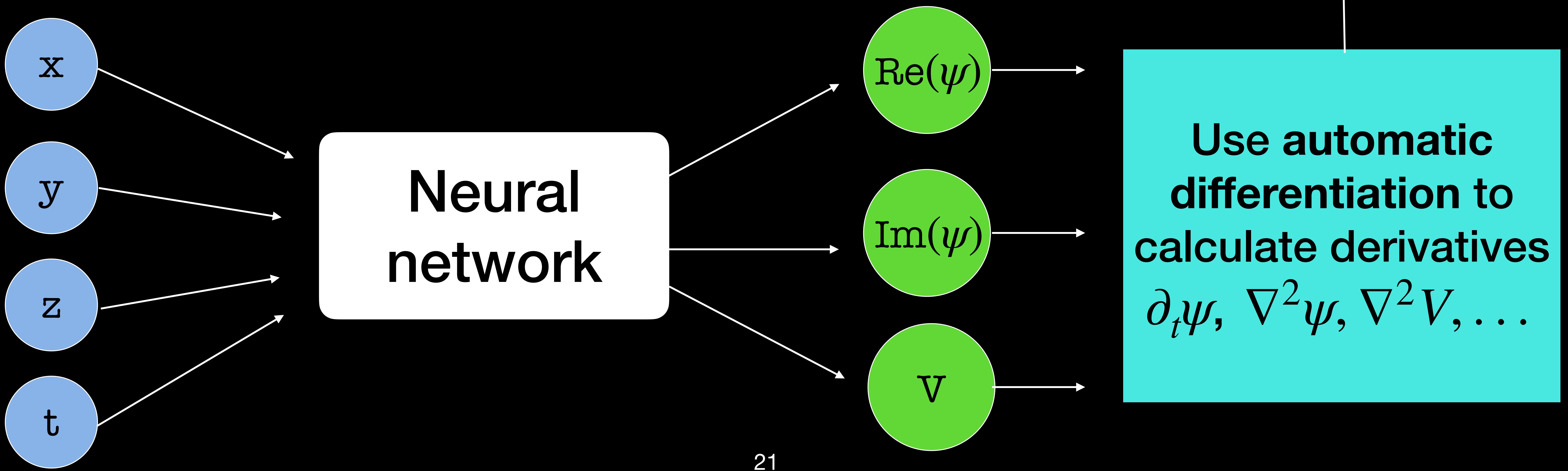
Schrödinger-Poisson Informed Neural Networks (SPINN)

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi$$

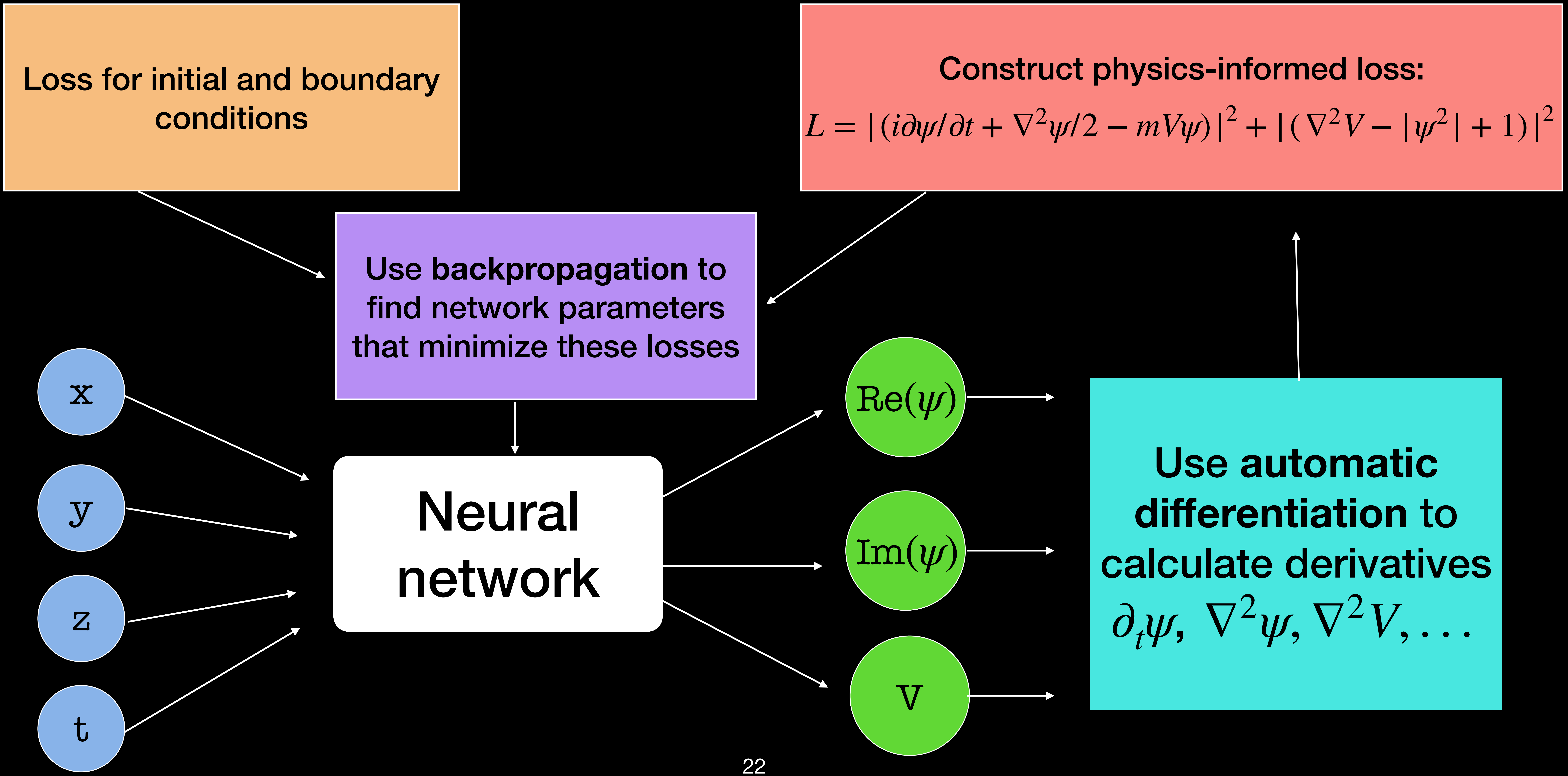
$$\nabla^2 V = 4\pi G m(|\psi|^2 - |\psi_0|^2)$$

Construct physics-informed loss:

$$L = |(i\partial\psi/\partial t + \nabla^2\psi/2 - mV\psi)|^2 + |(\nabla^2 V - |\psi^2| + 1)|^2$$

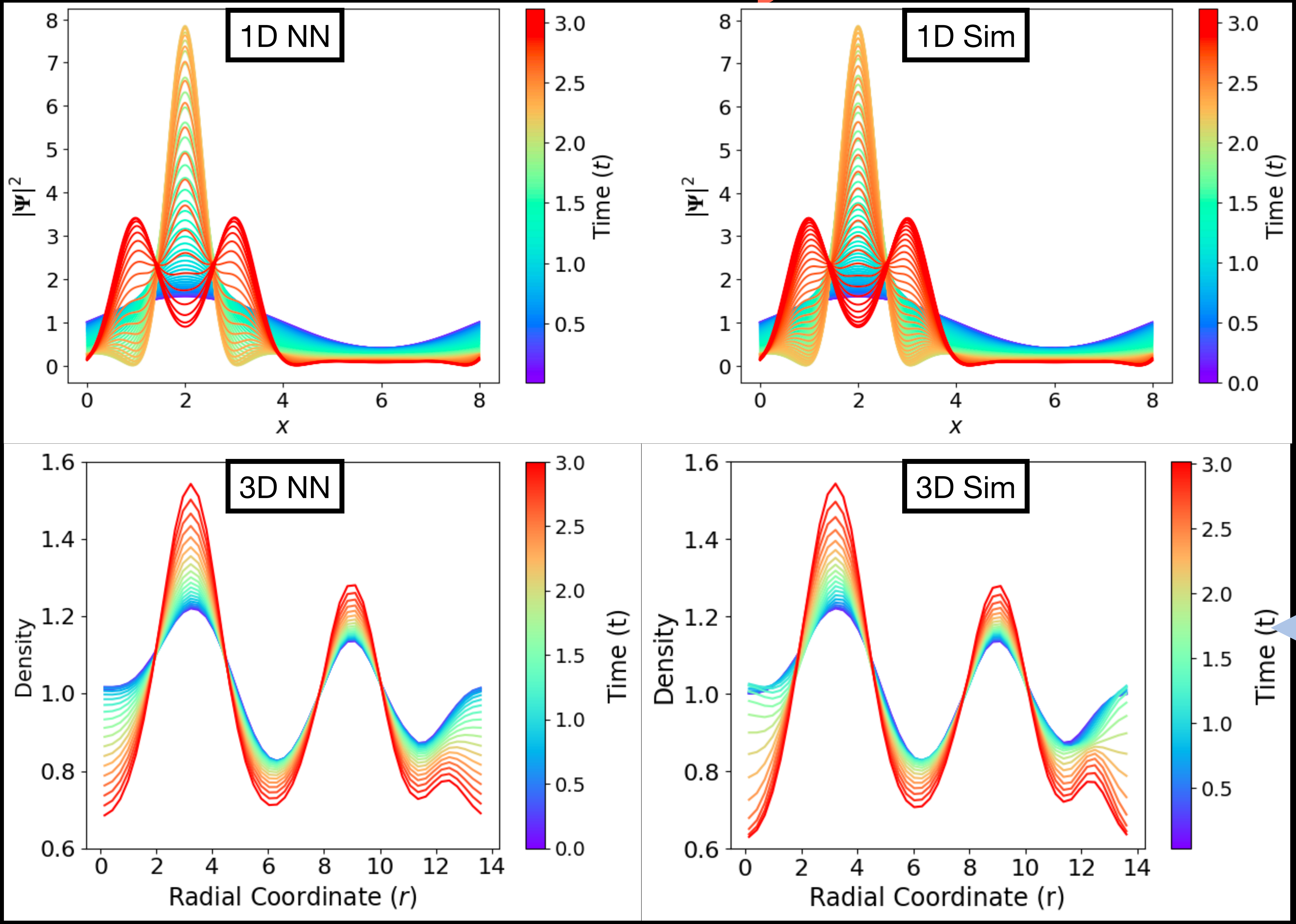


Schrödinger-Poisson Informed Neural Networks (SPINN)



