Main beam representation in non-regular arrays

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CALIM 2011 Workshop, Manchester, July 25-29, 2011

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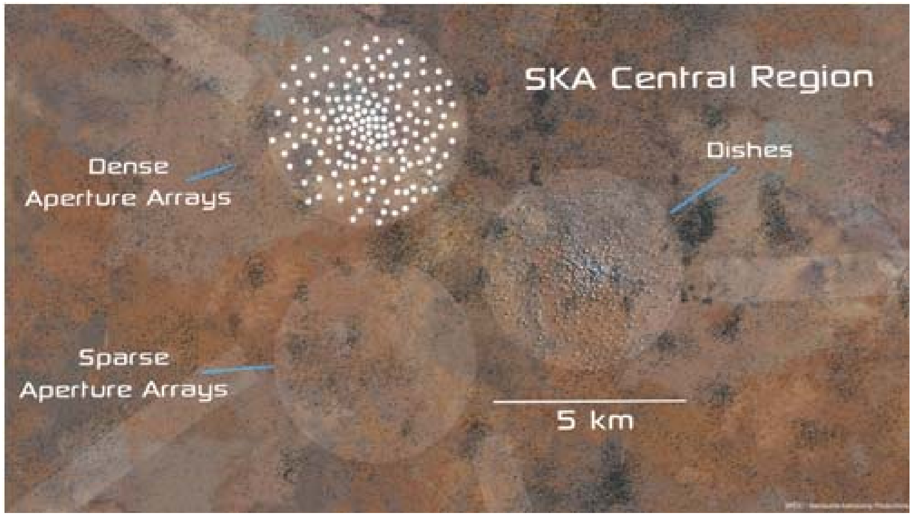
Context: SKA AA-lo

Type of element
Bowtie
Spiral
Log-periodic

Non-regular: max effective area with min nb. elts w/o grating lobes.

Parameter	Specification
Low frequency	70 MHz
High frequency	450 MHz
Nyquist sampling frequency	100 MHz
Number of stations	50 => 250
Antennas per station	10.000



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Problem statement

Goal: pattern representation for all modes of operation at station level.

Too many antennas vs. number of calibration sources

→ Calibrate the main beam and first few sidelobes
Suppress far inteferrers with interferometric methods (open).

→ Compact representations of patterns, inspired from radiation from apertures.
Expose problems related to mutual coupling

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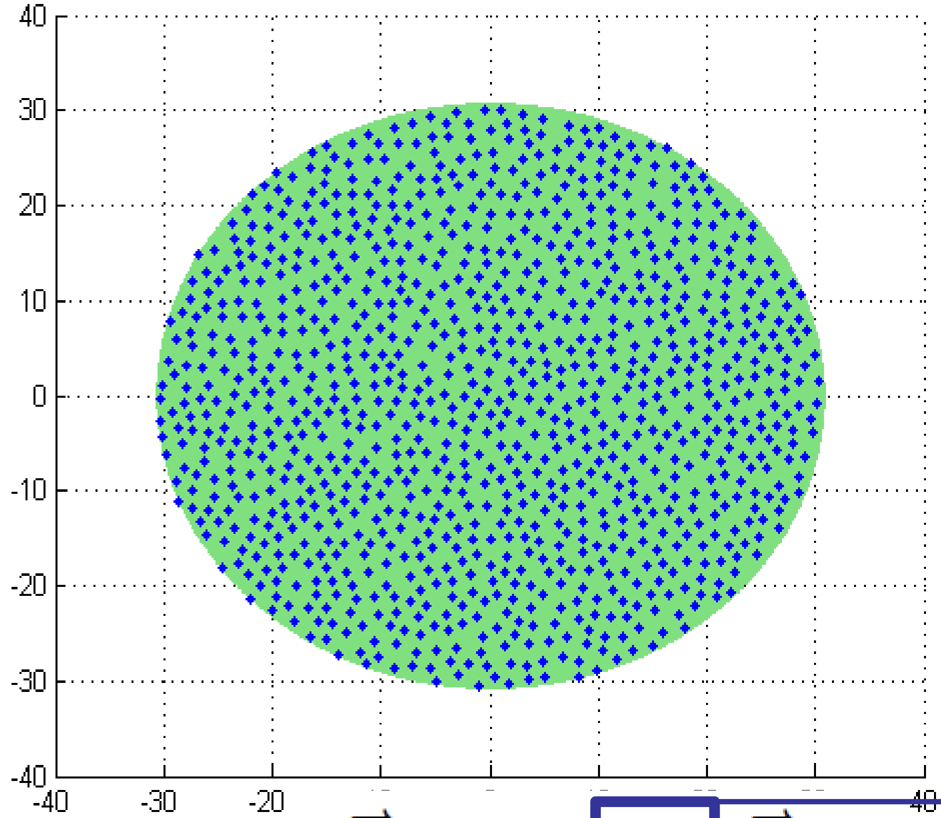
Outline

1. Apertures: continuous versus arrays
2. Array factors: series representations
3. Convergence analysis

} No MC (items 1–3)

4. Mutual coupling: computation, issues and partial solutions

Aperture sampling (1)

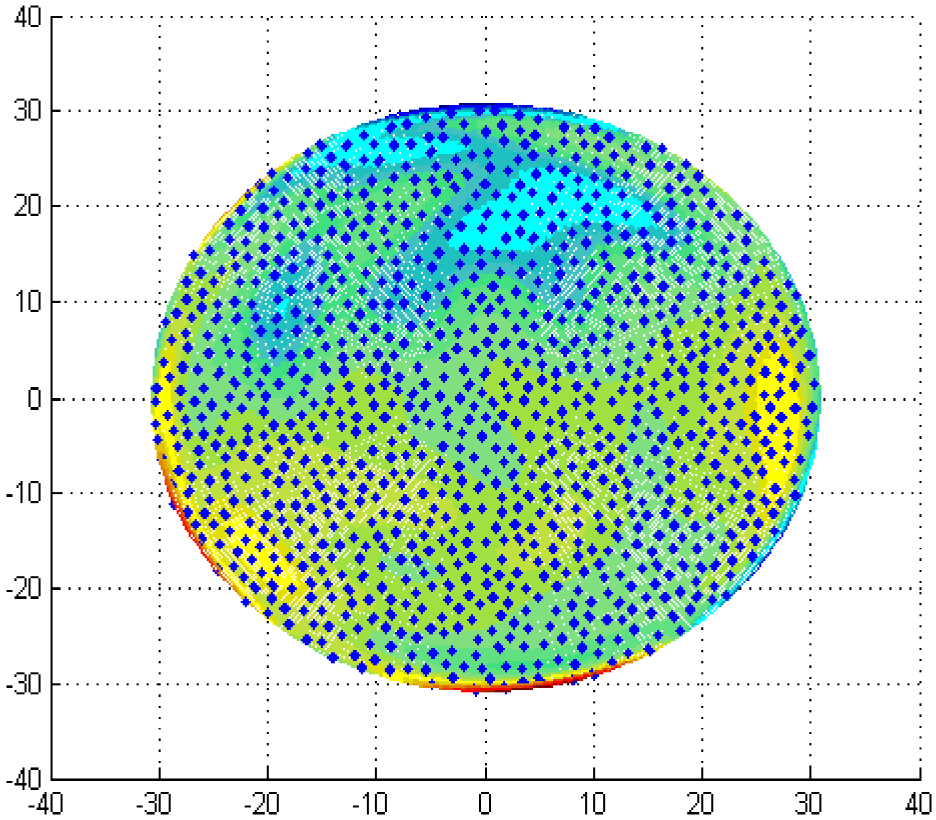


Change frequency: zoom in/out

No mutual coupling: $\vec{F}_{arr} = \boxed{F}\, \vec{F}_{elt}$

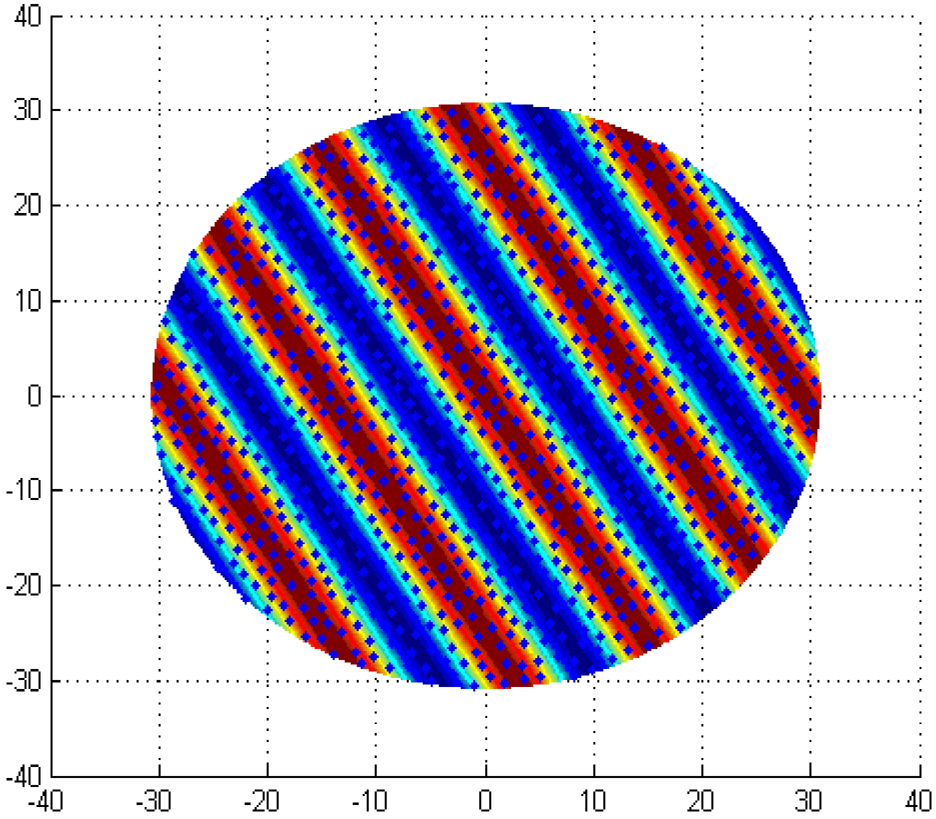
$$F(l,m)_{|l_\circ,m_\circ} = F(l-l_\circ, m-m_\circ)_{|0,0}$$

Aperture sampling (2)



Define a local density
(several definitions possible)

Aperture scanning (1)



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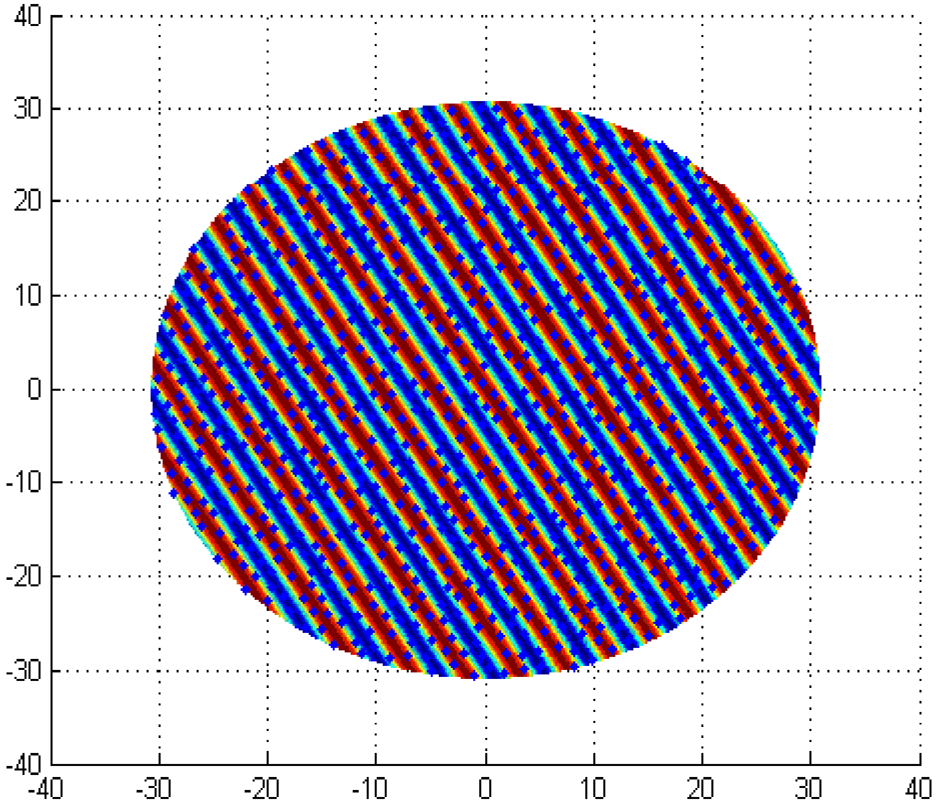
Aperture scanning (2)