Results

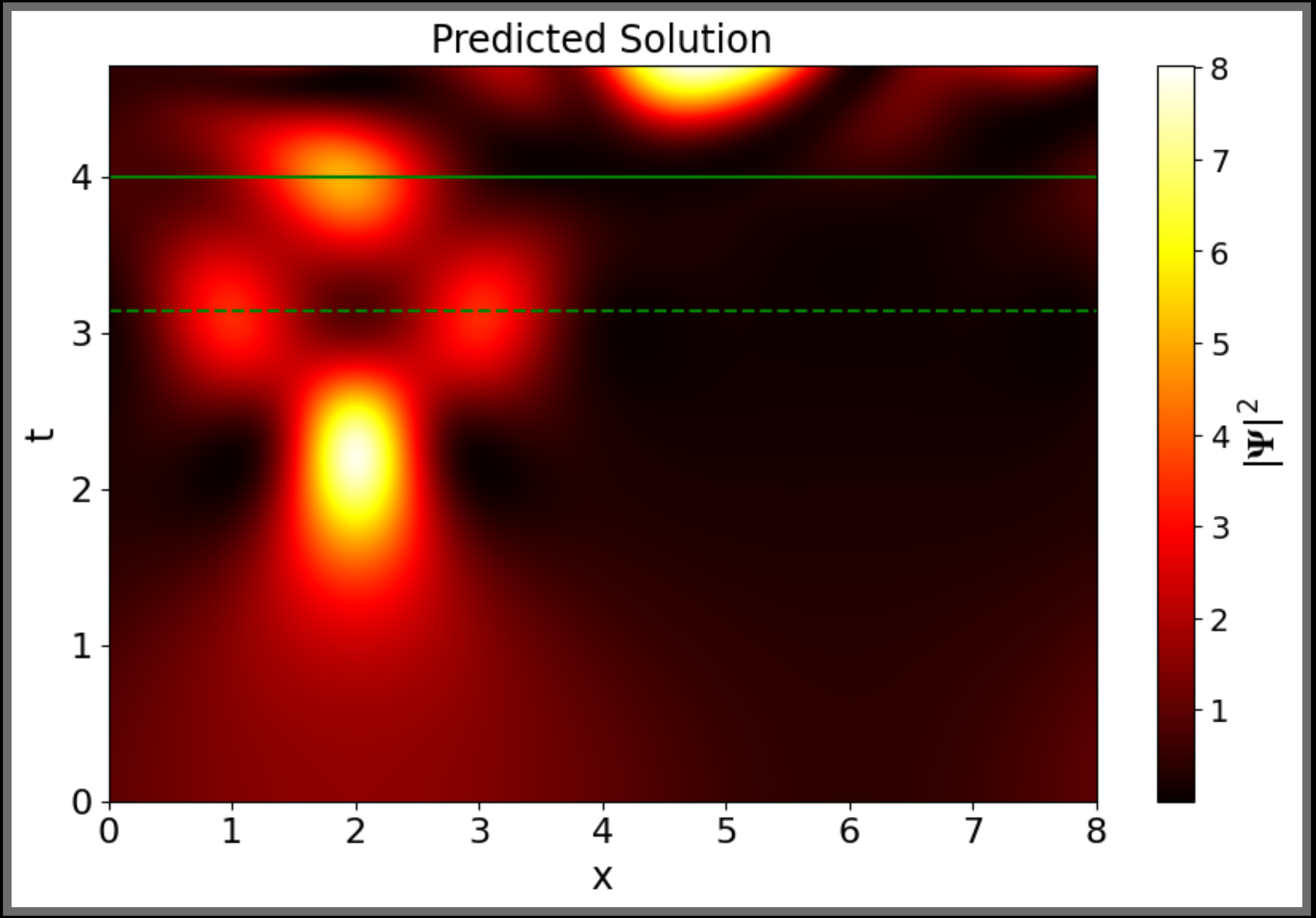
Density Predictions

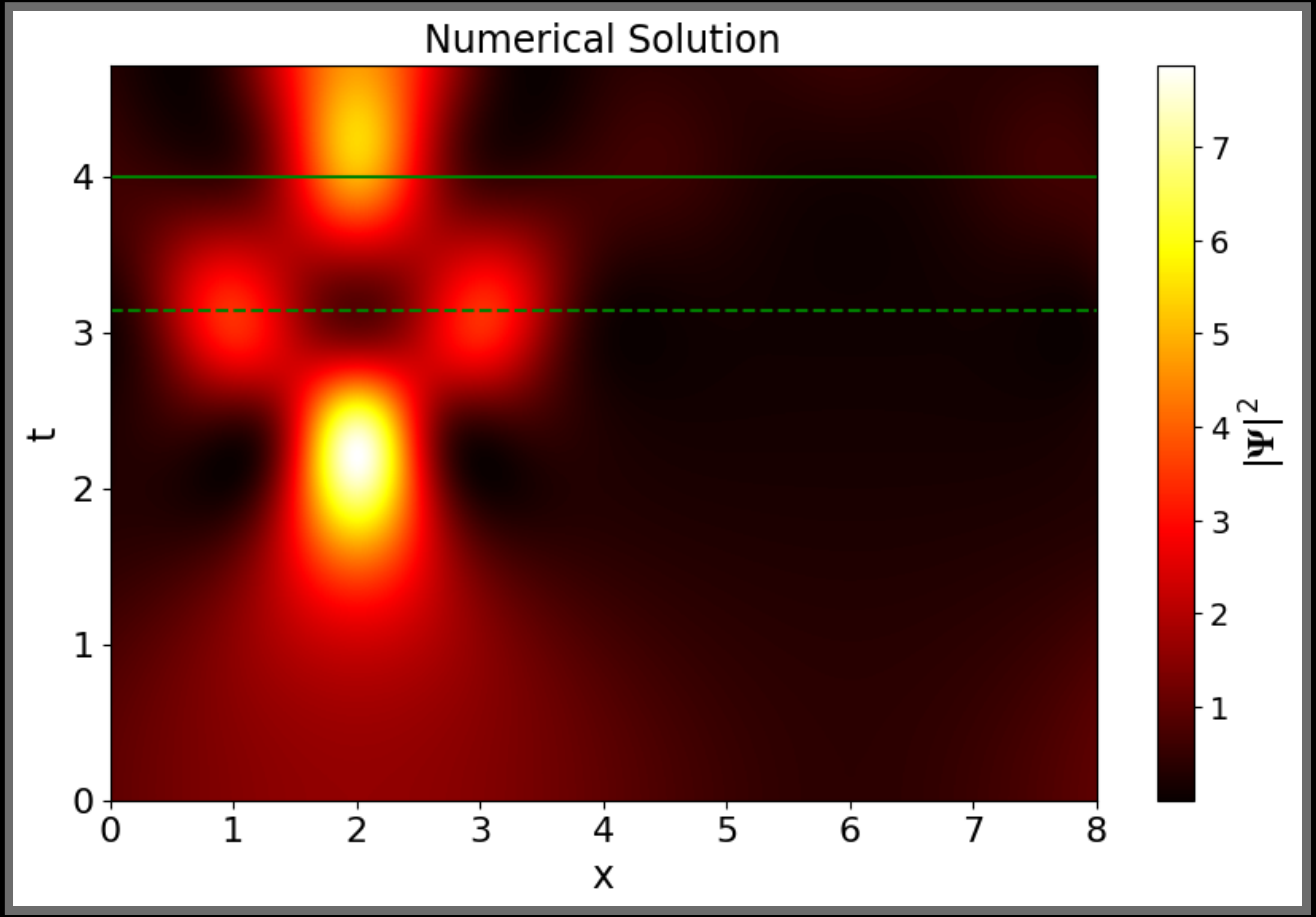


Mishra & Tolley, ApJ 2025 ✔

Unsupervised neural network accurately predicting FDM dynamics using just physics constraints and initial conditions

Extrapolation





Evidence of SPINNs' ability to generalize and predict beyond trained time intervals !