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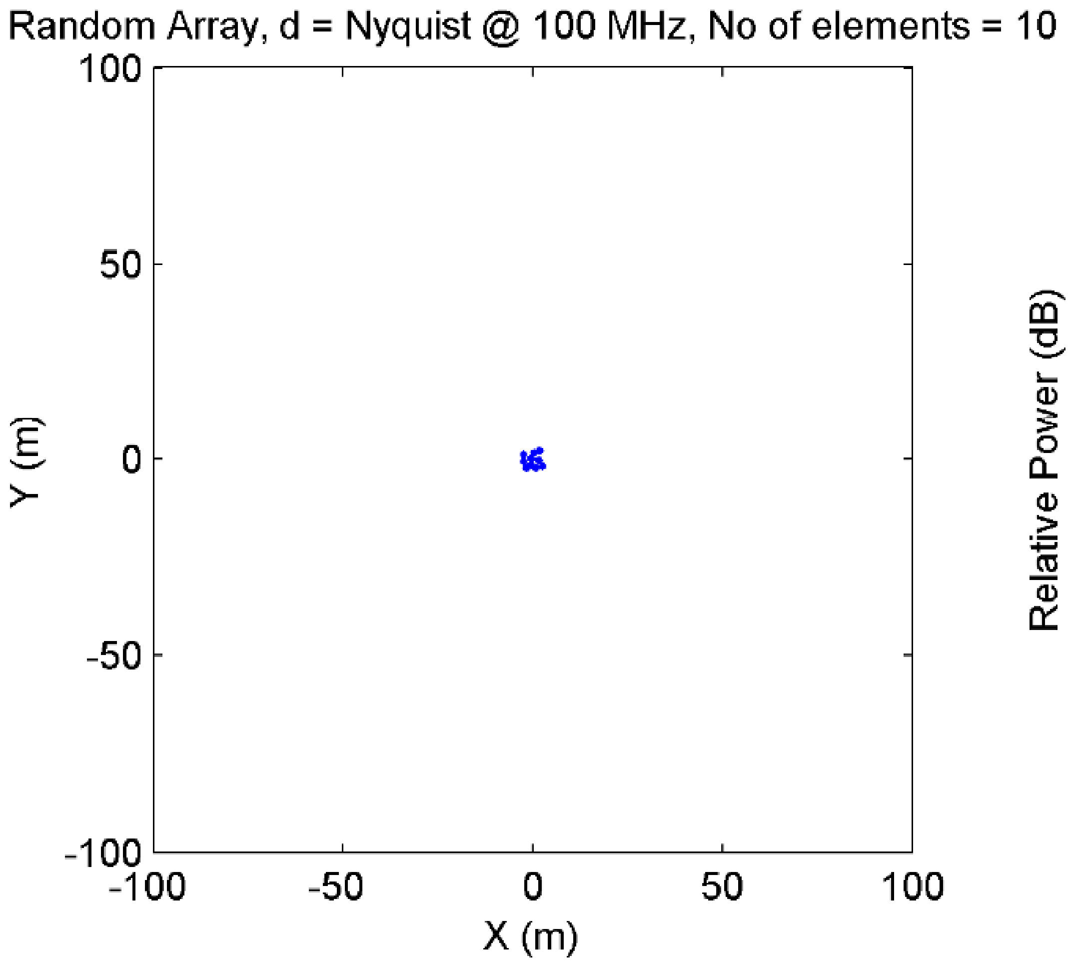
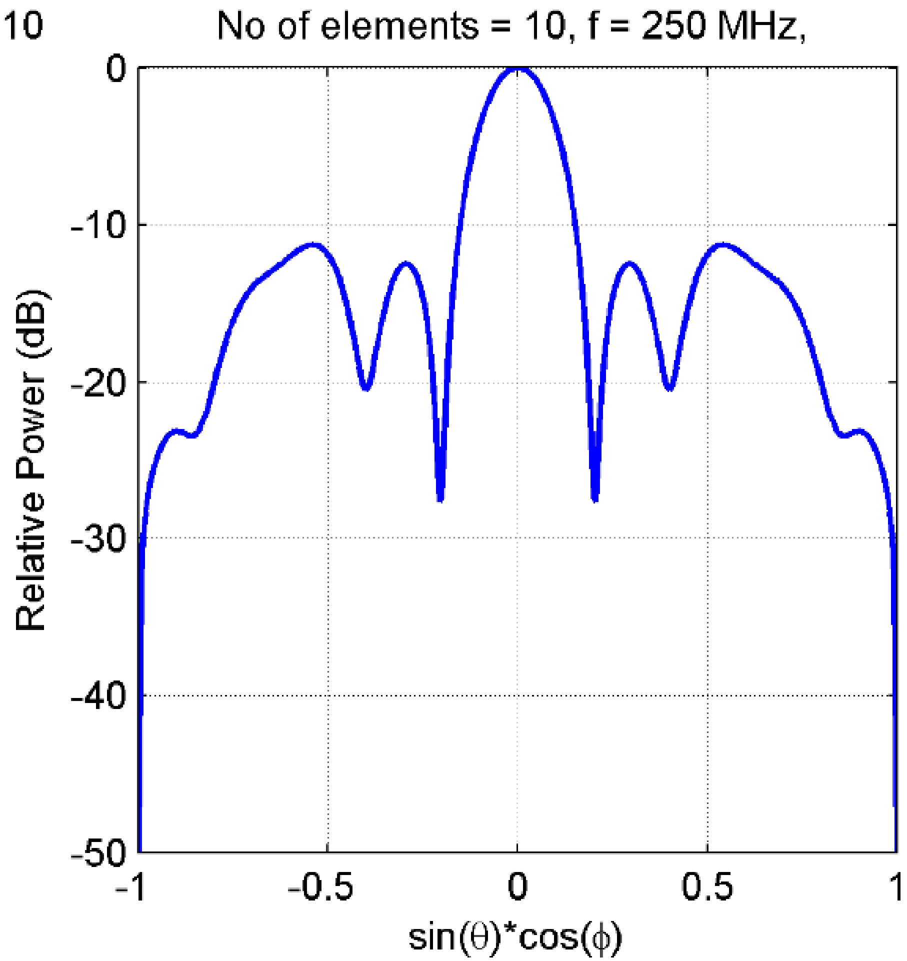
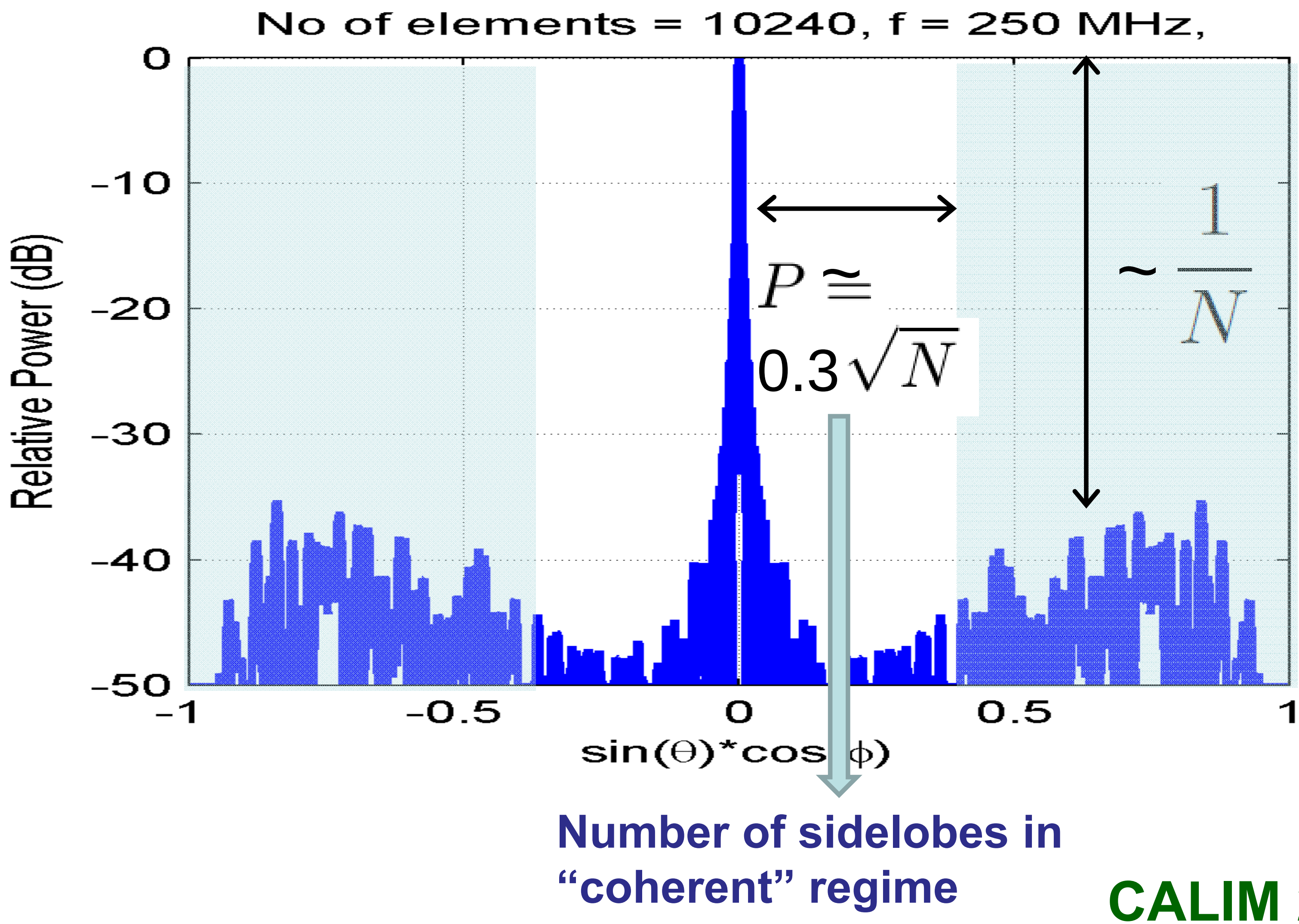
Patterns versus size of array





Coherent & incoherent regimes



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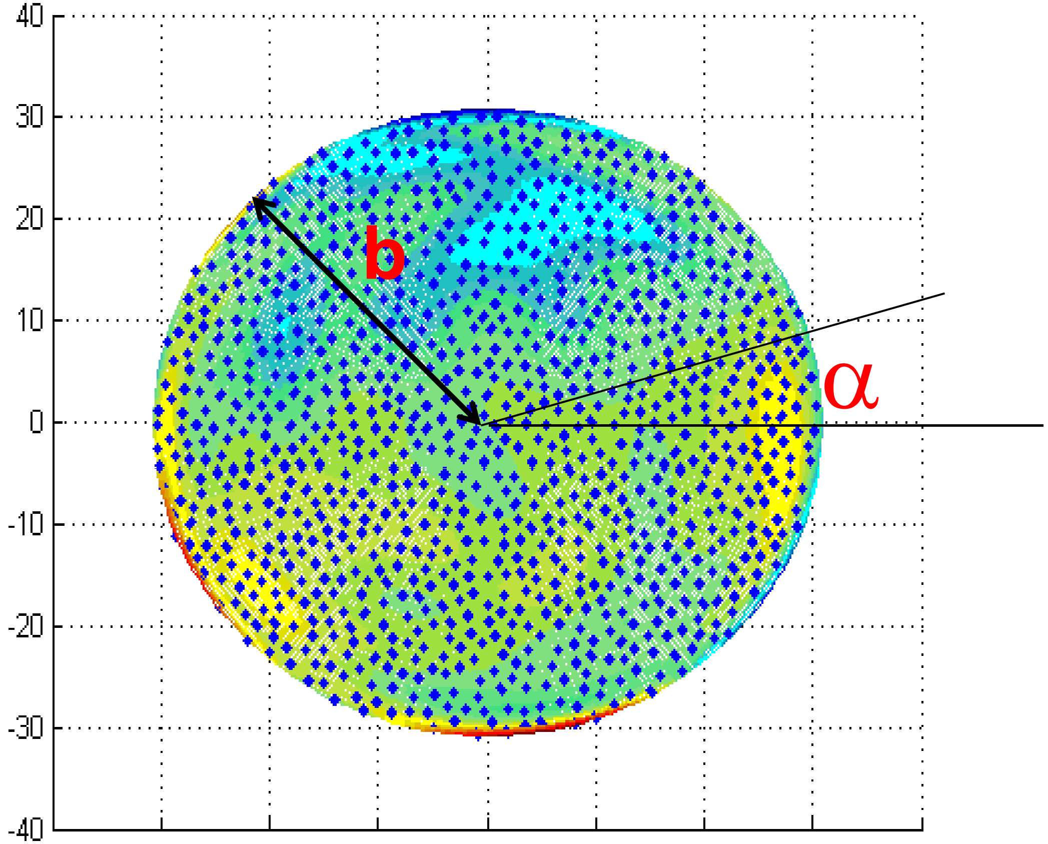
Aperture field representation



$$f(r,\alpha) \simeq \sum_{n=-N}^{N} a_n(r)\, e^{j\,n\,\alpha} \qquad a_n(r) = \frac{1}{2\,\pi}\int_0^{2\pi} f(r,\alpha)\, e^{-j\,n\,\alpha}\, d\alpha$$

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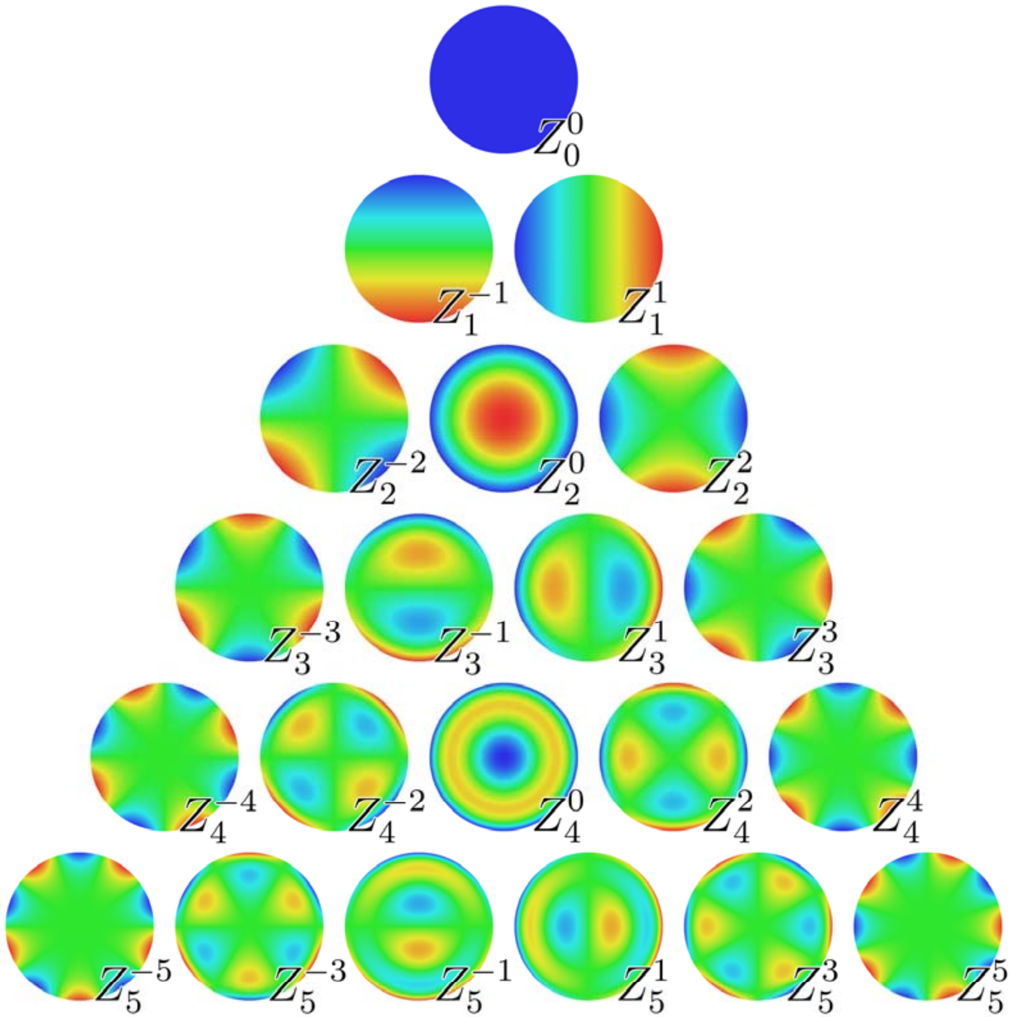
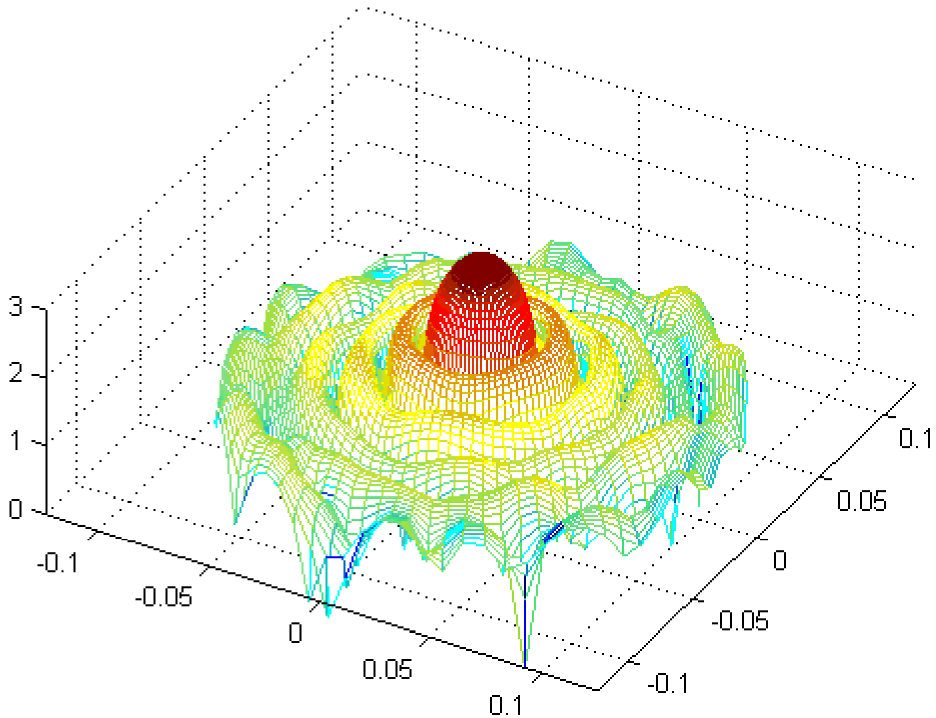
Zernike functions



Picture from Wikipedia

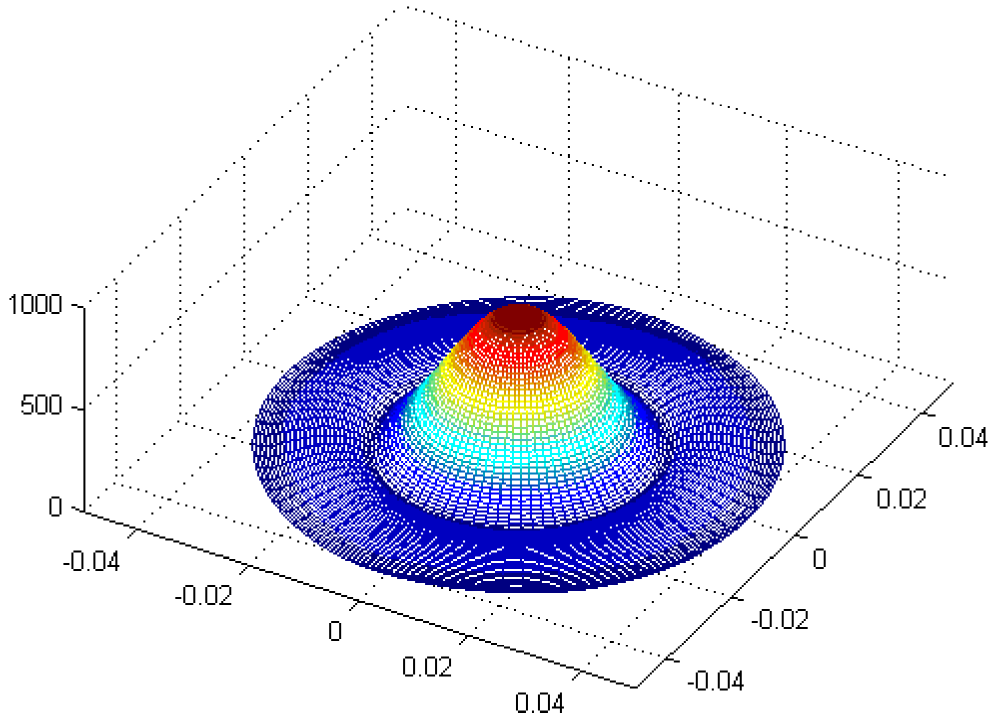
Polynomial	Fourier-Bessel	Zernike-Bessel
Fast functions	Good 1st order	Good 1st order
weaker at low orders	weak direct convergence	fast direct convergence

Array factor



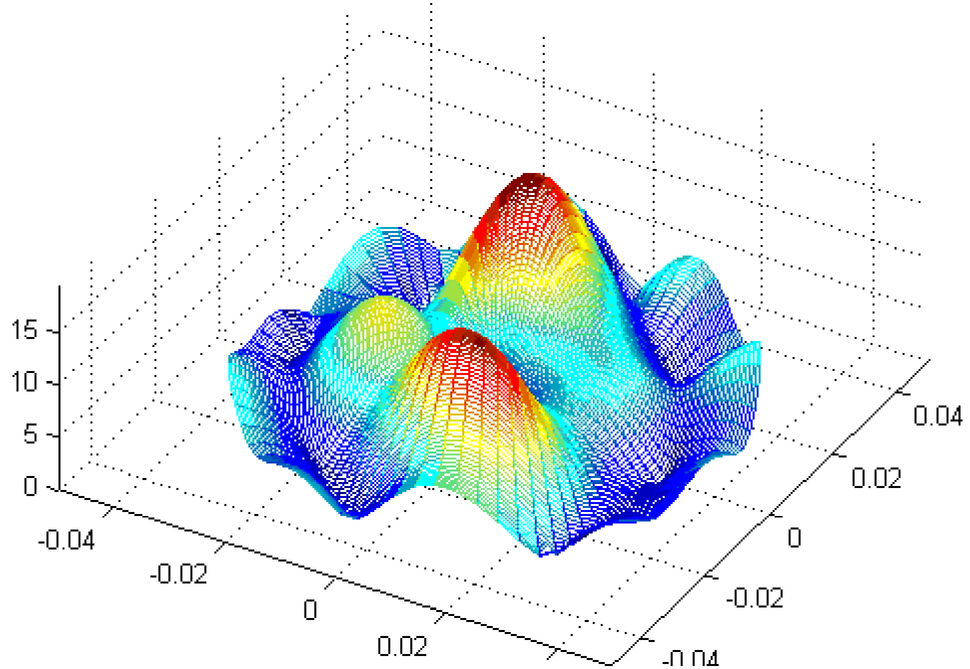
with apodization

Array factor



with apodization

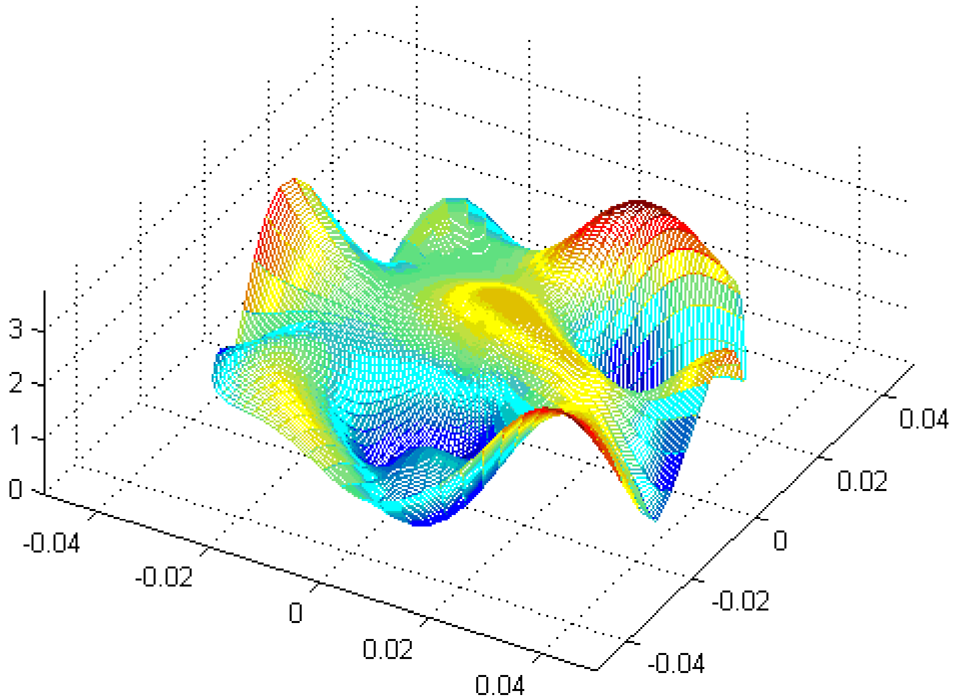
Apodization function *w(r)* extracted



$$F_0(\theta, \alpha) = 2\pi\, q \int_0^b w(r)\, J_0(k\, r\, \sin\theta)\, r\, dr$$

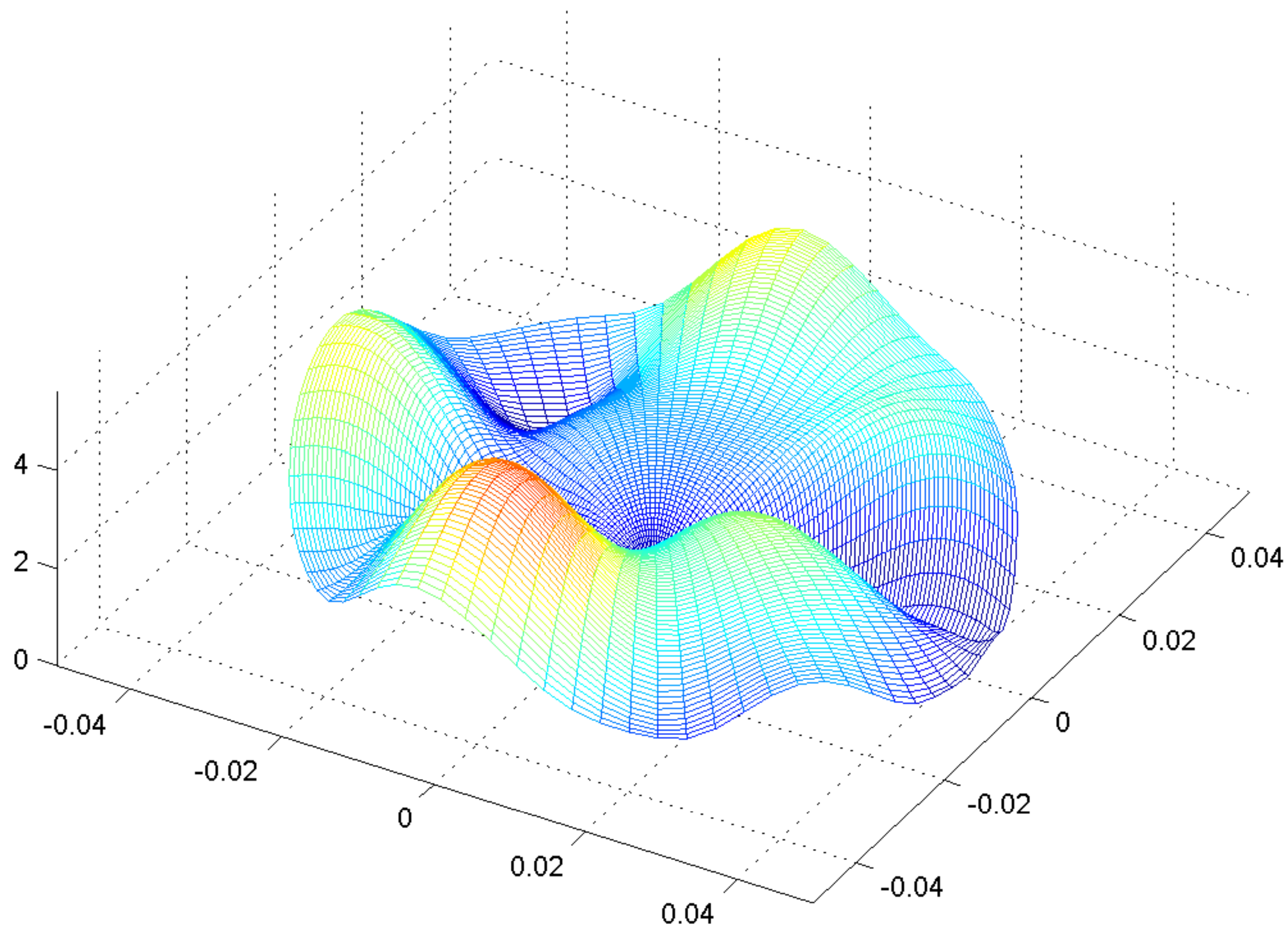
$$q = N/(\pi\, b^2)$$

Approximate array factor extracted



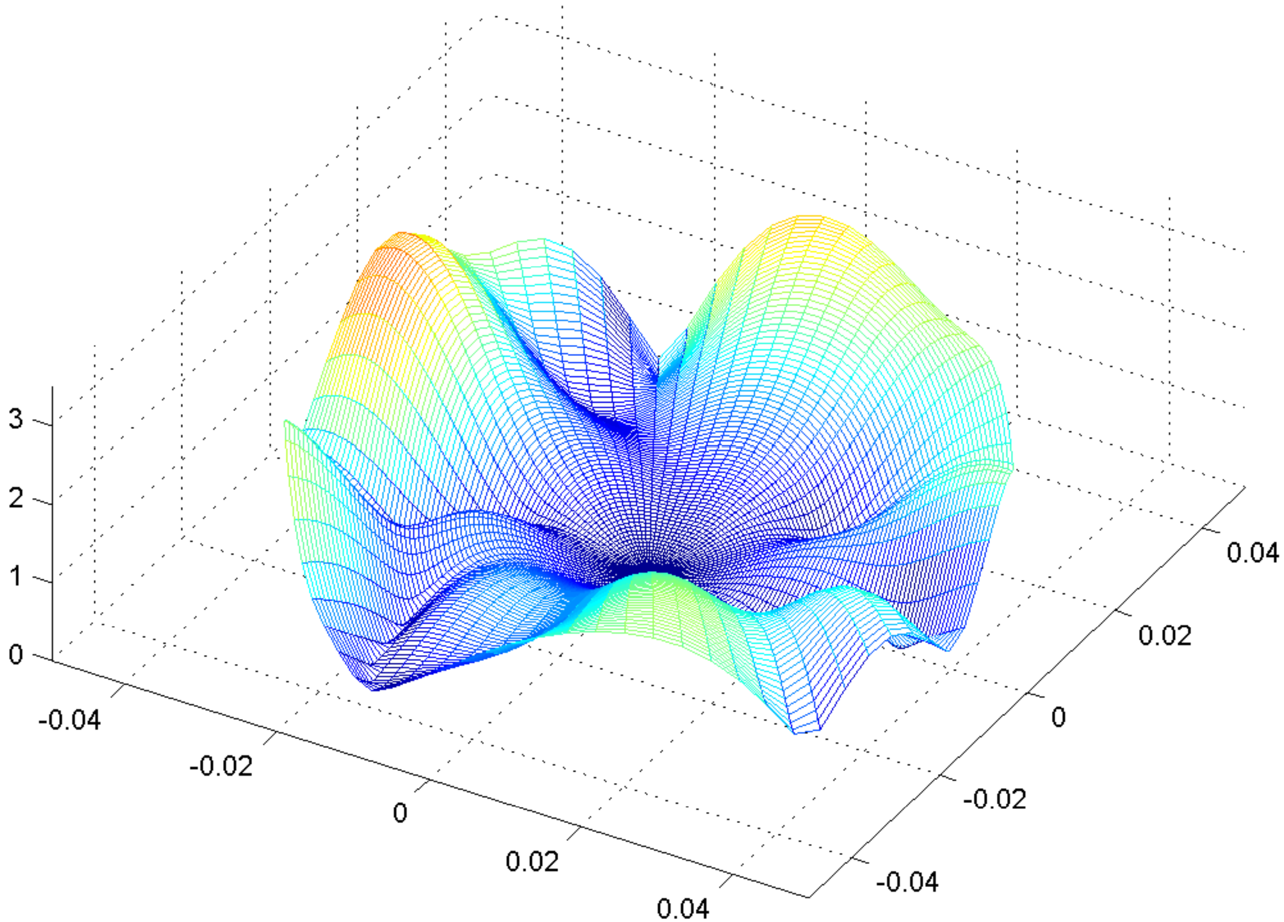
20 % error on amplitudes
λ/4 error on positions (at 300 MHz)