What about cosmological FDM simulations?

Approaches

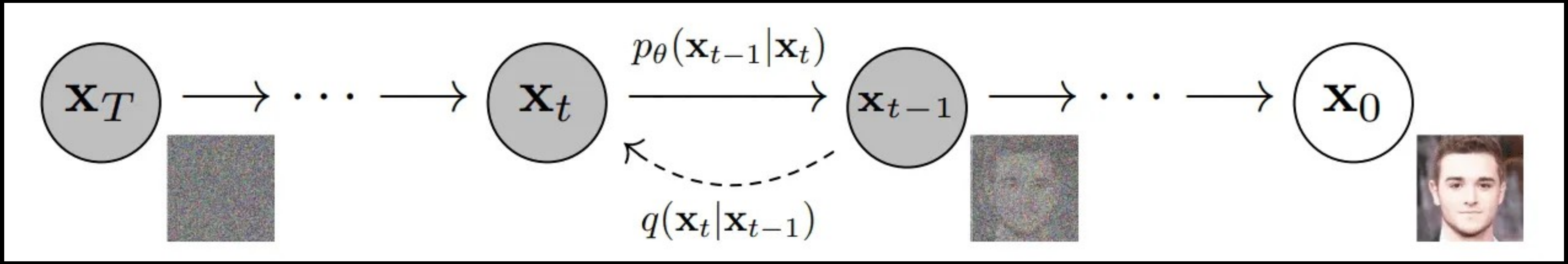
▷ Learning the CDM to FDM density mapping

$$\rho_{CDM}(x) \rightarrow \rho_{FDM}(x)$$

(Essence: Learn the small scale corrections with ML — where FDM diverges — while taking advantage of its numerical equivalence to CDM at large scales.)

▷ Use physics-informed Generative models to simulate/interpolate FDM boxes across redshifts

How do diffusion models work?



source: arXiv:1503.03585, 2015

Source: medium.com – Vadim Titko

Data for CDM to FDM Mapping I

Type	Resolution Elements	L/h^{-1} Mpc	mc^2 / eV	Resolution
FDM	8640^3	10	7×10^{-23}	1.16 h^{-1} kpc
FDM	4320^3	10	$(1, 3.5) \times 10^{-23}$	2.31 h^{-1} kpc
FDM	2400^3	2	2.5×10^{-22}	0.83 h^{-1} kpc
CDM	2048^3	10	-	9.69×10^3 h^{-1} $M_\odot$
CDM	512^3	2	-	4.13×10^3 h^{-1} $M_\odot$

Courtesy of Simon May

- We start with 10 Mpc simulation boxes for both Cold Dark Matter (CDM) and Fuzzy Dark Matter (FDM), covering the same region in space.

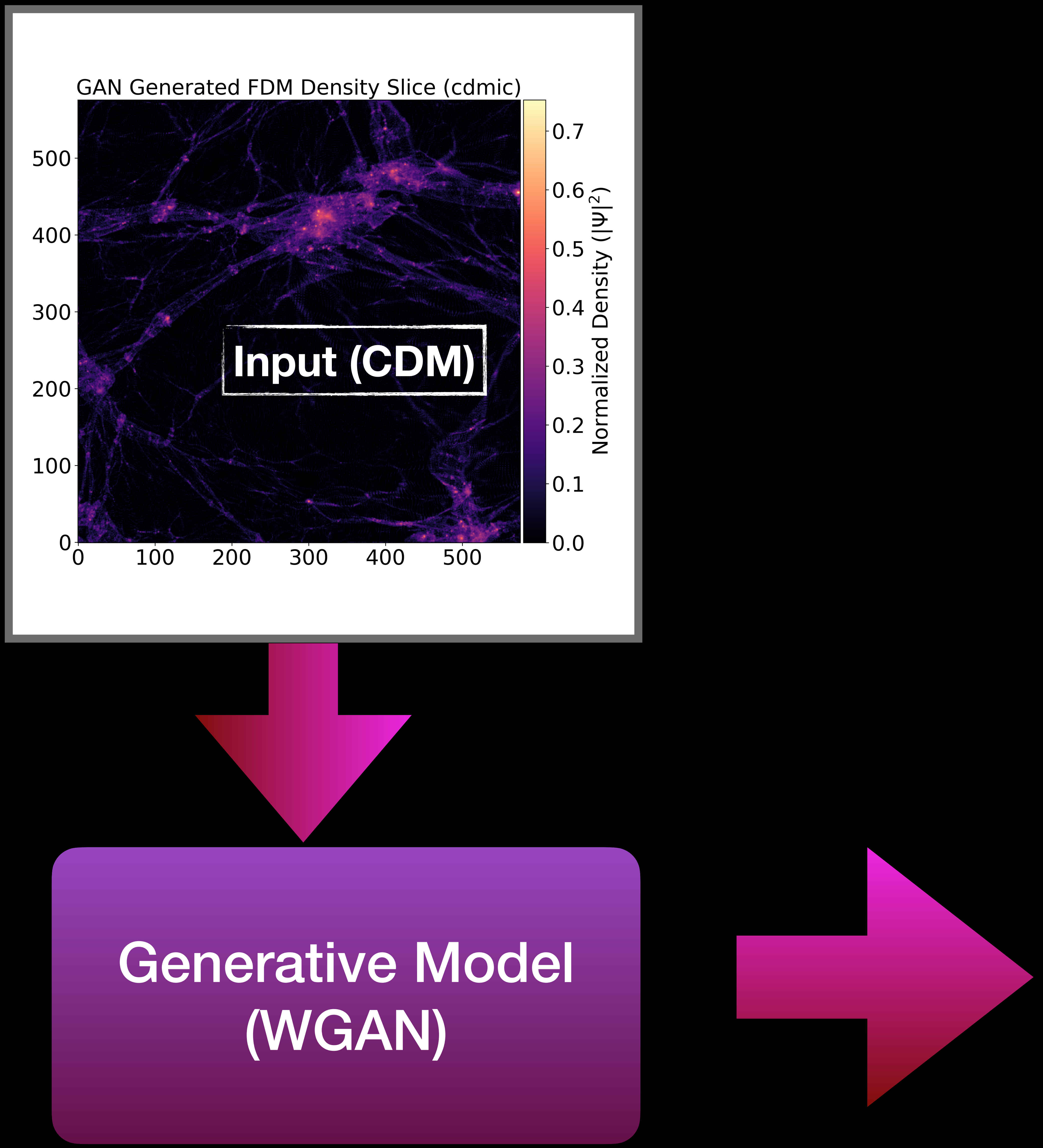
- Original FDM, CDM box resolution: $8640^3,\ 2048^3$

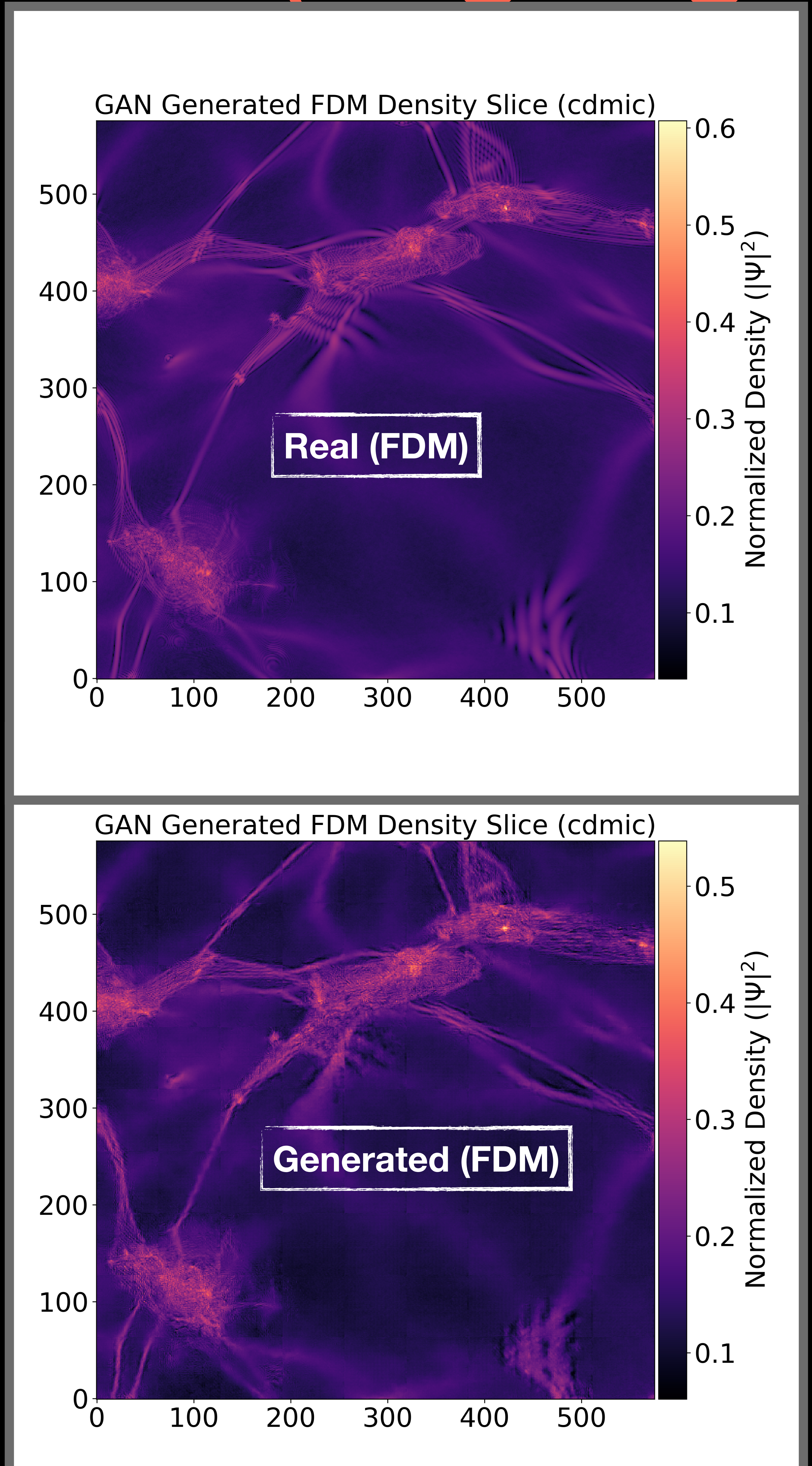
Data for CDM to FDM Mapping II

- For both the boxes, we extract a **sub-volume spanning 0 to 3 Mpc/h on each side**.