1

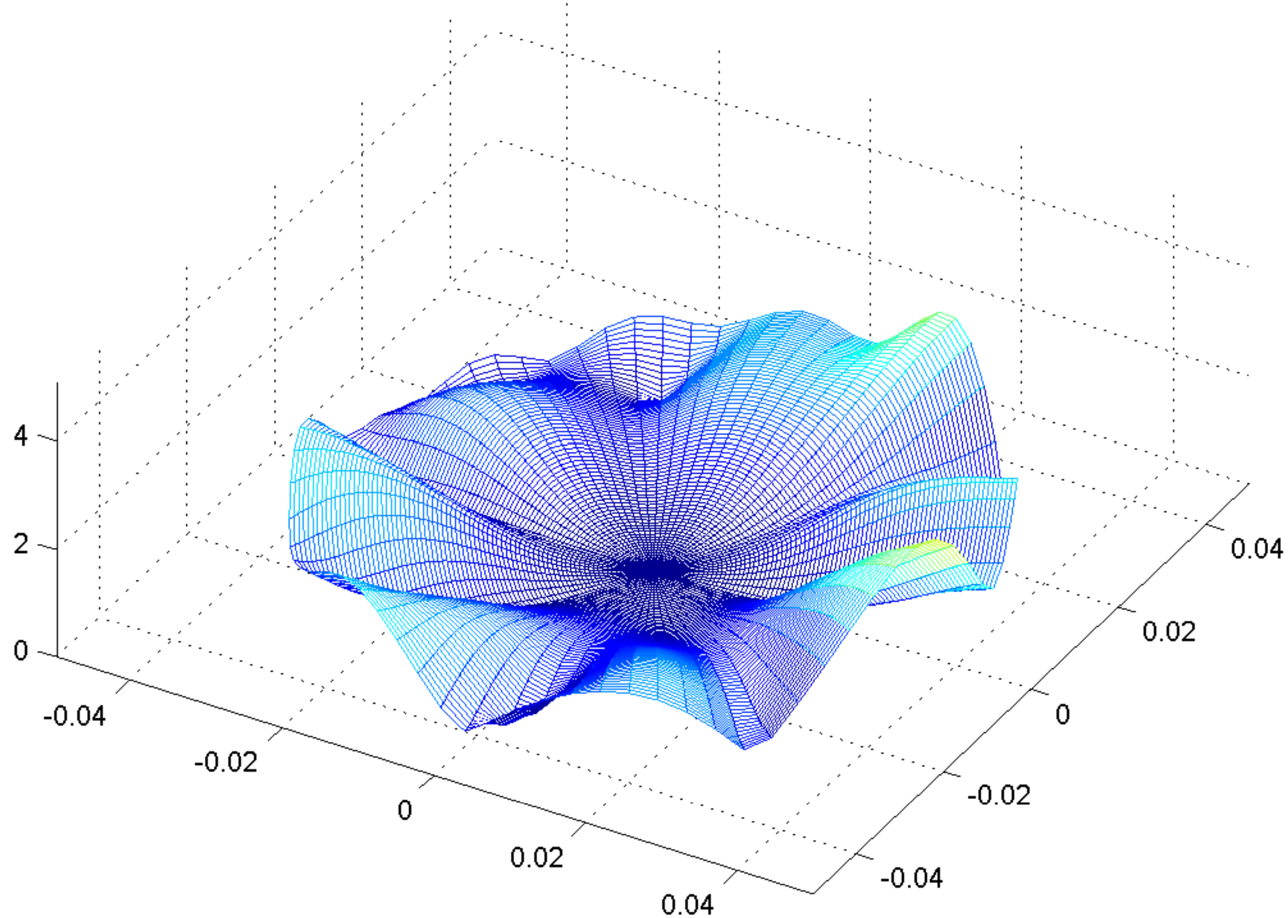


Residual error with Z-B approach

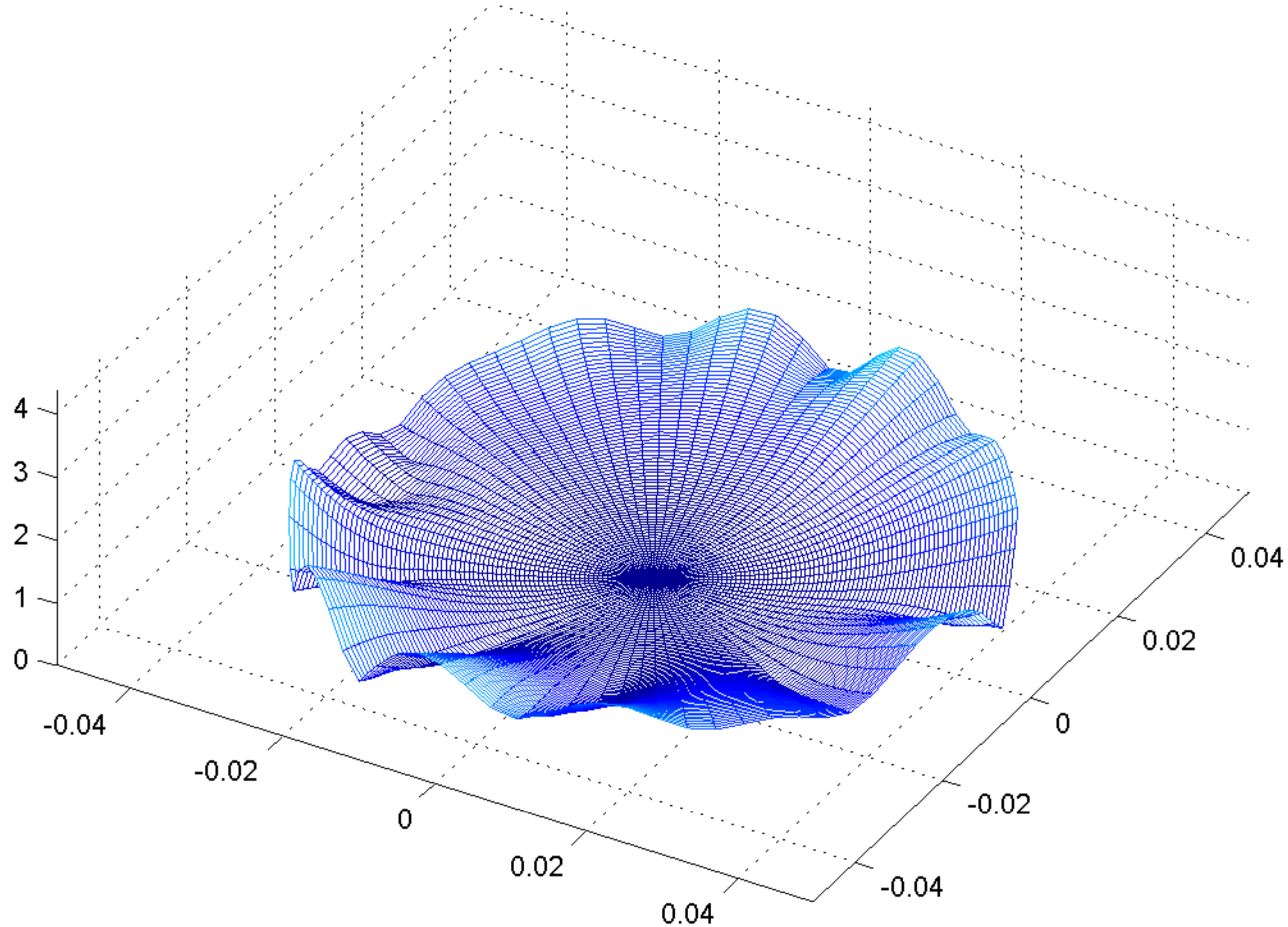
6



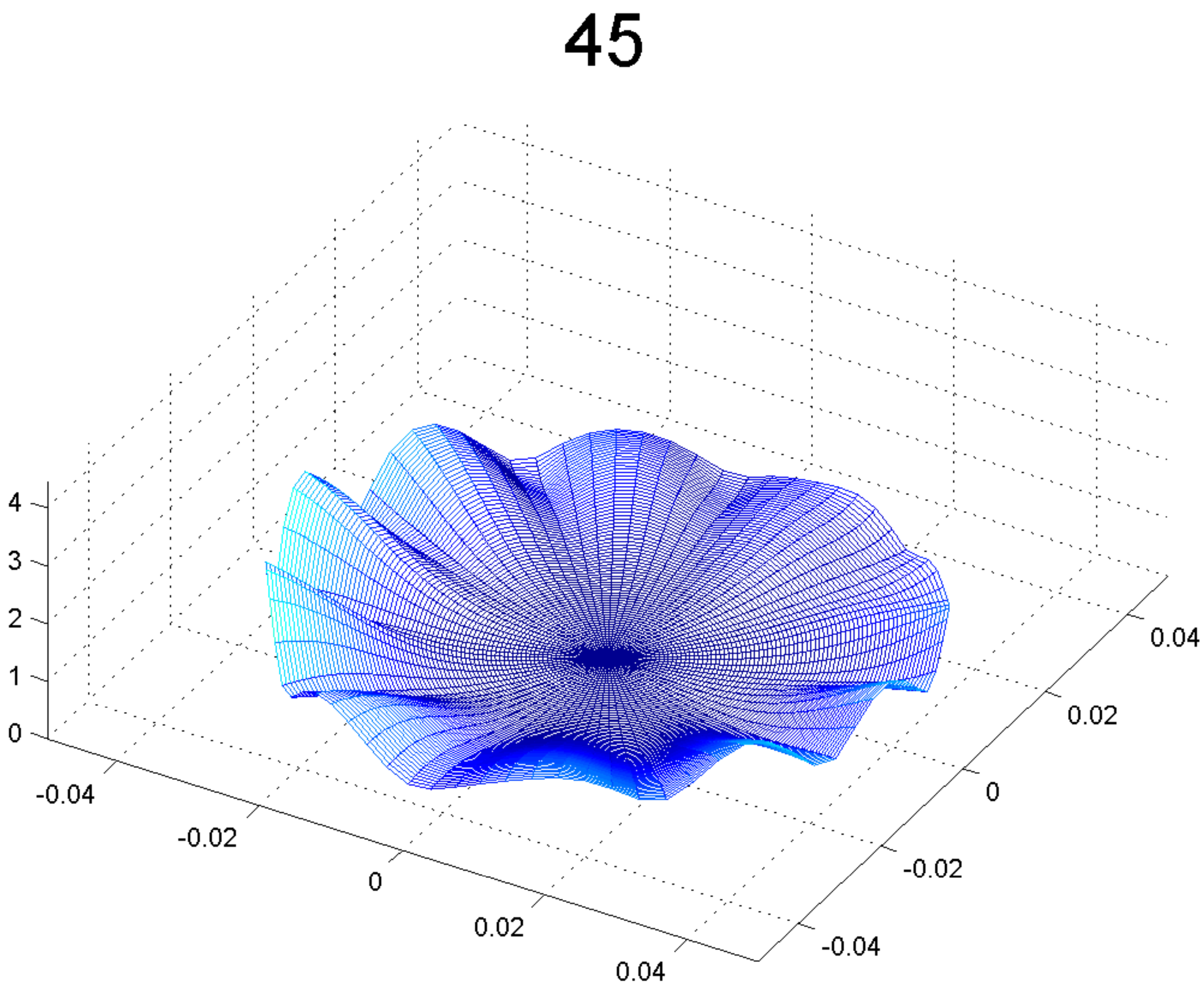
Residual error with Z-B approach



Residual error with Z-B approach



Residual error with Z-B approach



Residual error with Z-B approach

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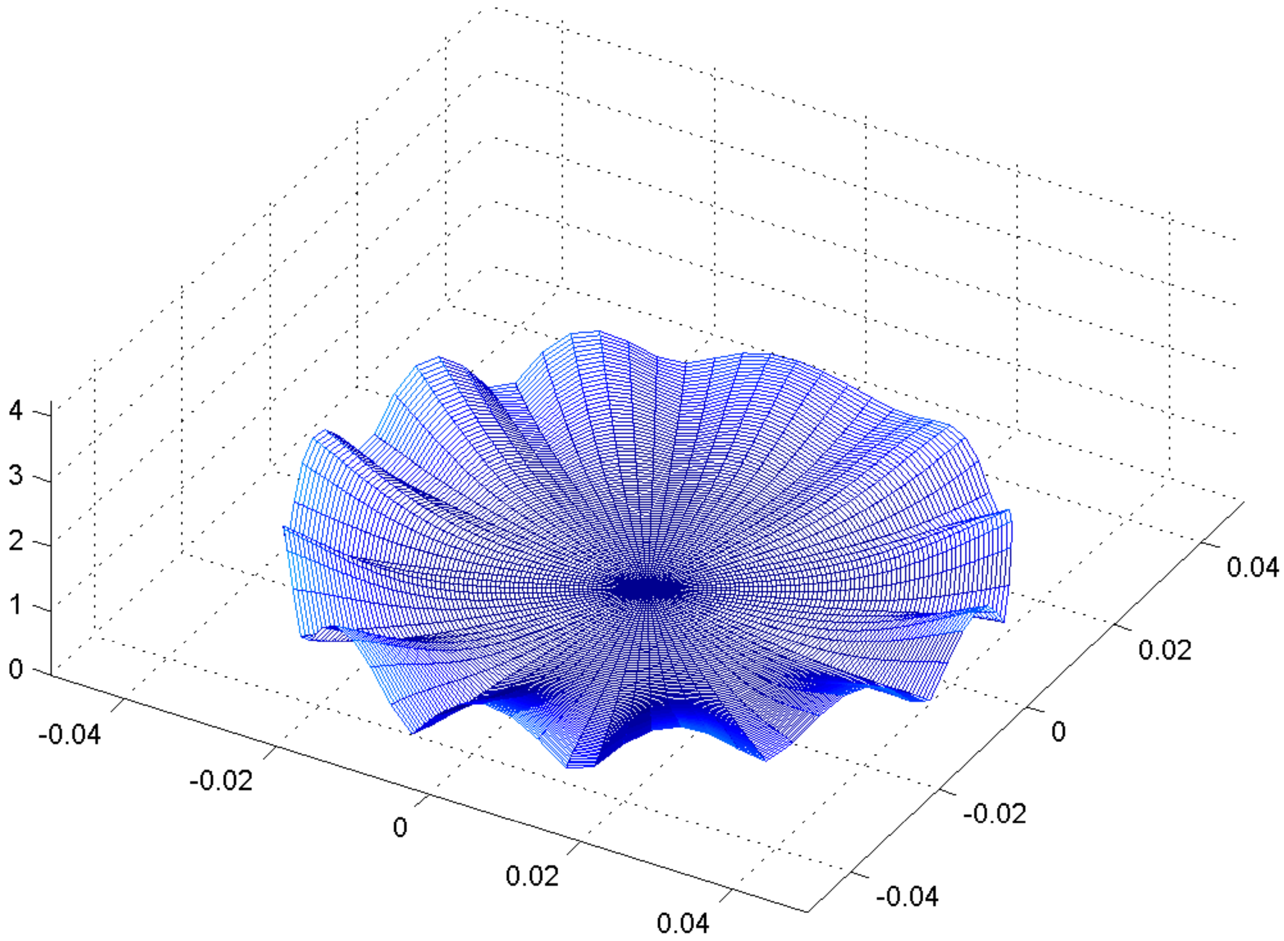
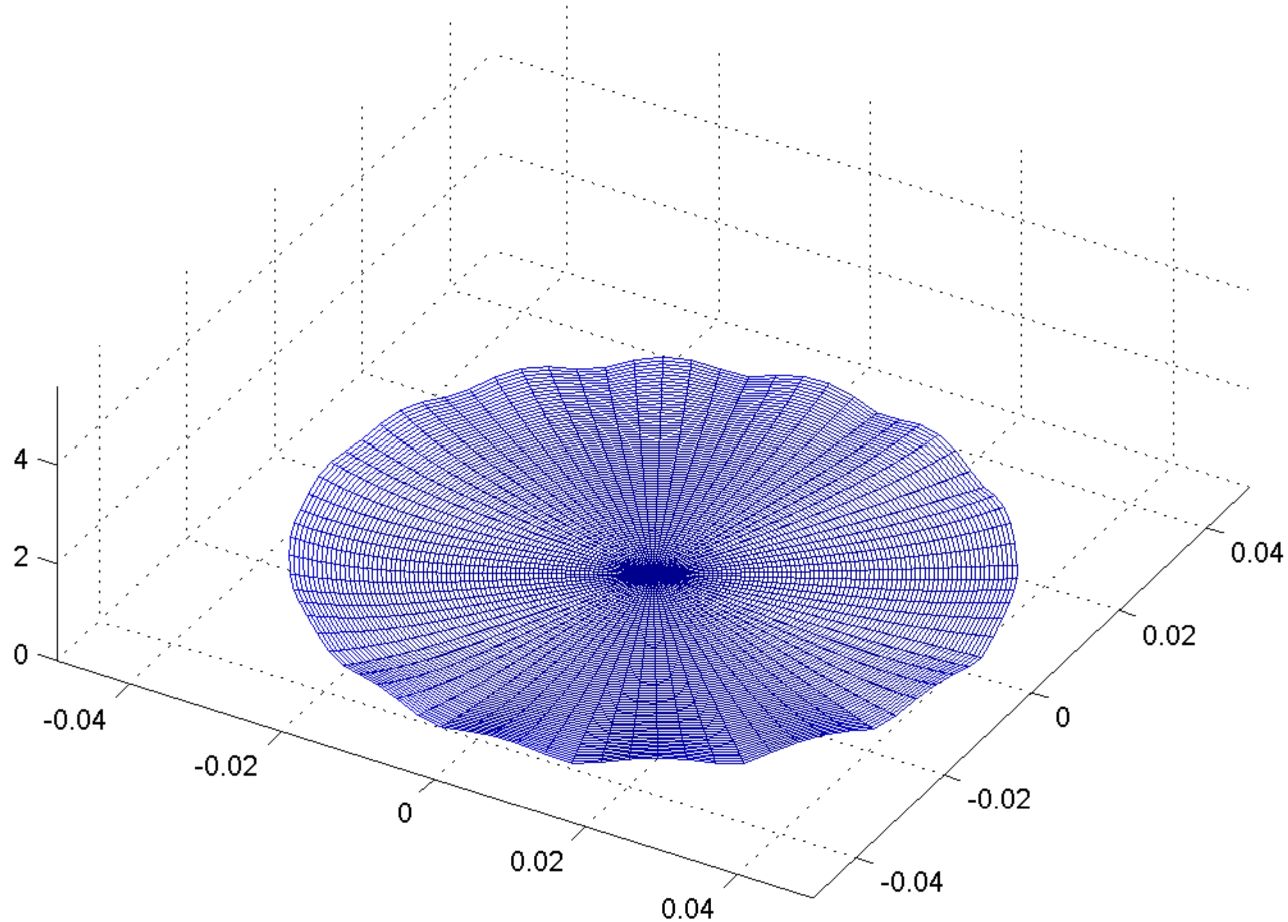


Residual error with Z-B approach



Residual error with Z-B approach

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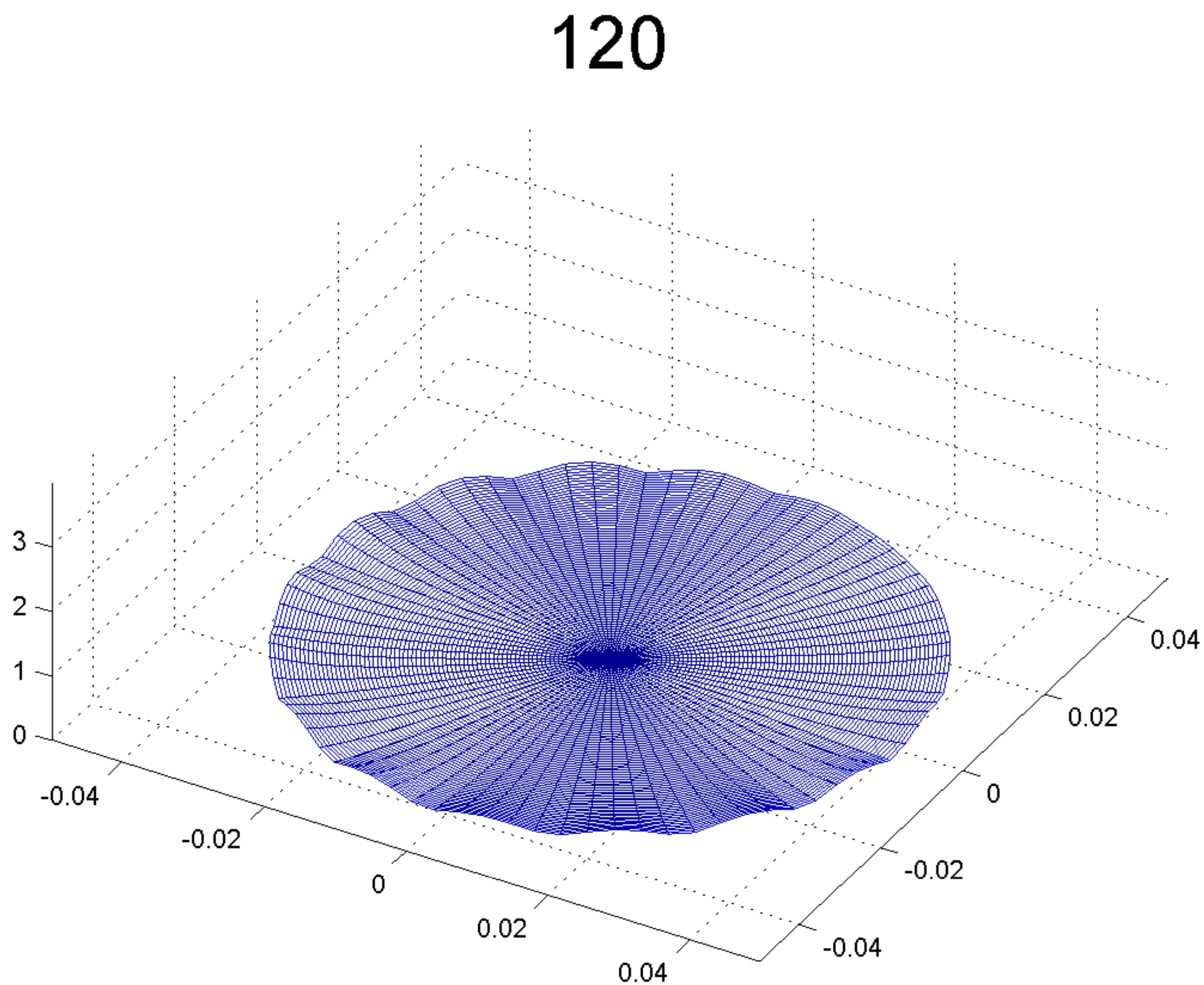

Residual error with Z-B approach

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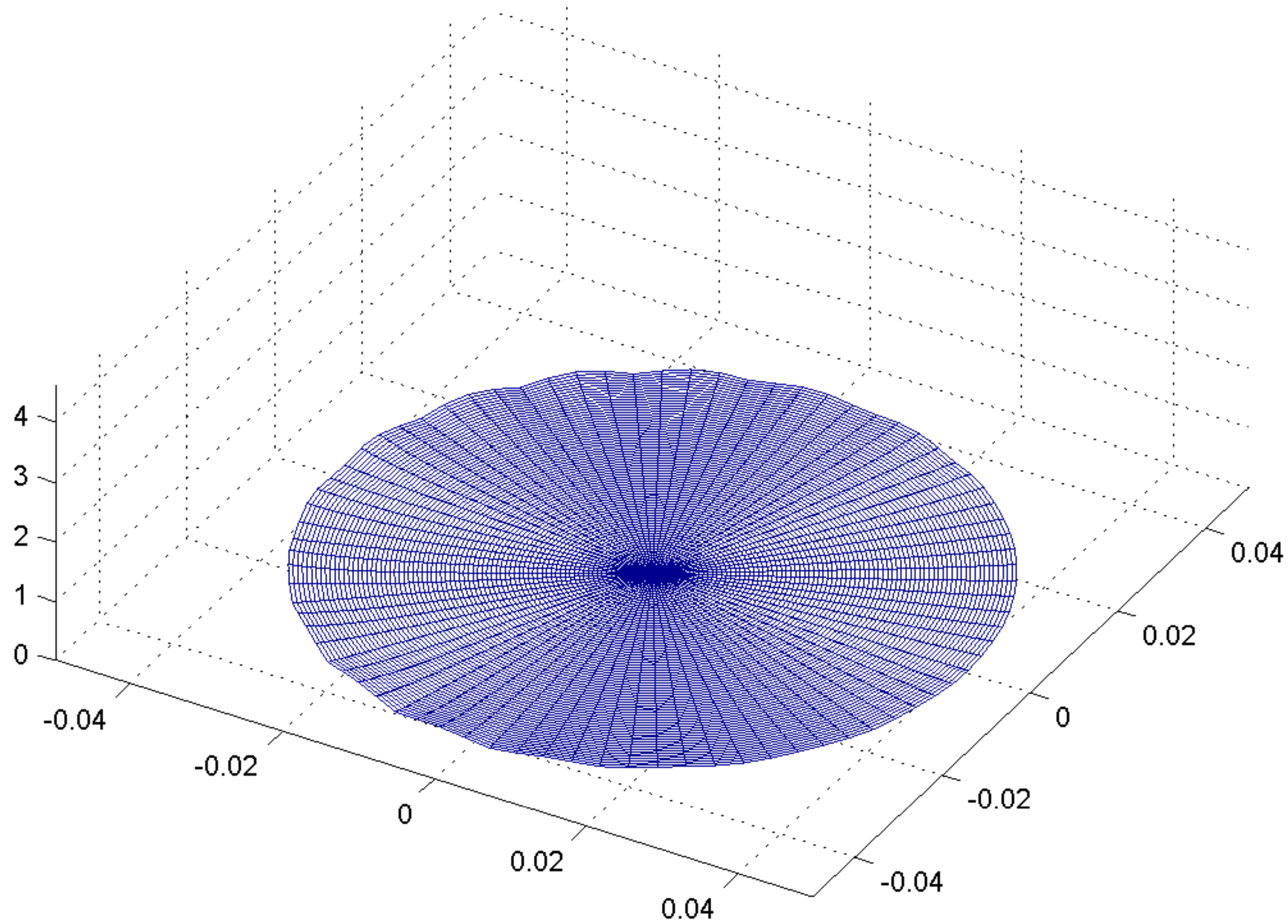
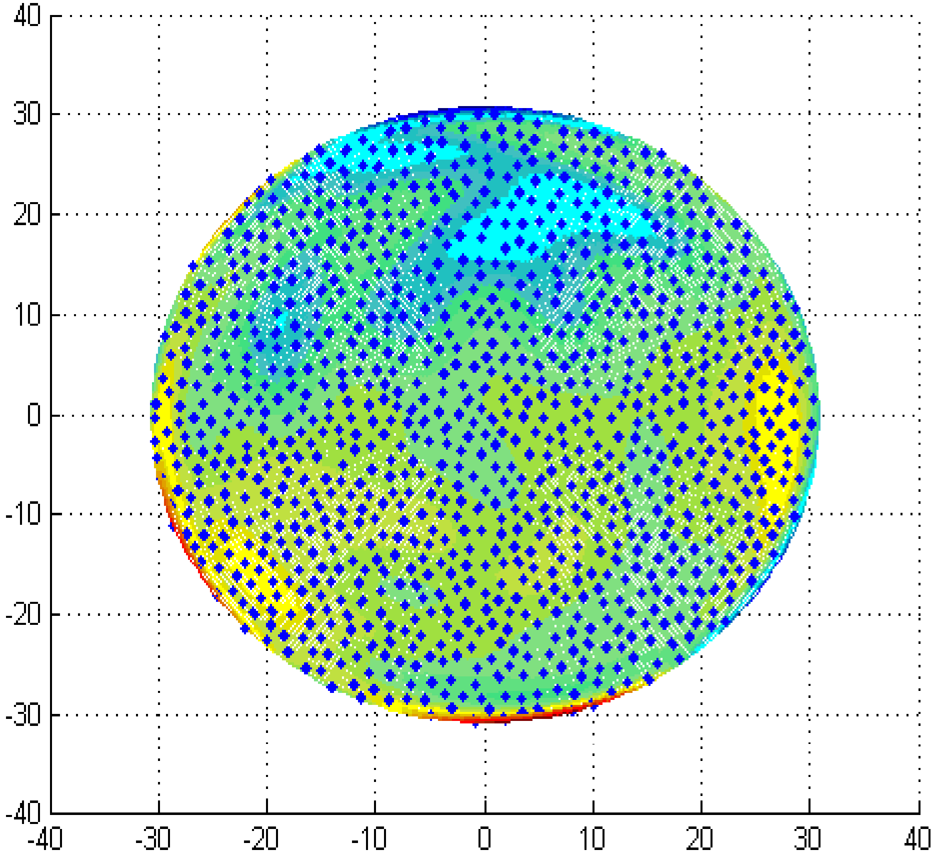


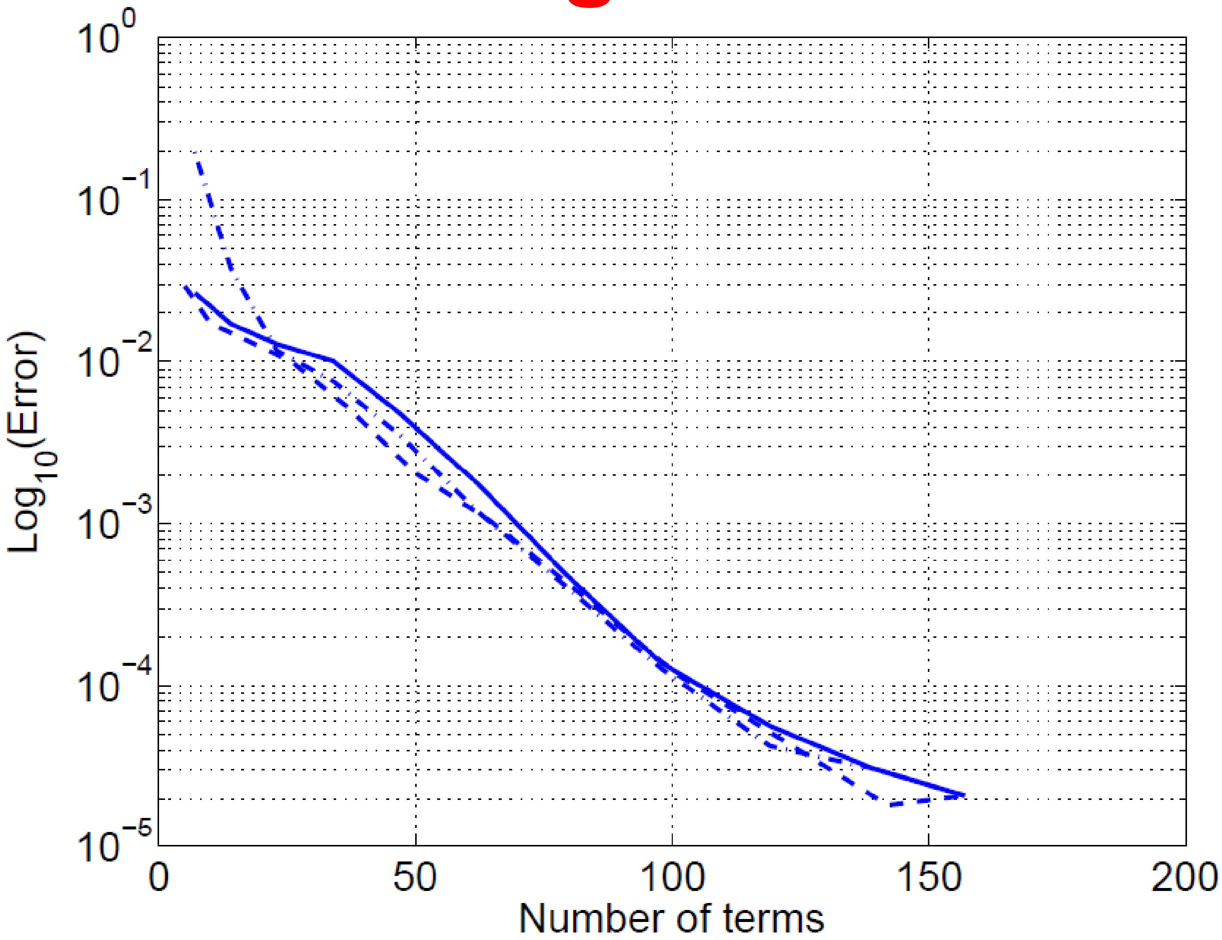
Residual error with Z-B approach

Density function



The number of terms tells the "resolution" with which density is observed

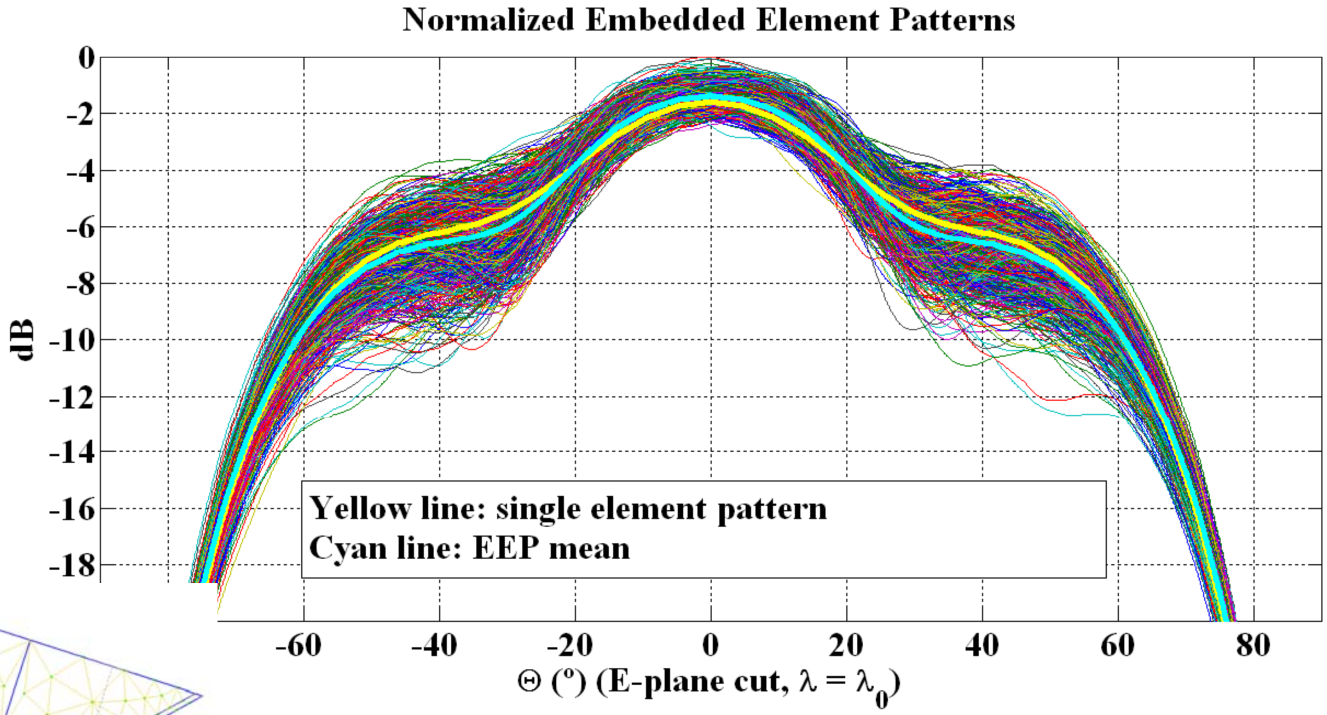
Convergence rate

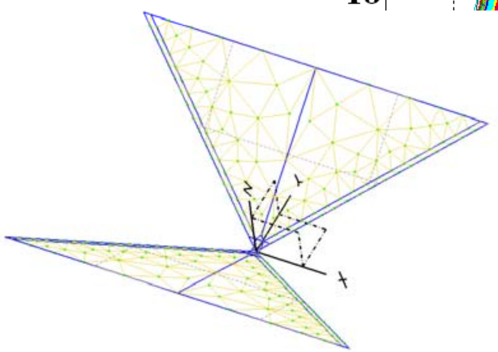


Errors versus number of terms for Fourier-Bessel (–), Zernike (-) and polynomial without "fundamental" (-.) approaches.

Effects of mutual coupling

$$\vec{F}_{arr} \neq F \ \vec{F}_{elt} \ !!!$$





Any compact representation ?!?

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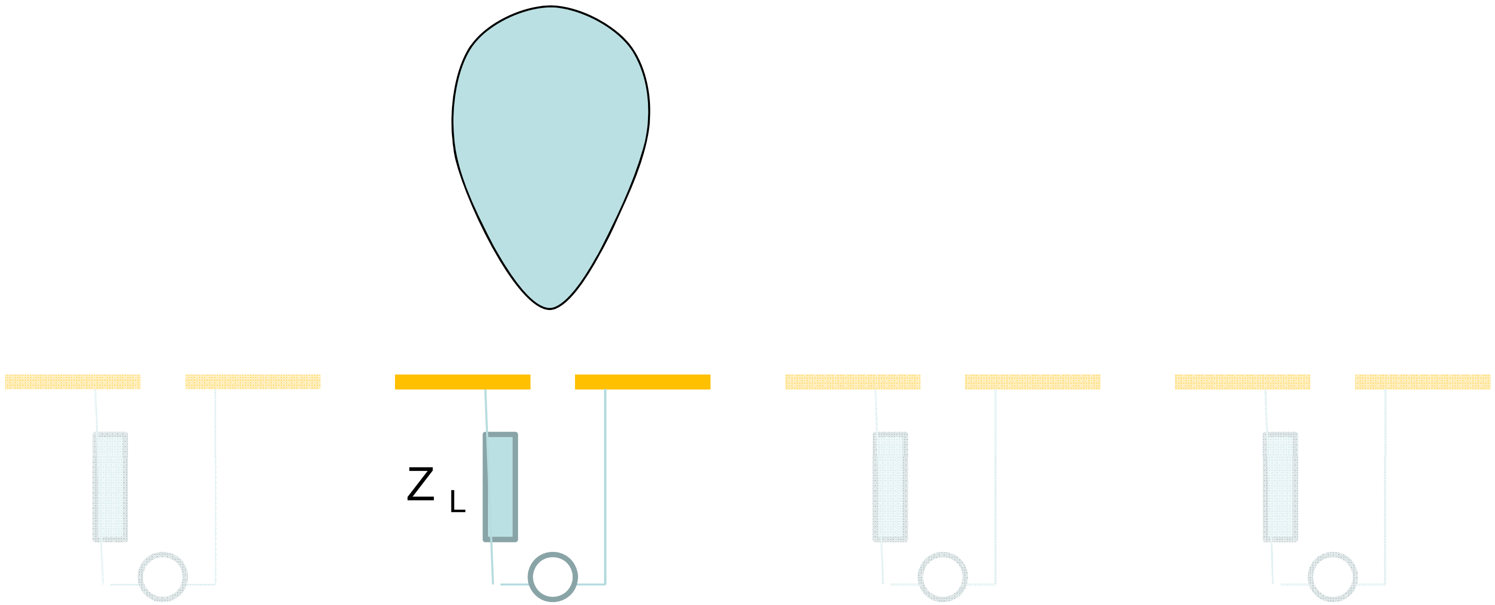
Isolated element pattern