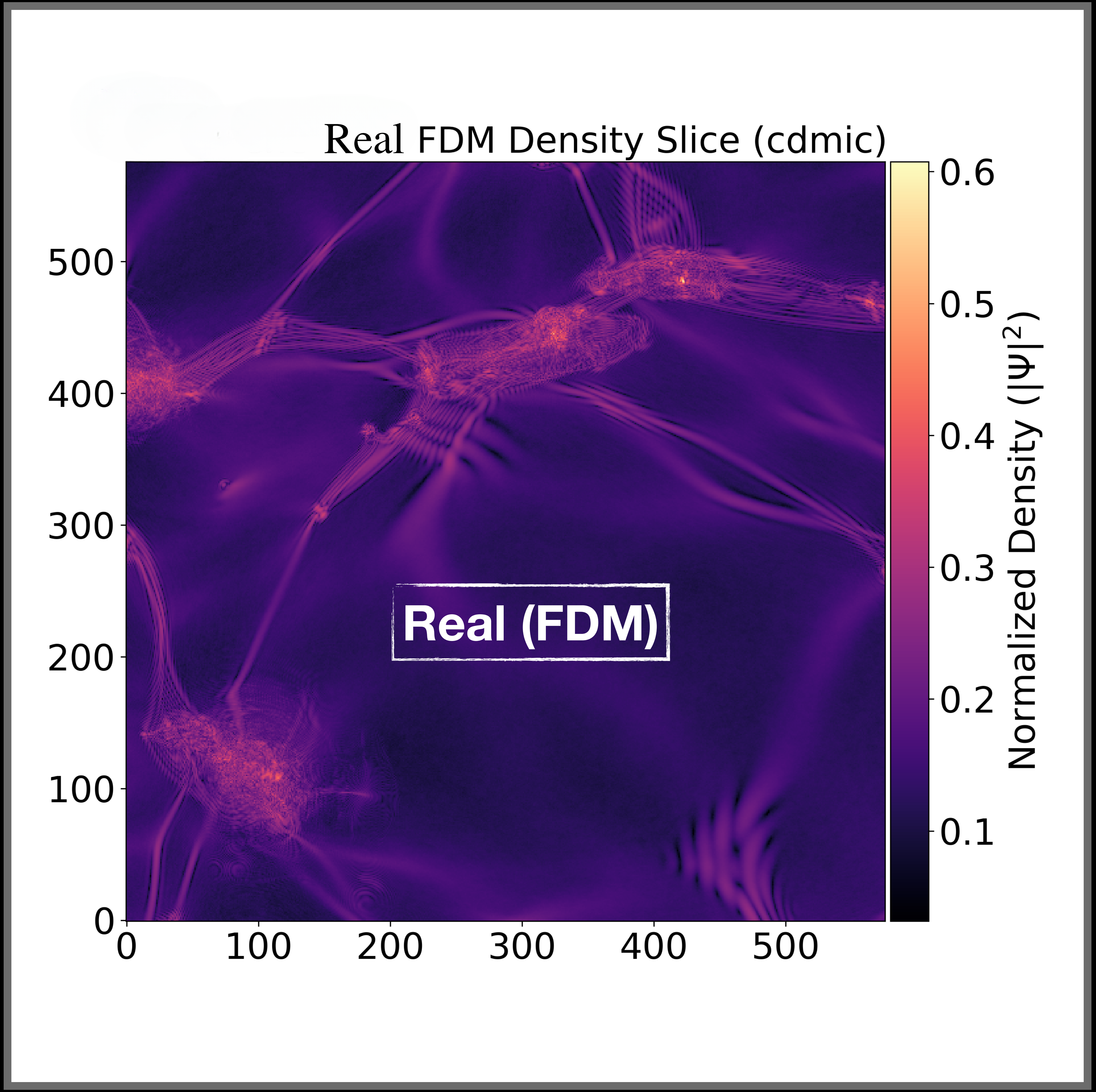
- This 3 Mpc region is then:
  - **Downsampled** to a resolution of **576^3 voxels**.
  - Further chunked into smaller 64^3 voxel boxes to fit within GPU memory constraints.
  - In total 729 pairs of CDM and FDM boxes!

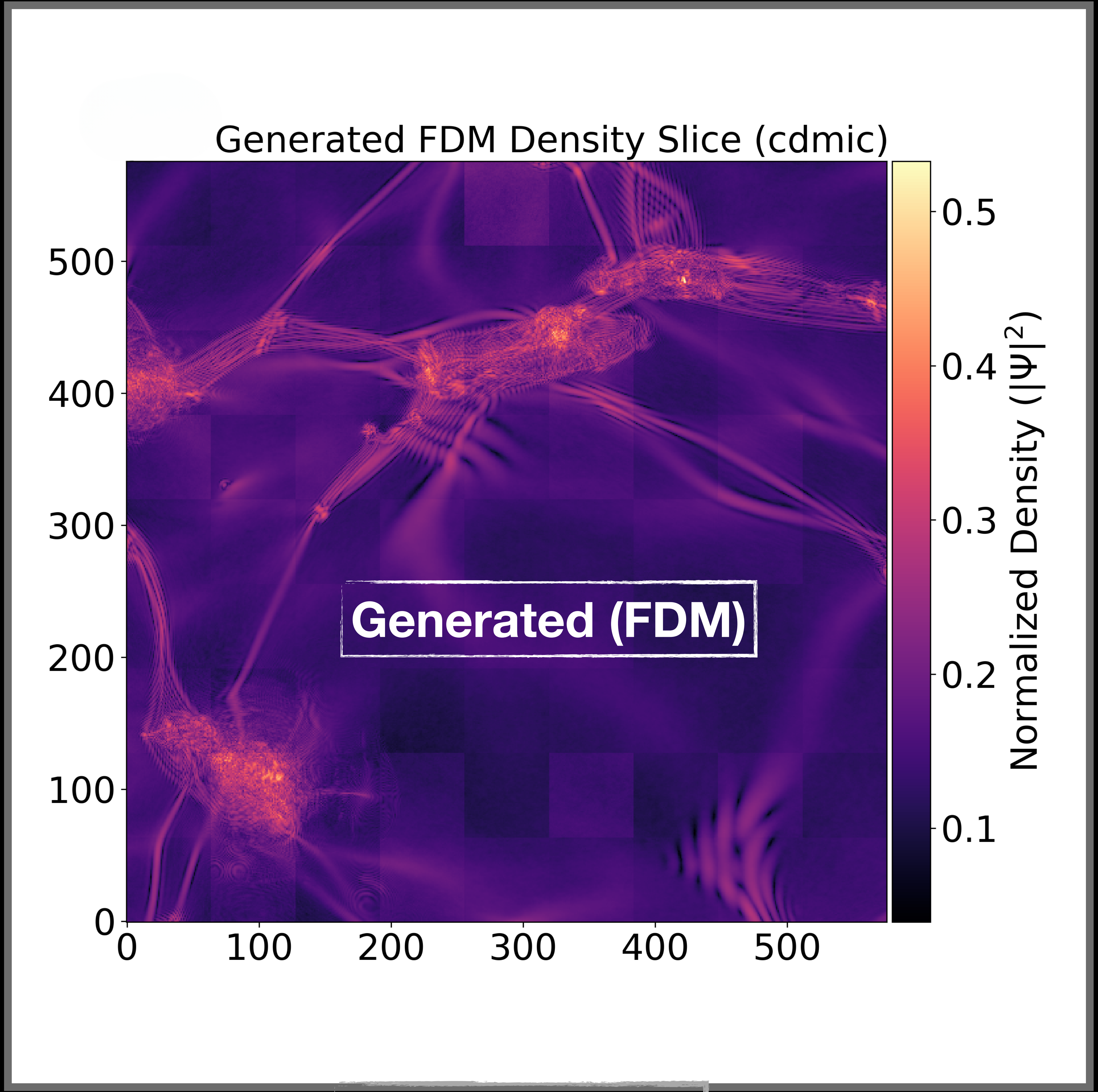
Current Hybrid Approach-WGAN (Ongoing work)





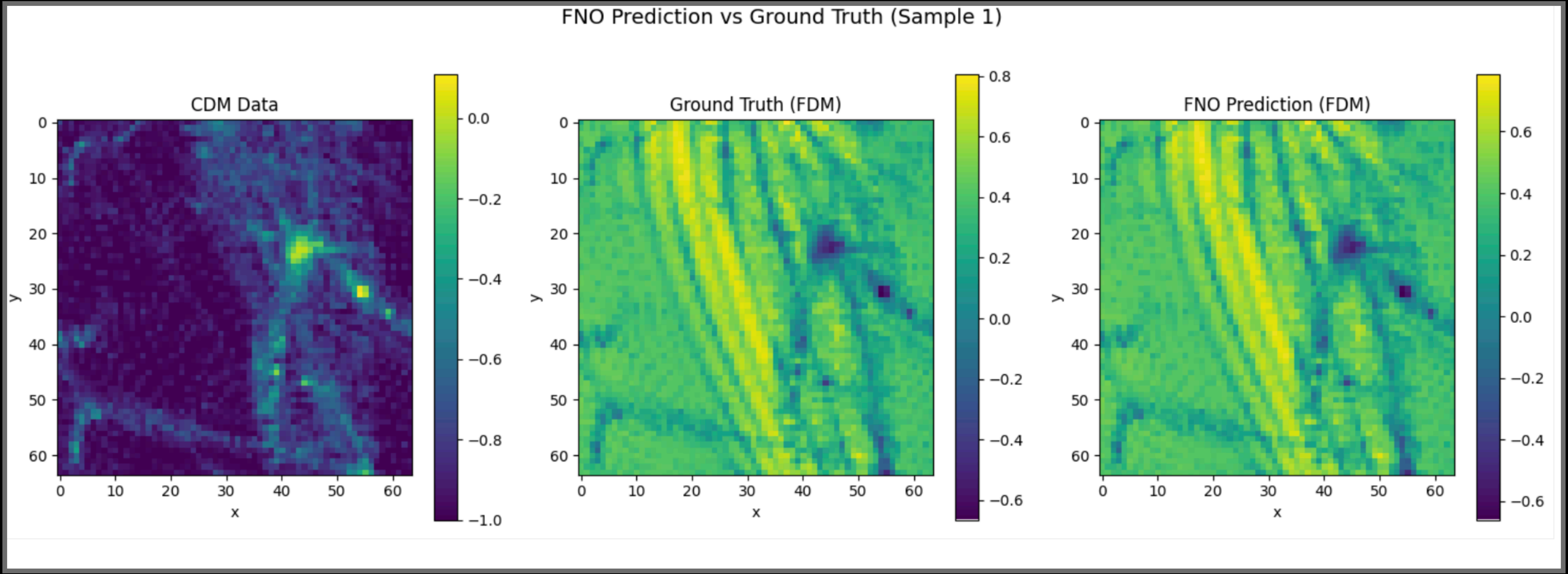
Current Hybrid Approach-DDPMs (Ongoing work)





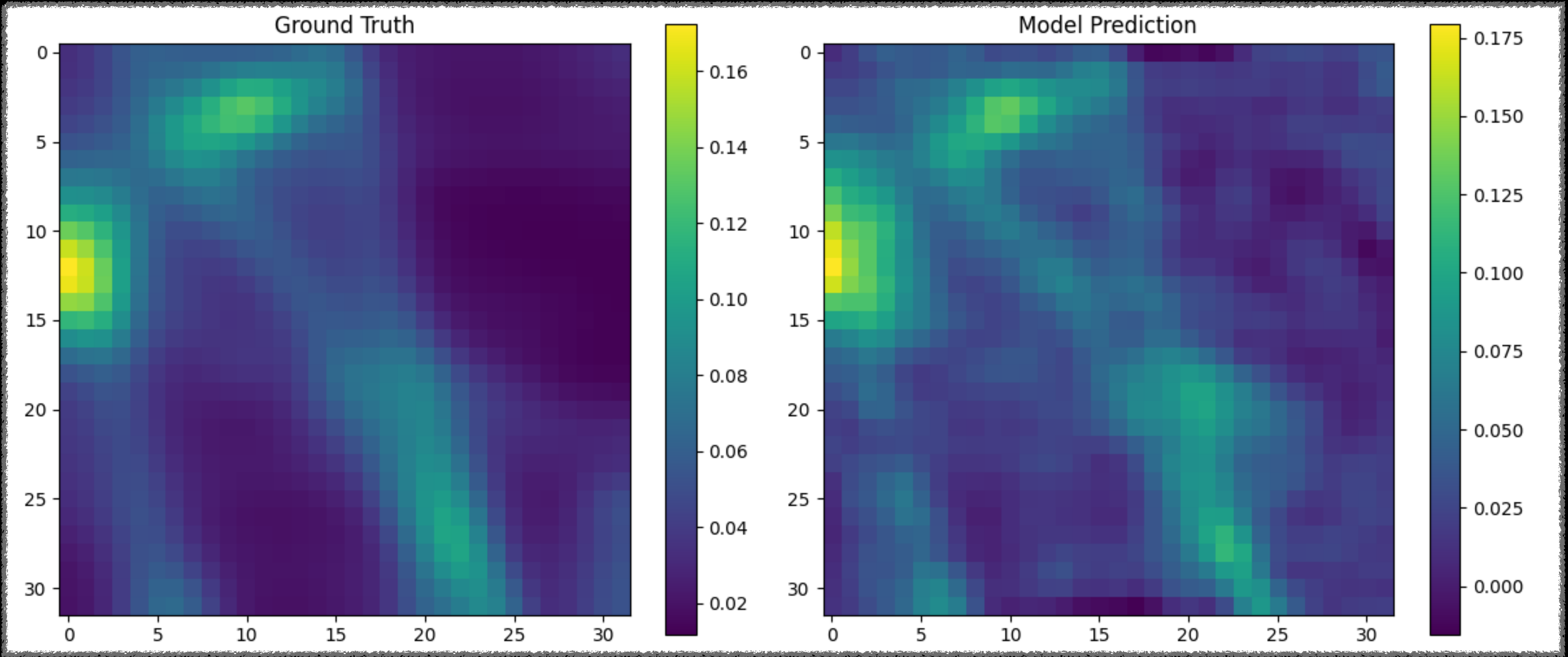
Much better!

Fourier Neural Operators First Results



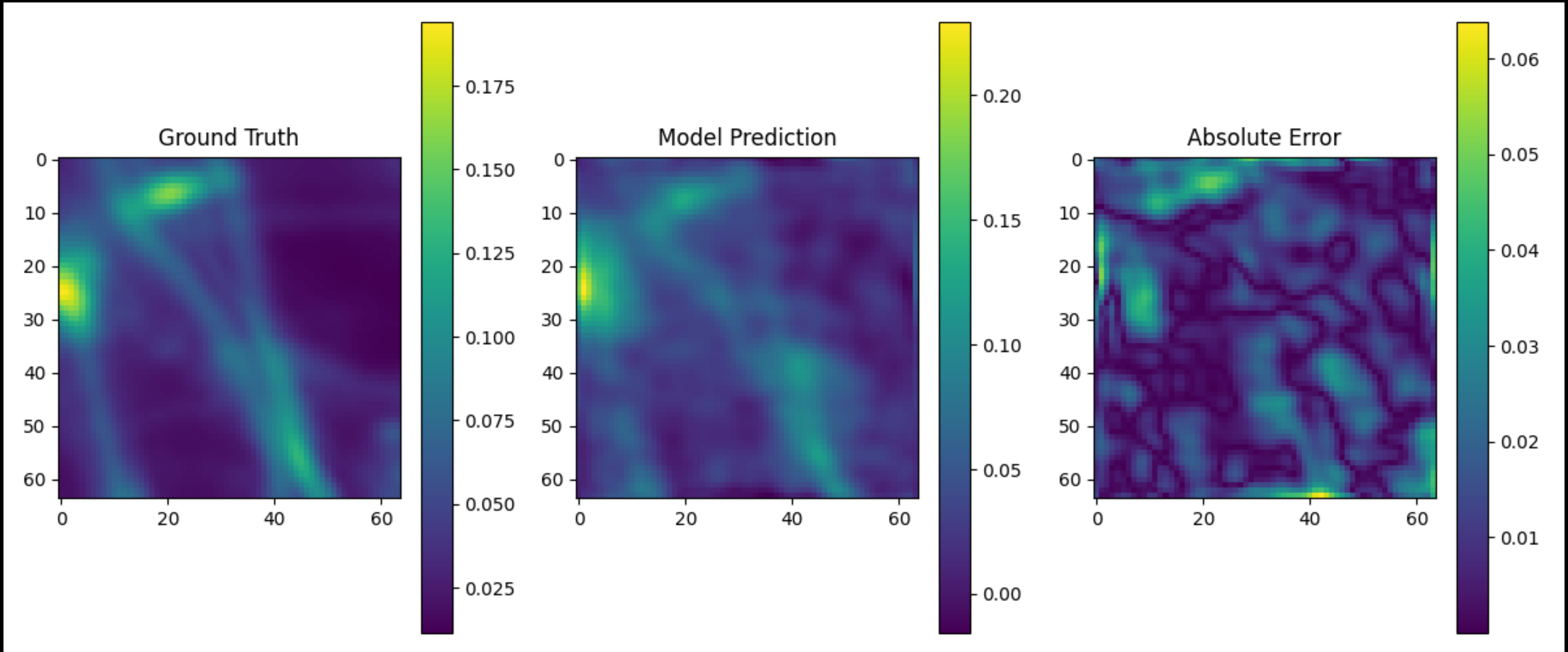
A prelimiary check!!