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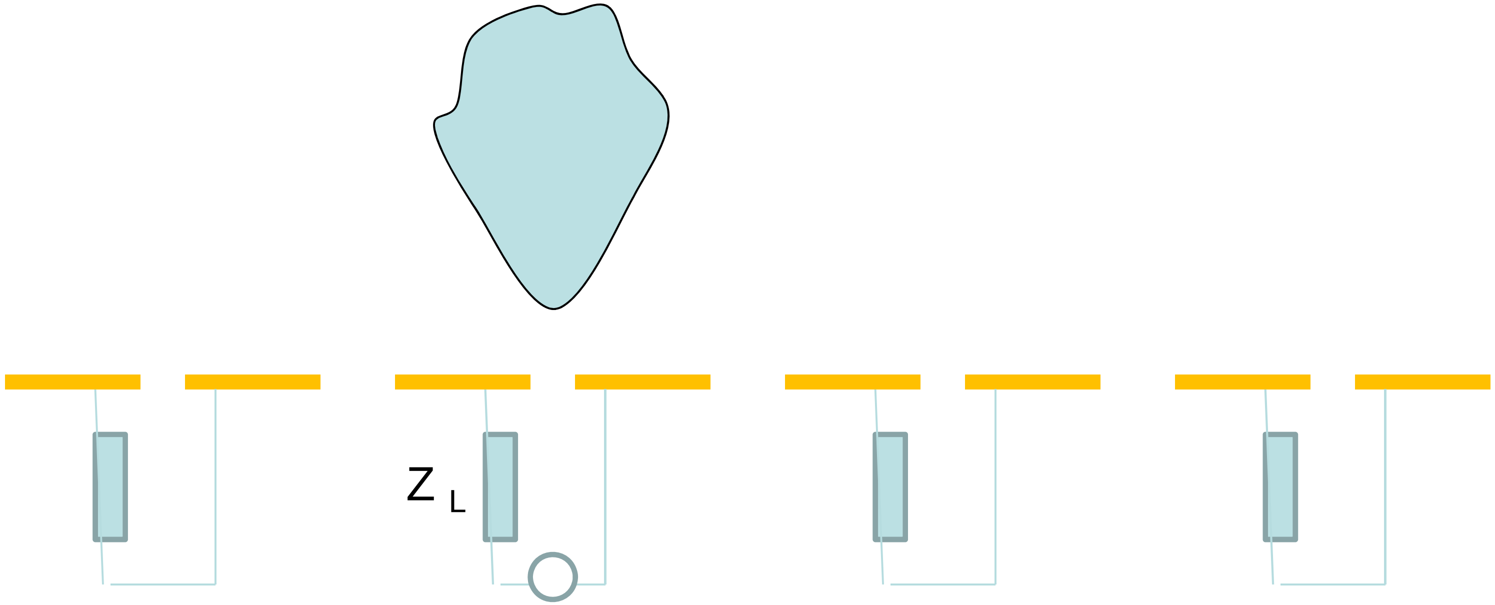
Embedded element pattern



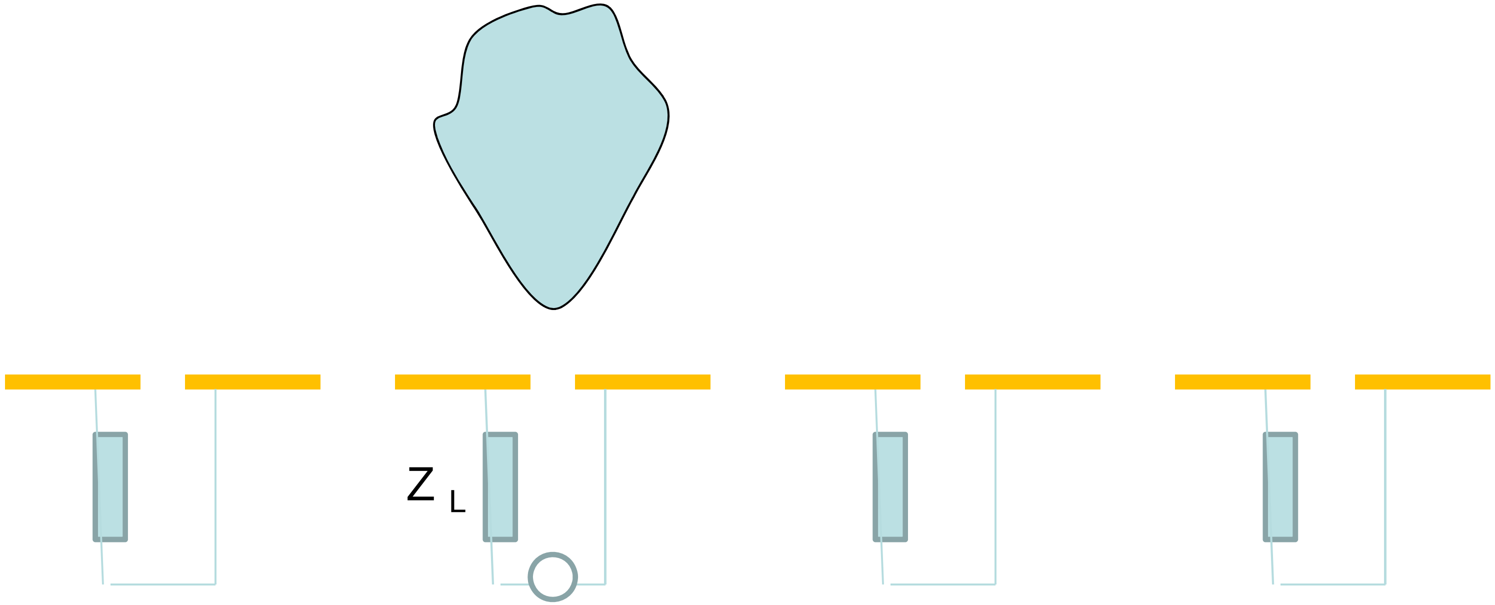
$$(\mathbf{Z}_d + \mathbf{Z}_L)\, \mathbf{g}^{nc} \simeq (\mathbf{Z} + \mathbf{Z}_L)\, \mathbf{g}^{e}$$

Impedance isolated elt

Array impedance matrix

To get voltages in uncoupled case: multiply voltage vector to the left by matrix

Embedded element pattern



After correction, we are back to original problem, with (zoomable, shiftable) array factor

Gupta, I., and A. Ksienski (1983), Effect of mutual coupling on the performance of adaptive arrays, IEEE Trans. Antennas Propag., 31(5), 785–791.

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Minimum-scattering antenna

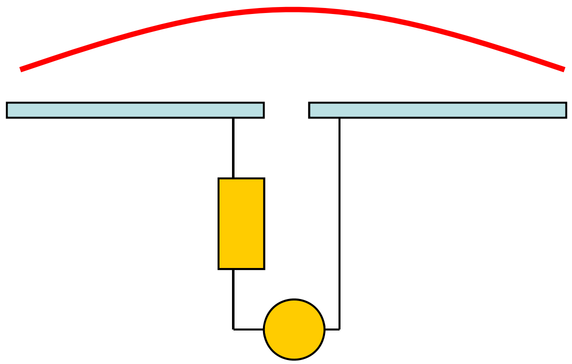
$$\mathbf{g}^{oc} = (\mathbf{Z} + \mathbf{Z}_L)\,\mathbf{g}^{e}$$

Approximation: o.c. pattern is uncoupled pattern, within constant factor:

$$\mathbf{g}^{oc} \simeq (\mathbf{Z}_d + \mathbf{Z}_L)\,\mathbf{g}^{nc}$$

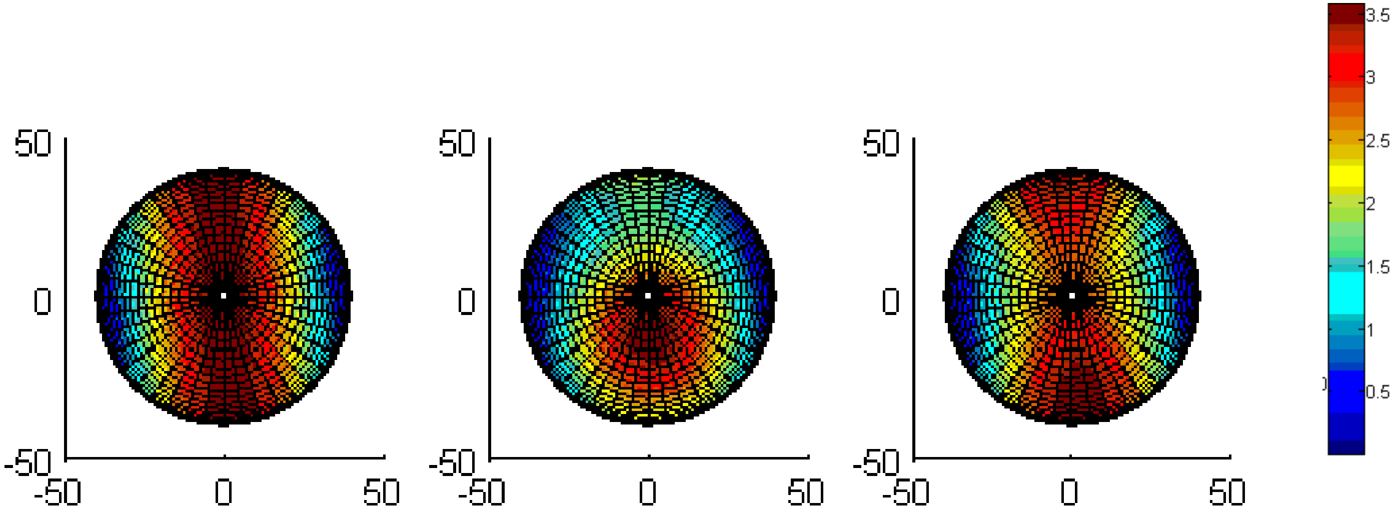
i.e. open antennas ~ invisible

Example: single-mode antenna:



Mutual coupling correction

Half-wave dipole

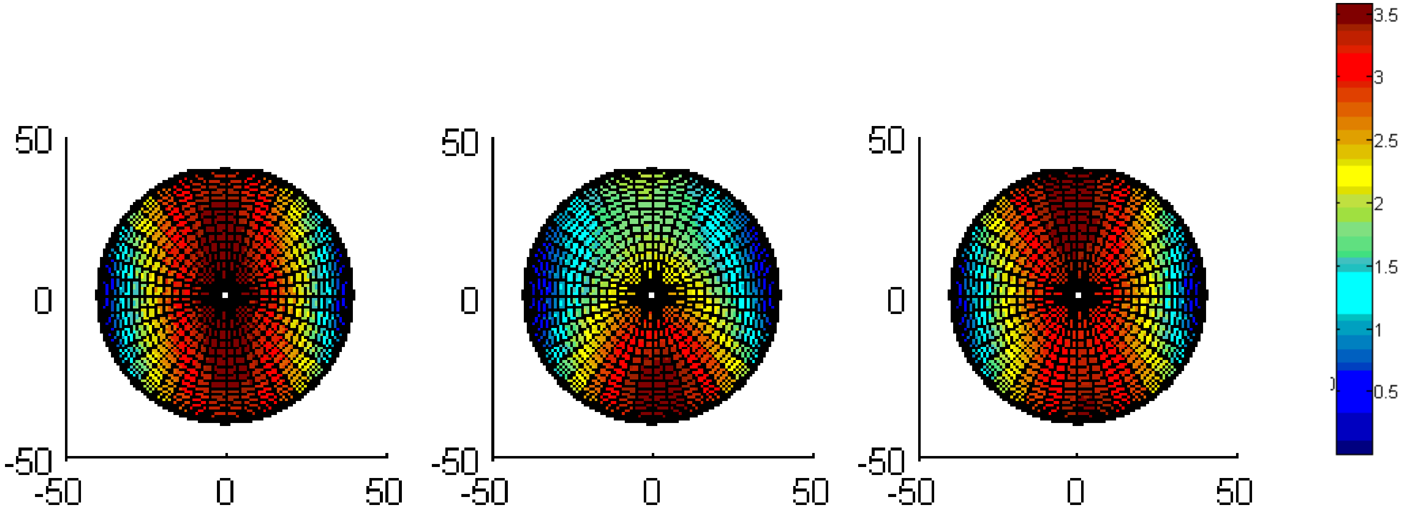


Isolated **Embedded** **Corrected**

Mutual coupling correction

Bowtie antenna

l=1.2 m, λ=3.5 m

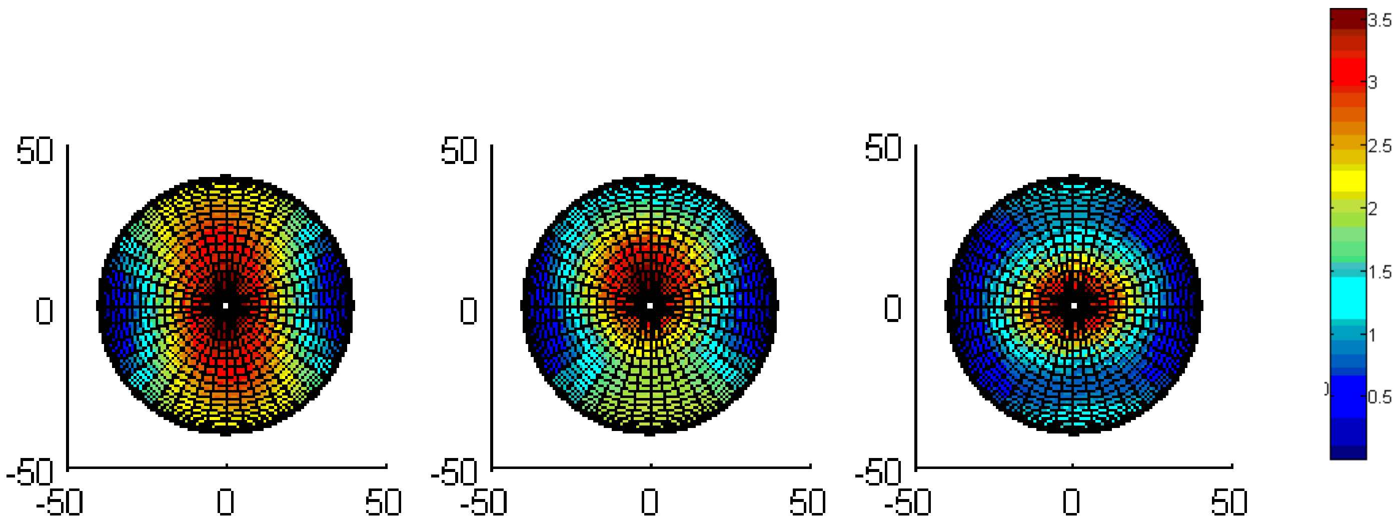


Isolated **Embedded** **Corrected**

Mutual coupling correction

Bowtie antenna

l=1.2 m, λ=1.5 m



Isolated **Embedded** **Corrected**

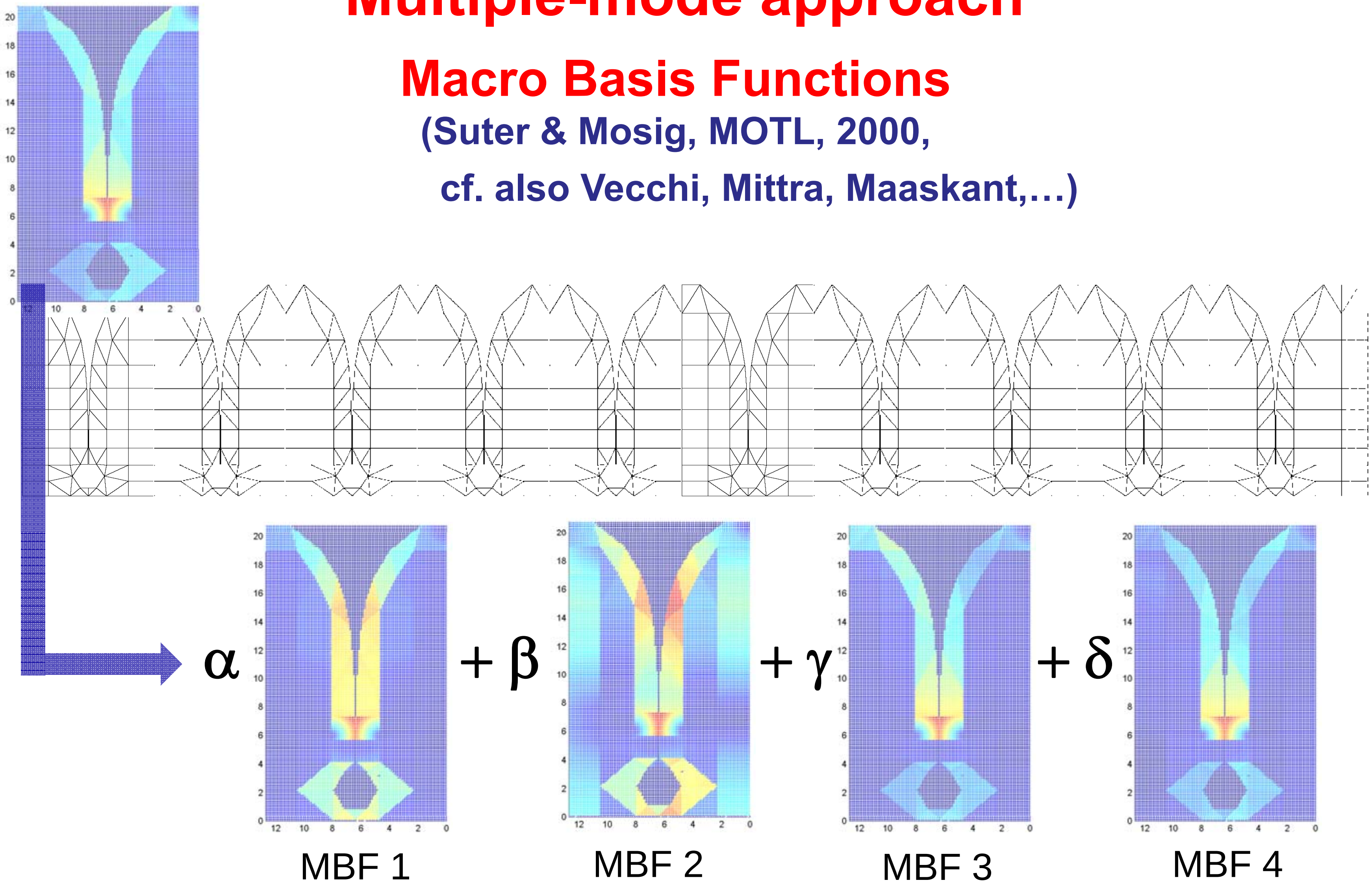
Single-mode assumption not valid for l ≲ λ/2