Fourier Neural Operators - Test images (with training resolution)



Test Samples for FNO trained on 32^3 (gaussian-smoothed) CDM-FDM boxes

Fourier Neural Operators - Test images (with double the training resolution)



Superresolution: Test Samples for FNO trained on 32^3 (gaussian-smoothed) CDM-FDM boxes but tested on 64^3 boxes !!

FDM Simulation with finer time steps (1Mpc Boxes)

Projected FDM density snapshots shown across time (from z = 127 on the left to z = 0 on the right)

Data for Physics-Informed UNet

- 1 Mpc/h sized FDM boxes at redshifts from $z = 127$ to $z = 0$ are simulated using *Jaxion code (from P. Mocz)*
- These are fine-steps simulations with da = 1e-5 (Absolutely essential!)
- Each of these boxes' original resolution is 256^3
- This chosen subset of samples is then:
  - **Downsampled** to a resolution of 64^3 **voxels**.

Cosmo-SPINN trained on 1Mpc Boxes

Cosmological SP equations with comoving quantities

$$i\hbar\partial_t\psi_c(t,\mathbf{x}) = -\frac{1}{2ma(t)^2}\nabla_c^2\psi_c(t,\mathbf{x}) + \frac{m}{a(t)}\Phi_c\psi_c(t,\mathbf{x})$$

$$\nabla_c^2\Phi_c(t,\mathbf{x}) = 4\pi Gm\left(\left|\psi_c(t,\mathbf{x})\right|^2 - \left\langle\left|\psi_c\right|^2\right\rangle(t)\right)$$

Introduce supercomoving time : $d\tau = a^{-2}dt$, & Redefine $V \equiv a\Phi_c$

$$i\hbar\partial_\tau\psi_c(\tau,\mathbf{x}) = -\frac{1}{2m}\nabla_c^2\psi_c(\tau,\mathbf{x}) + mV\psi_c(\tau,\mathbf{x})$$

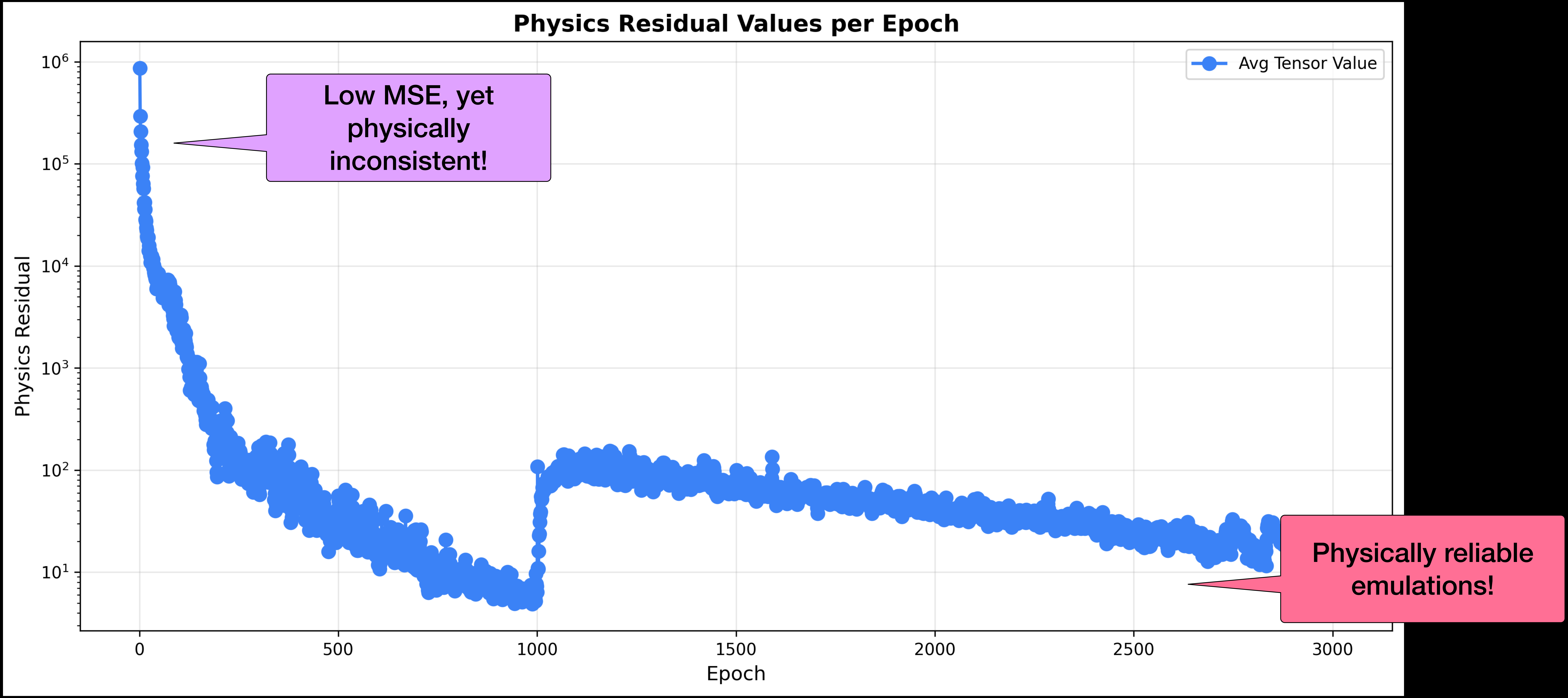
$$\nabla_c^2\Phi_c(t,\mathbf{x}) = 4\pi Gam\left(\left|\psi_c(t,\mathbf{x})\right|^2 - \left\langle\left|\psi_c\right|^2\right\rangle(t)\right)$$

Scale factor range for training : $a_{sing} \in [0.149, 0.157]$, $a_{multi} \in [0.0798, 0.803]$

Cosmo-SPINN : Training and Test Losses

Better generalizability with inclusion of physics loss!

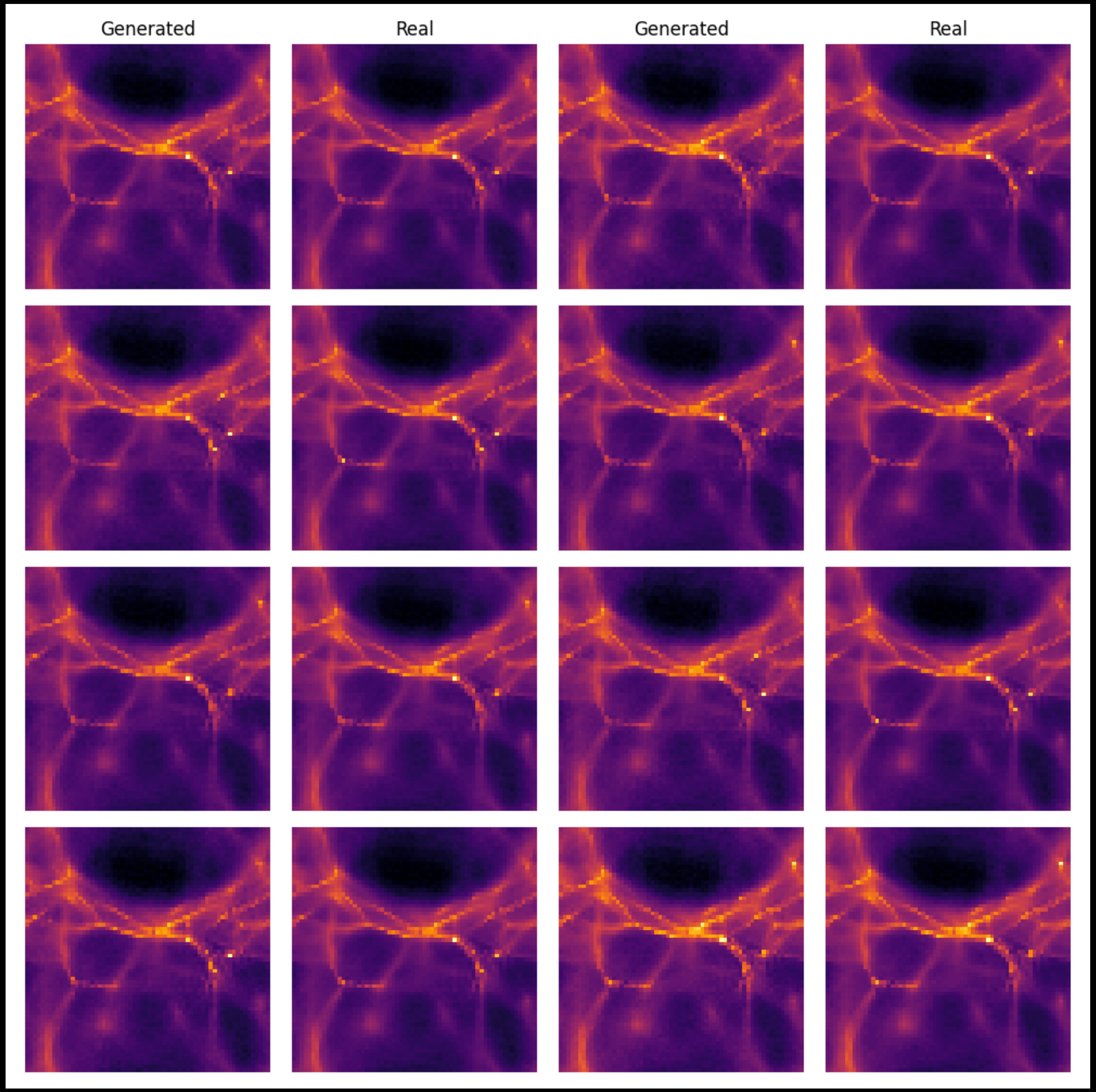
Cosmo-SPINN: Physics Residuals



Cosmo-SPINN trained on 1Mpc Boxes (Single Realization)