Multiple-mode approach

Macro Basis Functions

(Suter & Mosig, MOTL, 2000,

cf. also Vecchi, Mittra, Maaskant,…)



MBF interactions (1)

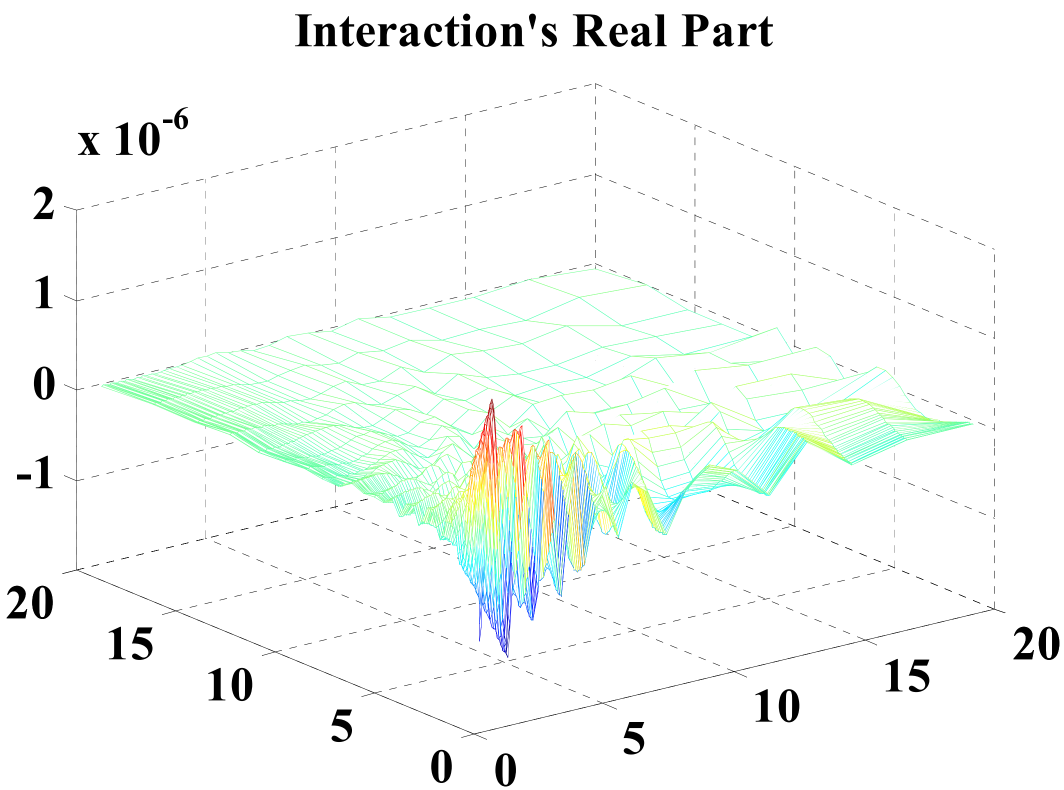
Subtracting the far field component

↓

Dividing by the phase term

↓

Change of variable



MBF interactions (2)

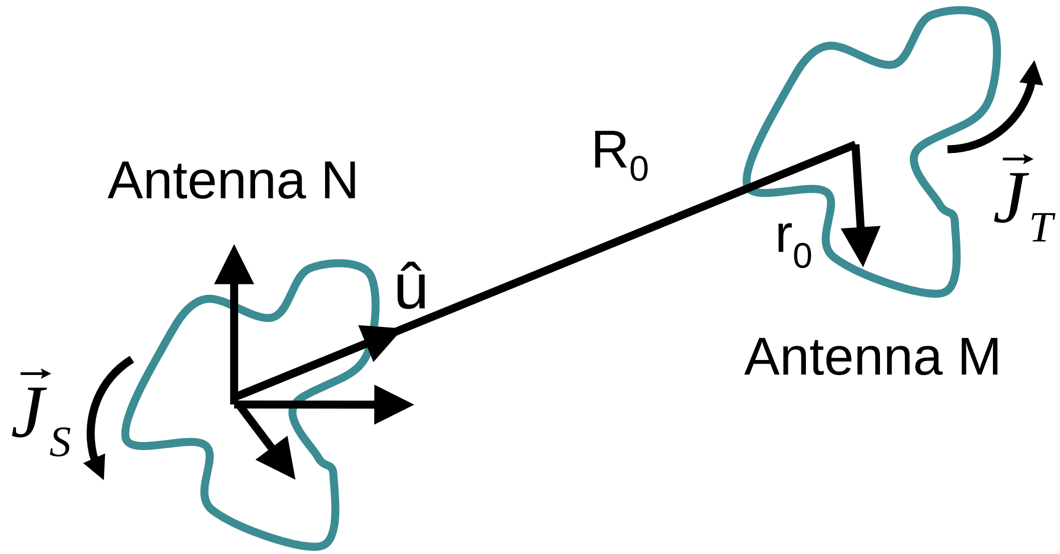
Subtracting the far field component

↓

Dividing by the phase term

↓

Change of variable



$$Z_{MN}(T,S) = \underline{j}_T^T \underline{\underline{Z}}_{MN} \underline{j}_S \approx \frac{-j\omega\mu}{4\pi R_0} e^{-jkR_0} \overline{F}_T(-\hat{u}) \cdot \overline{F}_S(\hat{u})$$

MBF interactions (1)

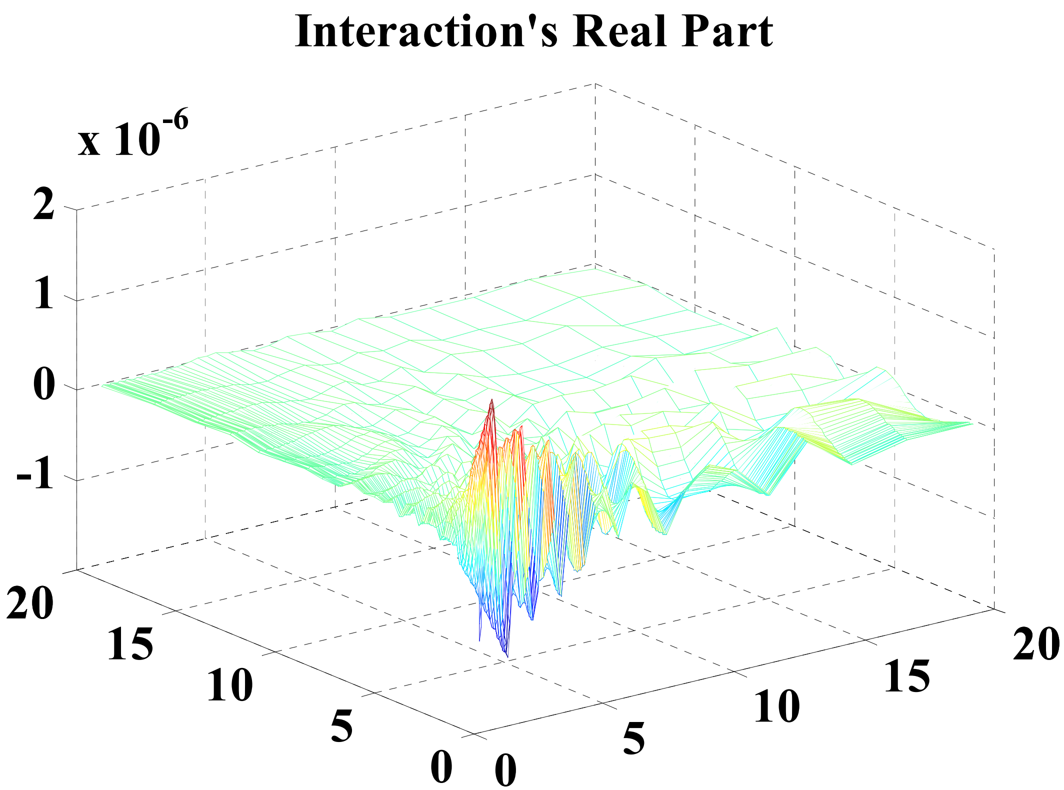
Subtracting the far field component

↓

Dividing by the phase term

↓

Change of variable



MBF interactions (2)

Subtracting the far field component

↓

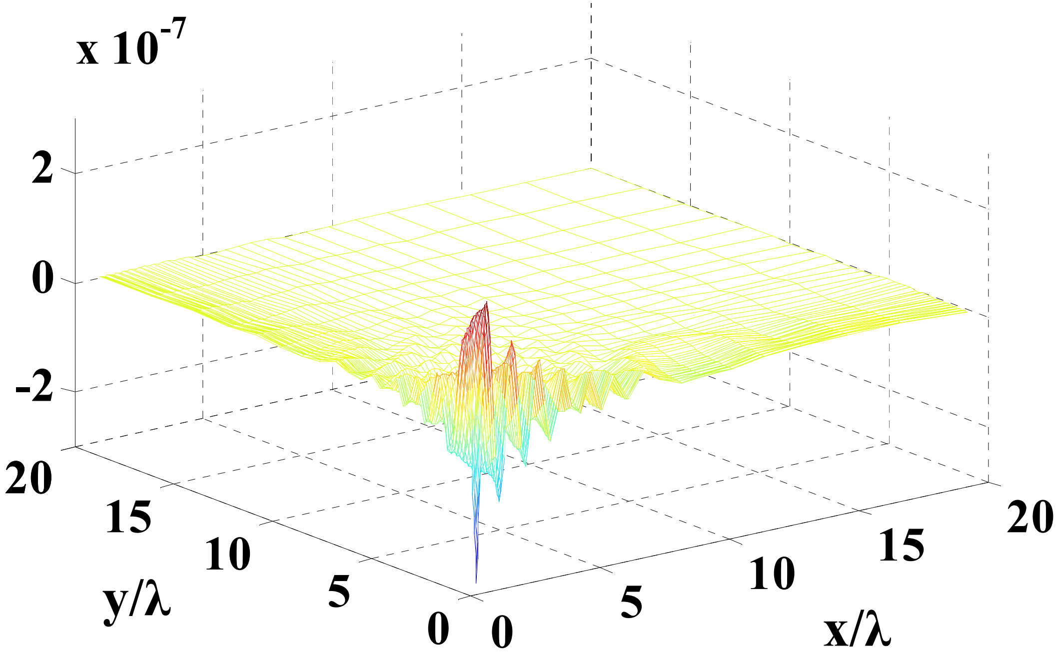
Dividing by the phase term

↓

Change of variable

$$Z_{MN}^{EXACT}(T,S) - Z_{MN}^{APPROX}(T,S)$$

Interaction's real part after FF extraction



MBF interactions (3)

Subtracting the far field component

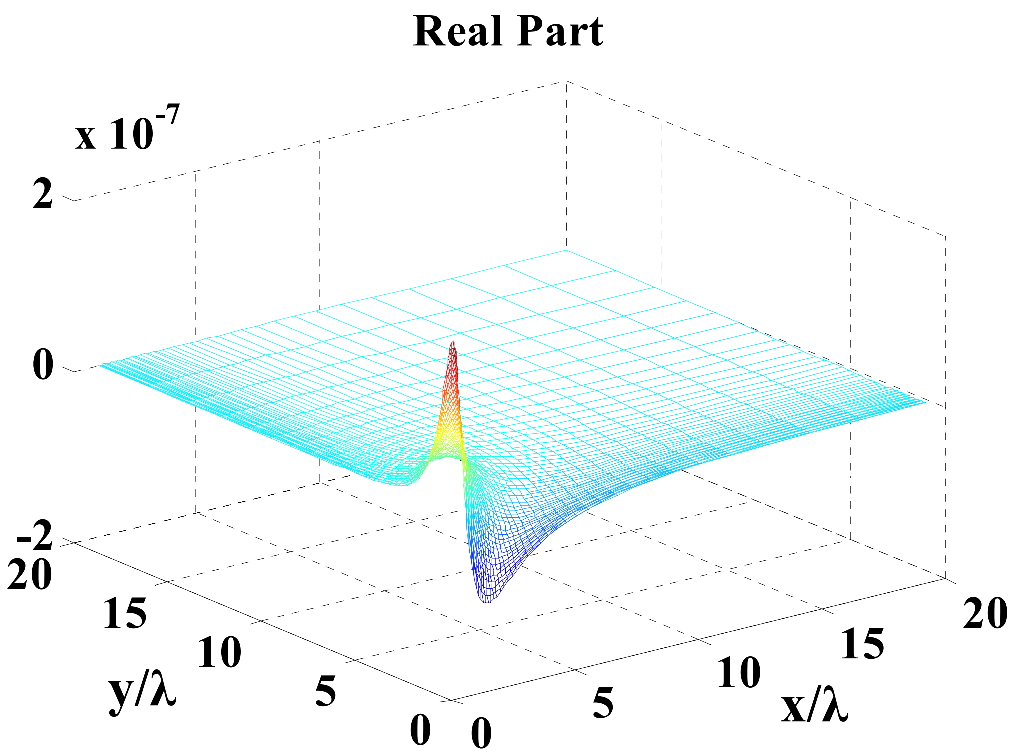
↓

Dividing by the phase term

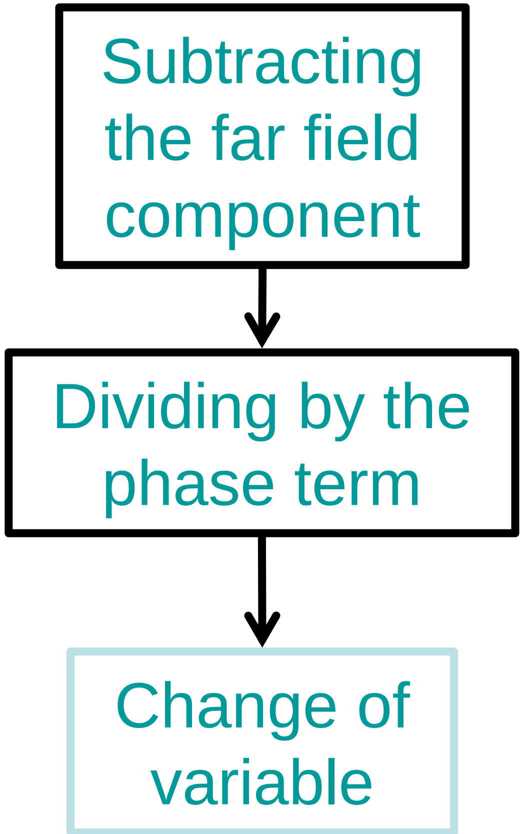
↓

Change of variable

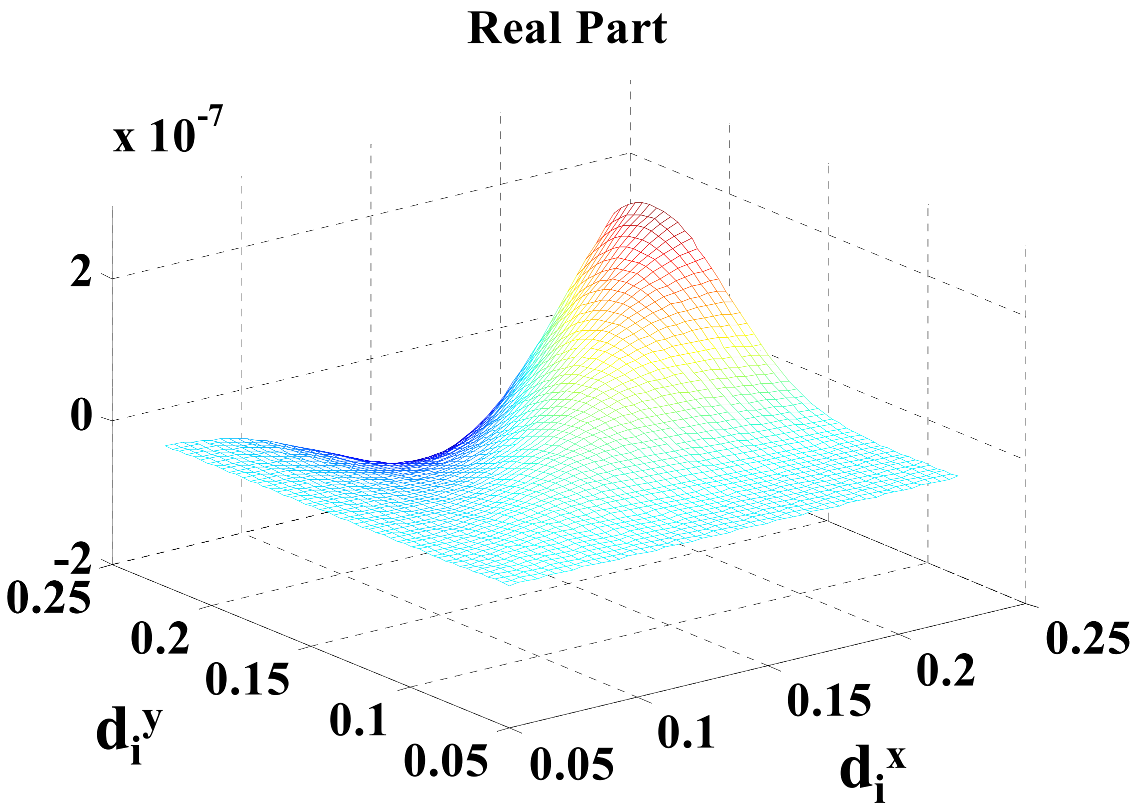
D. Gonzalez-Ovejero, C. Craeye, "Interpolatory Macro Basis Functions analysis of non-periodic arrays," IEEE Trans. Antennas Propagat., Aug. 2011.