Predicted R, I , V given the redshift and initial condition (this is w. physics informed loss after some initial warm up)

Just 20% Training data + physics loss

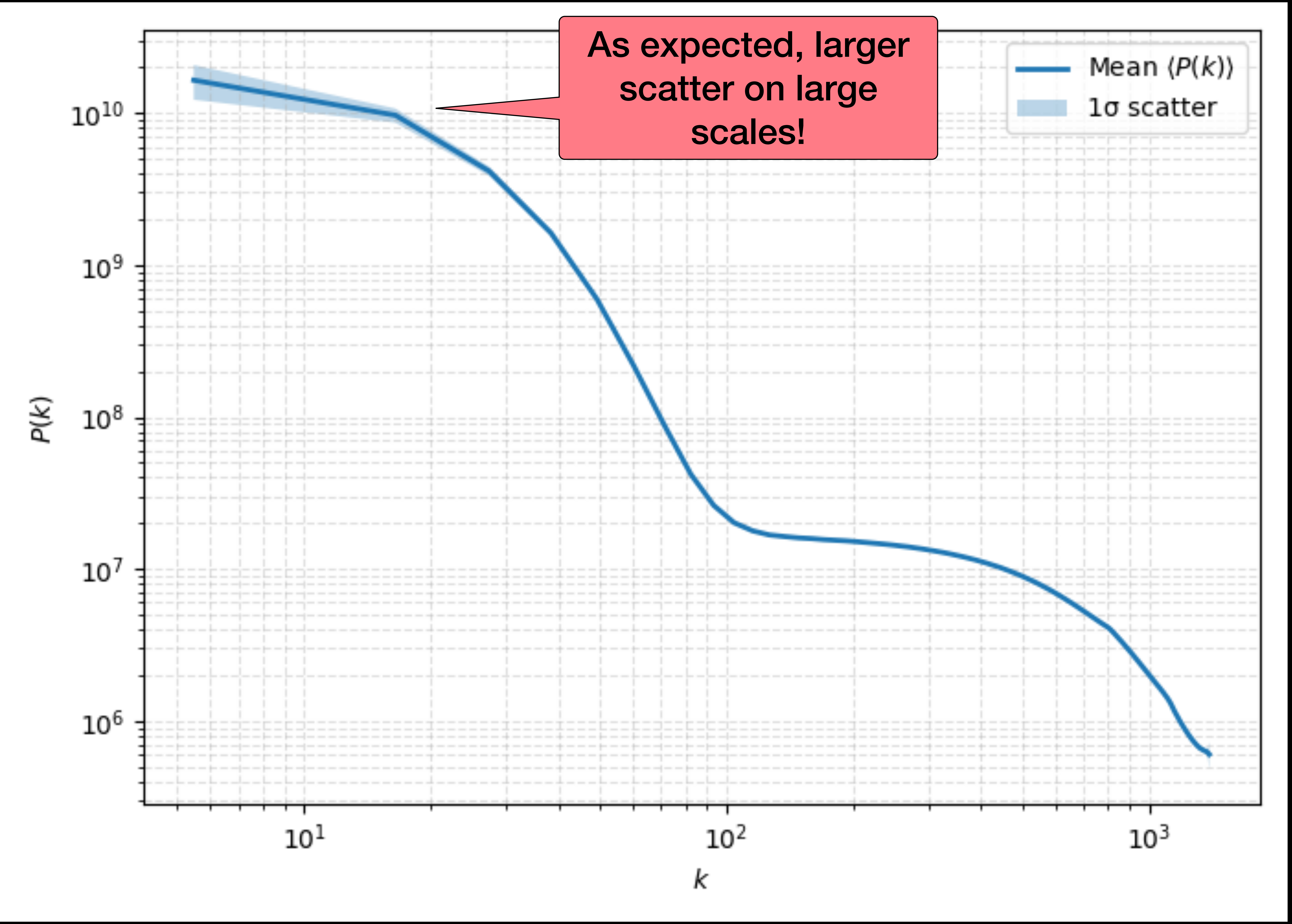


What about its extension to multi-realizations of the ICs?

1) ***AxionCAMB*** for Power-Spectrum $(m_a = 2.5 \times 10^{-22} eV)$

2) ***JaxPM*** for displacement and velocity fields (x,v) using Lagrangian Perturbation theory (LPT) 2nd Order

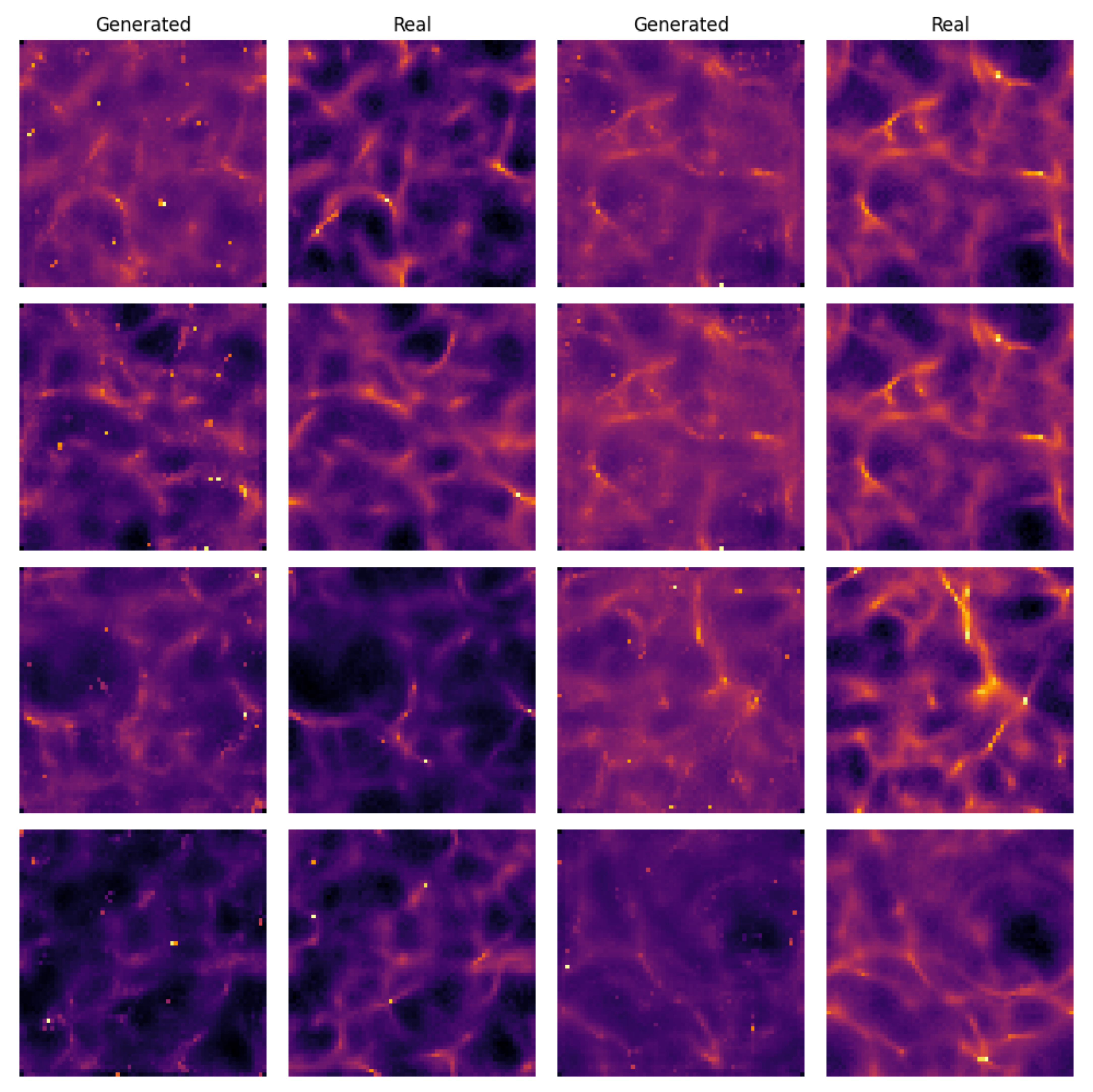
3) Custom code to convert it into wave function for the FDM Ψ



Cosmo-SPINN trained on 1Mpc Boxes (Multiple Realizations)

Predicted R, I , V given the redshift and different initial conditions (this is w/o physics informed loss after some initial warm up)

Just good over-all large scale but some hallucinations at small scales!