MBF interactions (4)



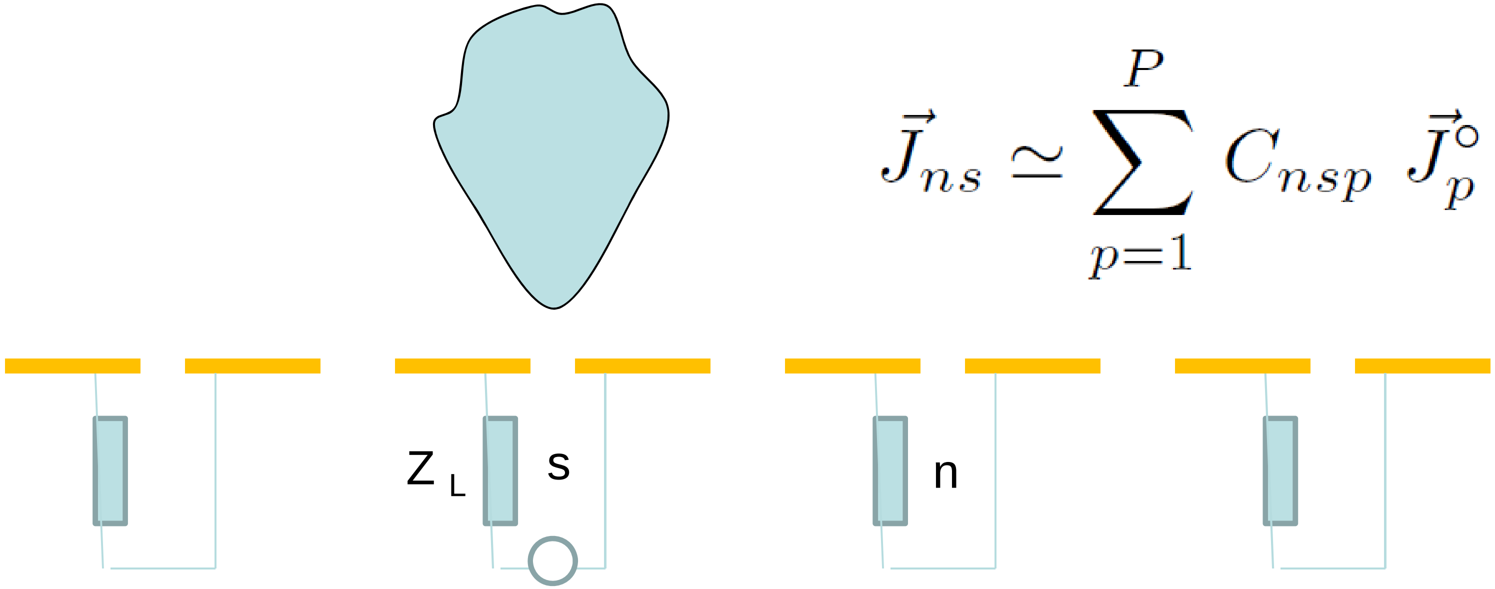
Smooth and easy to interpolate function!!



Comp. time indep. from complexity of antenna !!!

Array factorisation

$$\vec{J}_{ns} \simeq \sum_{p=1}^{P} C_{nsp}\ \vec{J}_p^{\circ}$$



	Antenna index		
MBF 1	C_{111}	C_{211}	C_{311}
MBF 2	C_{122}	C_{222}	C_{322}
⋮			

→ Coefficients for **IDENTICAL** current distribution

$$\vec{F}_s \simeq \sum_{p=1}^{P} \sum_{s=1}^{N} A_s \sum_{n=1}^{N} C_{nsp}\, e^{j\,k\,\hat{u}.\vec{r}_n}\ \vec{F}_p^{\circ}$$

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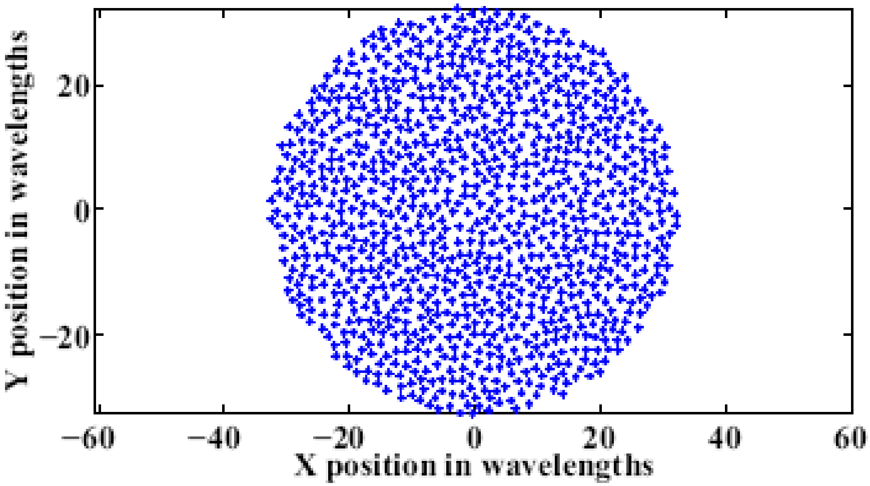
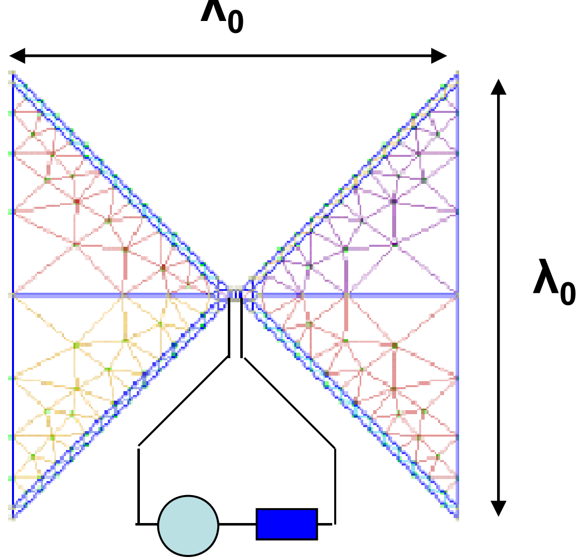
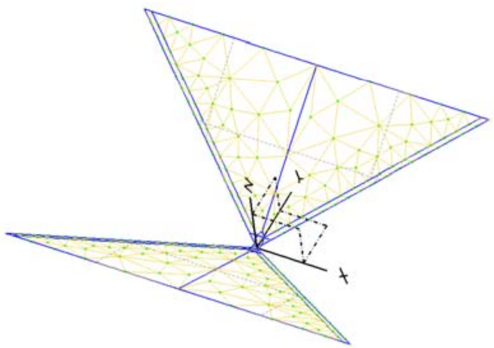
Example array



- Array radius = $30\lambda_0$.
- Number of elements = 1000.



- Distance to ground plane = $\lambda_0/4$.
- No dielectric.



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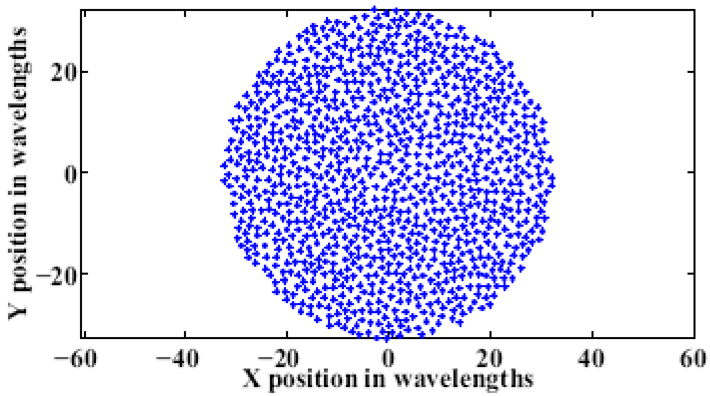
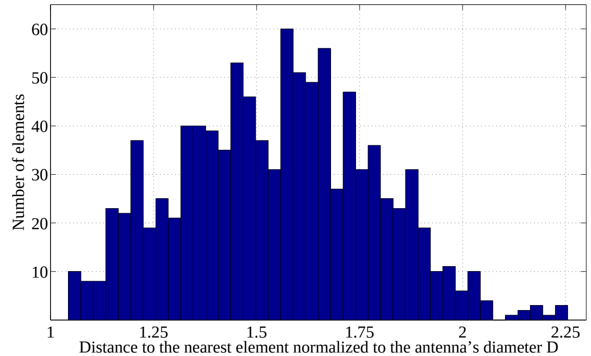
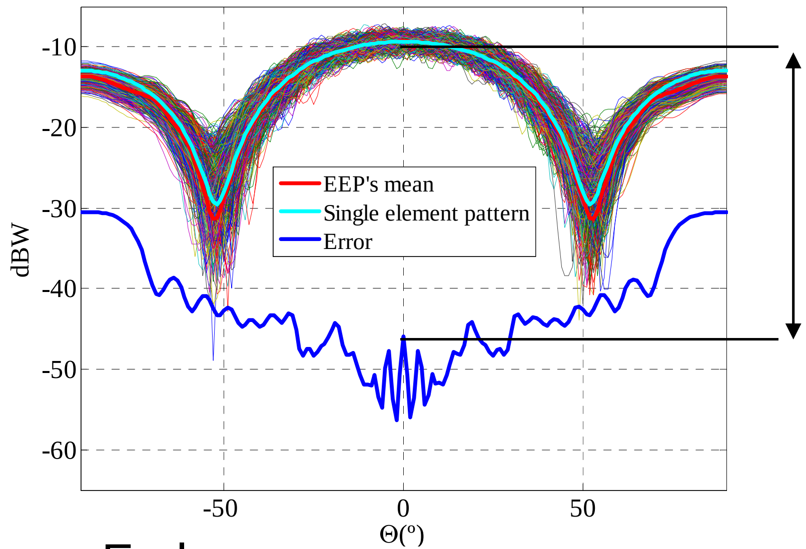
Random arrangement

Random configuration

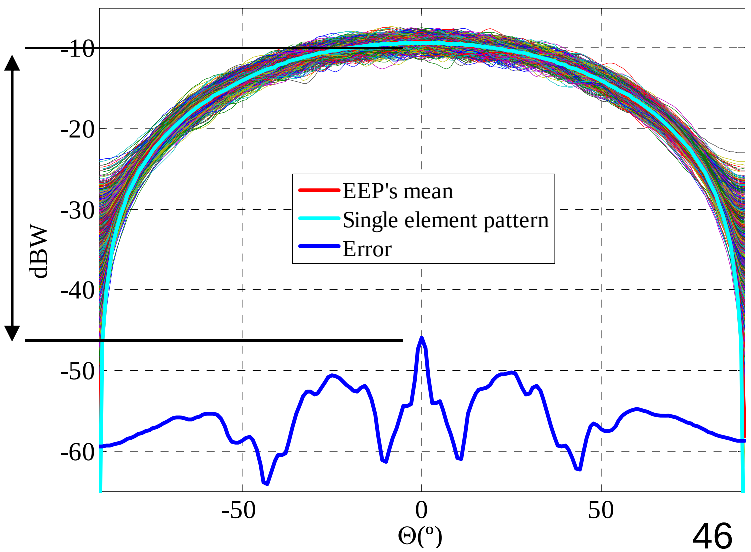






E-plane

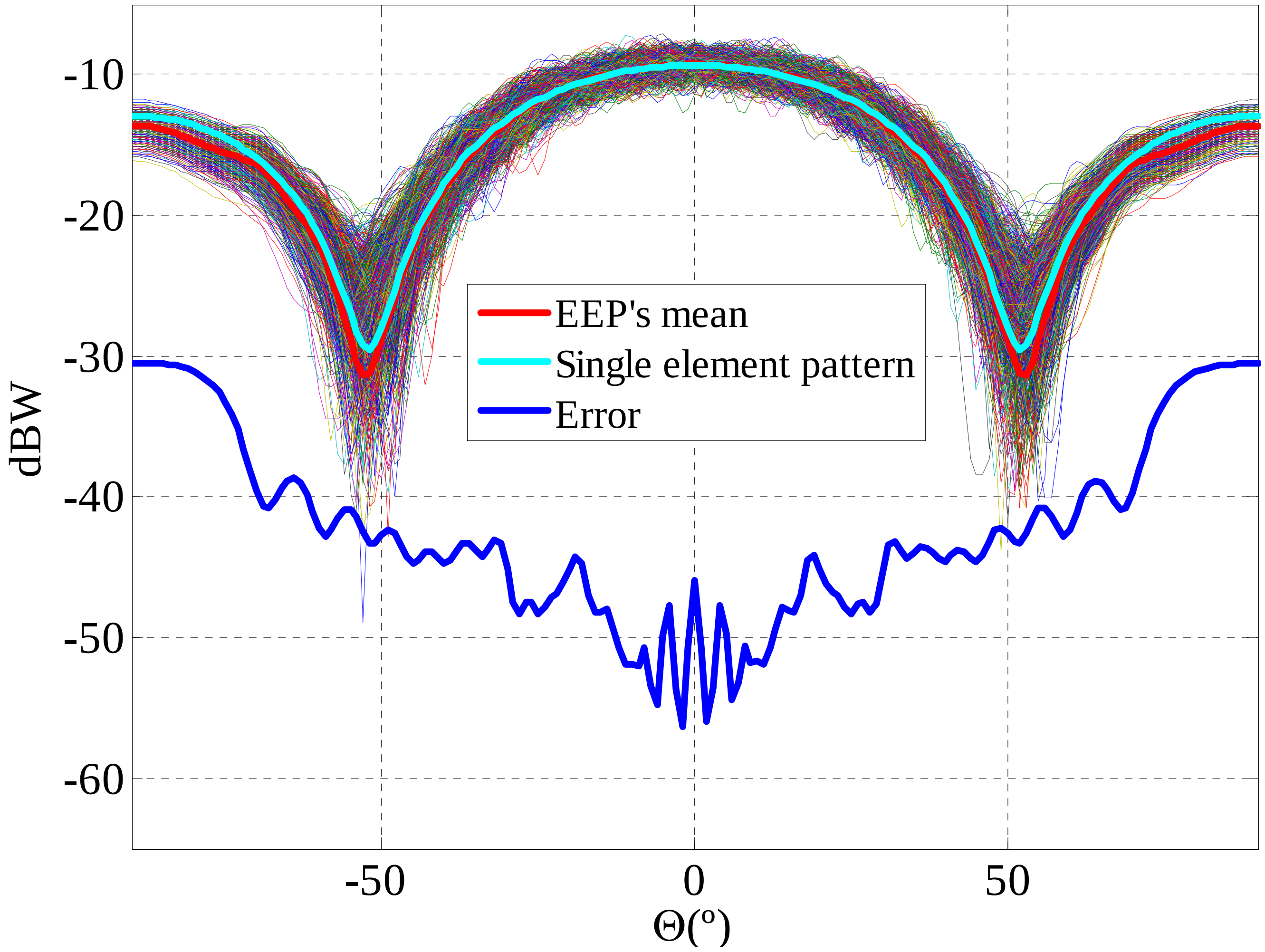
~ 35 dB



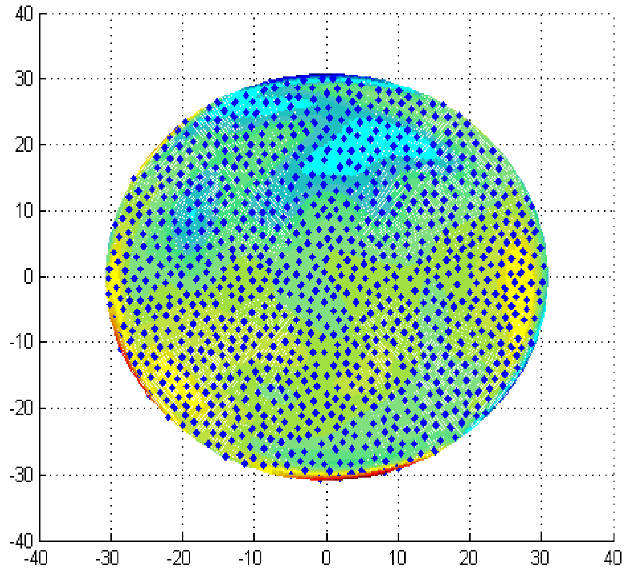
H-plane

$$e = 10\log_{10}\left(\left|\vec{E}_{mean}(\theta,\phi) - \vec{E}_{sin\,gle}(\theta,\phi)\right|^2\right)$$

Quasi-random arrangement



What's the problem ?



- No problem with **"smoothness"** of first few lobes,
  in view of **finiteness** of the "aperture"
- The challenge: can we simply **"shift" the array factor** upon scanning ?
  Strictly speaking, no because { **variable** embedded patterns
  need for **several** array factors
  (too many coefficients).

Search for simple approximate solution,
like average embedded element pattern.

Conclusions

1. Near main beam: array ~ smooth aperture
2. Aperture distributions with close-form F.T.
3. Exponential convergence – extraction of approximate array
4. Mutual coupling:
   - Evaluate average element pattern concept:

   Which calibration dynamic range can be achieved for given array size ?

   - Proceed with efforts on more compact representations