Test Samples after 50 epochs

Work in Progress!
(Still to get to large boxes with PINNs)

01 Neural Operators for consecutive time step predictions (Auto-regressive approach)

02 Physics-informed neural operators for painting in FDM small-scale features

03 Extending Cosmo-SPINNs to larger boxes and larger time intervals

04 Reproducing Core-Halo Relations for FDM with PINNs

THANK YOU!

Question?

Ashutosh Kumar Mishra
Email: ashutosh.mishra@epfl.ch

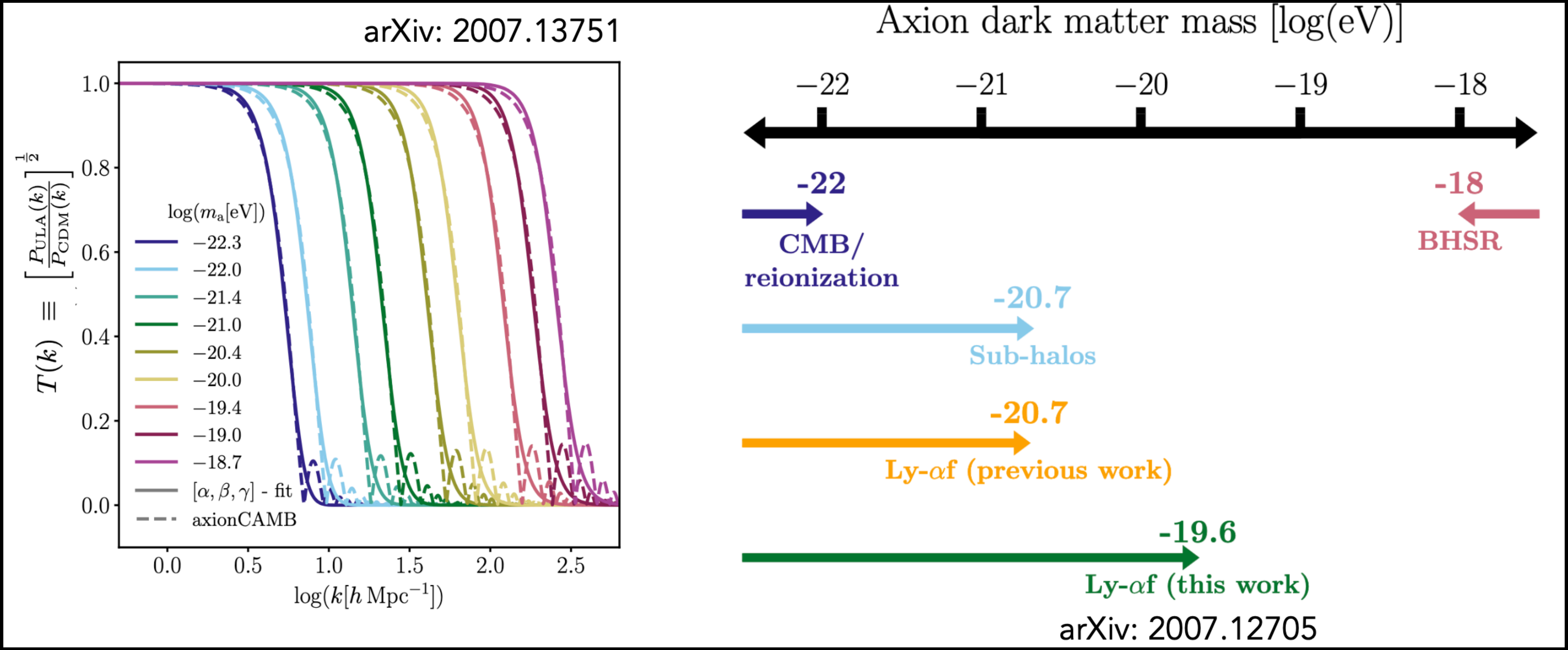
SCAN ME

Arxiv Link

Back-up

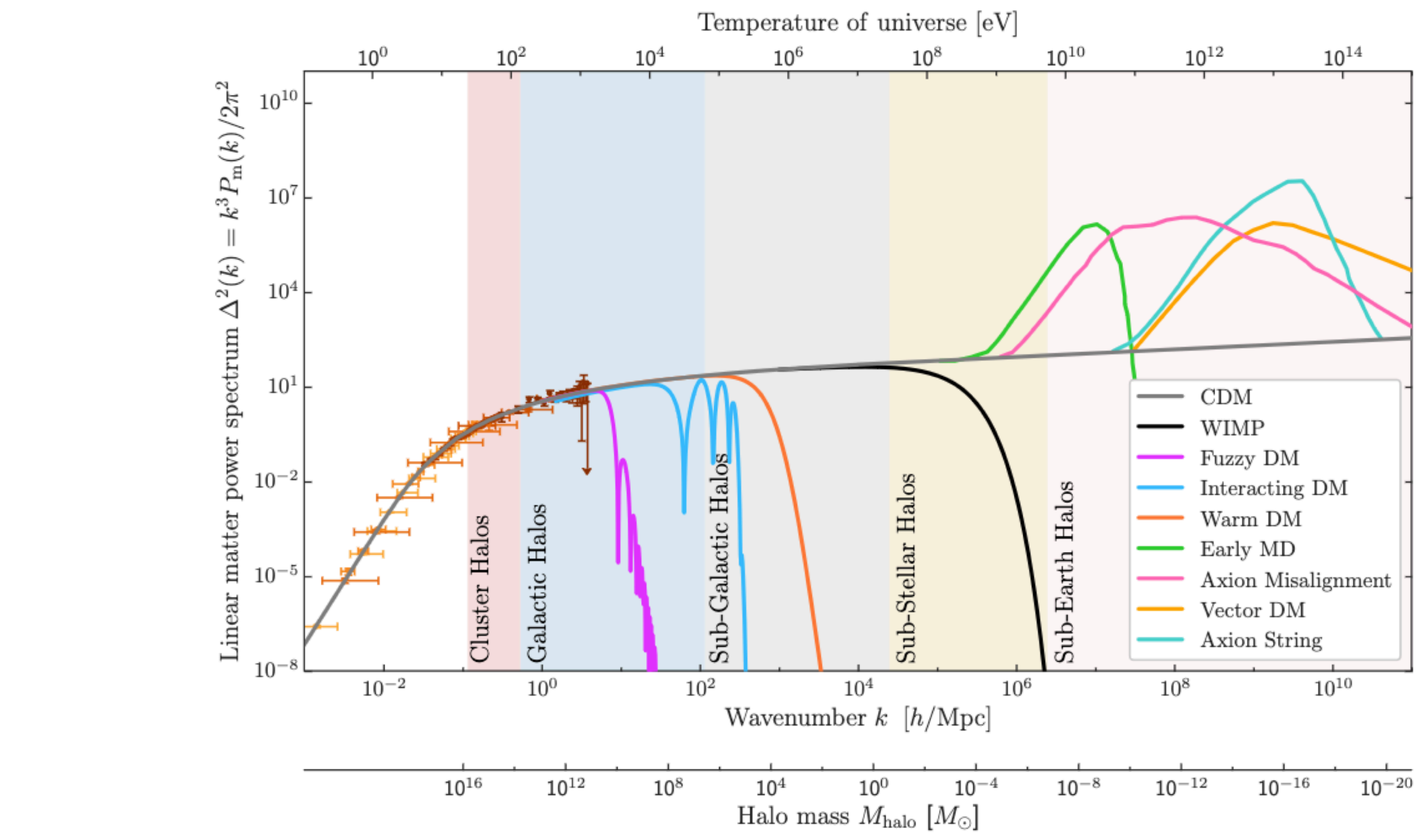
Fuzzy Dark Matter

Linear theory predicts sharp cutoff in power spectrum due to quantum pressure



Direct probe of DM distribution

Power spectrum, halo mass profile



Bechtol et al. 2023

Numerical Method (Mocz et. al. 2017)

2nd Order Unitary Spectral Method

- ✦ Calculate potential:

$$V = \text{IFFT}\left(-\frac{1}{k^2}\text{FFT}\left(4\pi G m(|\psi|^2 - |\psi_0|^2)\right)\right)$$

- ✦ Half-Step 'Kick':

$$\psi \leftarrow exp[-i(m/\hbar)(\Delta t/2)V]\psi$$ **Kick**

- ✦ Full-Step 'Drift' in Fourier Space:

$$\psi \leftarrow \textbf{IFFT}\left(exp[-i\Delta t(\hbar/m)k^2/2]\textbf{FFT}(\psi)\right)$$ **Drift**

- ✦ Update the potential:

$$V \leftarrow \textbf{IFFT}\left(-\frac{1}{k^2}\textbf{FFT}\left(4\pi G m(|\psi|^2 - |\psi_0|^2)\right)\right)$$

- ✦ Another Half-Step 'Kick':

$$\psi \leftarrow exp[-i(m/\hbar)(\Delta t/2)V]\psi$$ **Kick**