Beam Quality and Stability of PAF Systems

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Beam quality
Warnick et al., IEEE TAP, accepted 2011
Ivashina et al., IEEE TAP, Jun 2011

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The ideal polarimetric beamformer should

- provide maximum sensitivity
- preserve polarimetric properties of observed signal

→ **optimal beamformer**

Other concerns (a.o.)

- polarimetric behavior over FoV
- side lobe level
- beam symmetry

→ **beam shaping using constraints**

PAF calibration

Wijnholds et al., IEEE TAP, in prep.

With orthogonally polarized far field reference sources

- optimal
- max-SNR (signal-to-noise)
- max-SLNR (signal-to-leakage-and-noise)
- correction for imperfect reference sources

With unpolarized far field reference source

- eigenvector method (with bi-scalar correction)
- bi-scalar

Green: sensitivity equivalent to optimal method

Generic model of a phased array

Ivashina, Maaskant & Woestenburg, IEEE AWPL, 2008
Ivashina et al., IEEE TAP, Jun 2011

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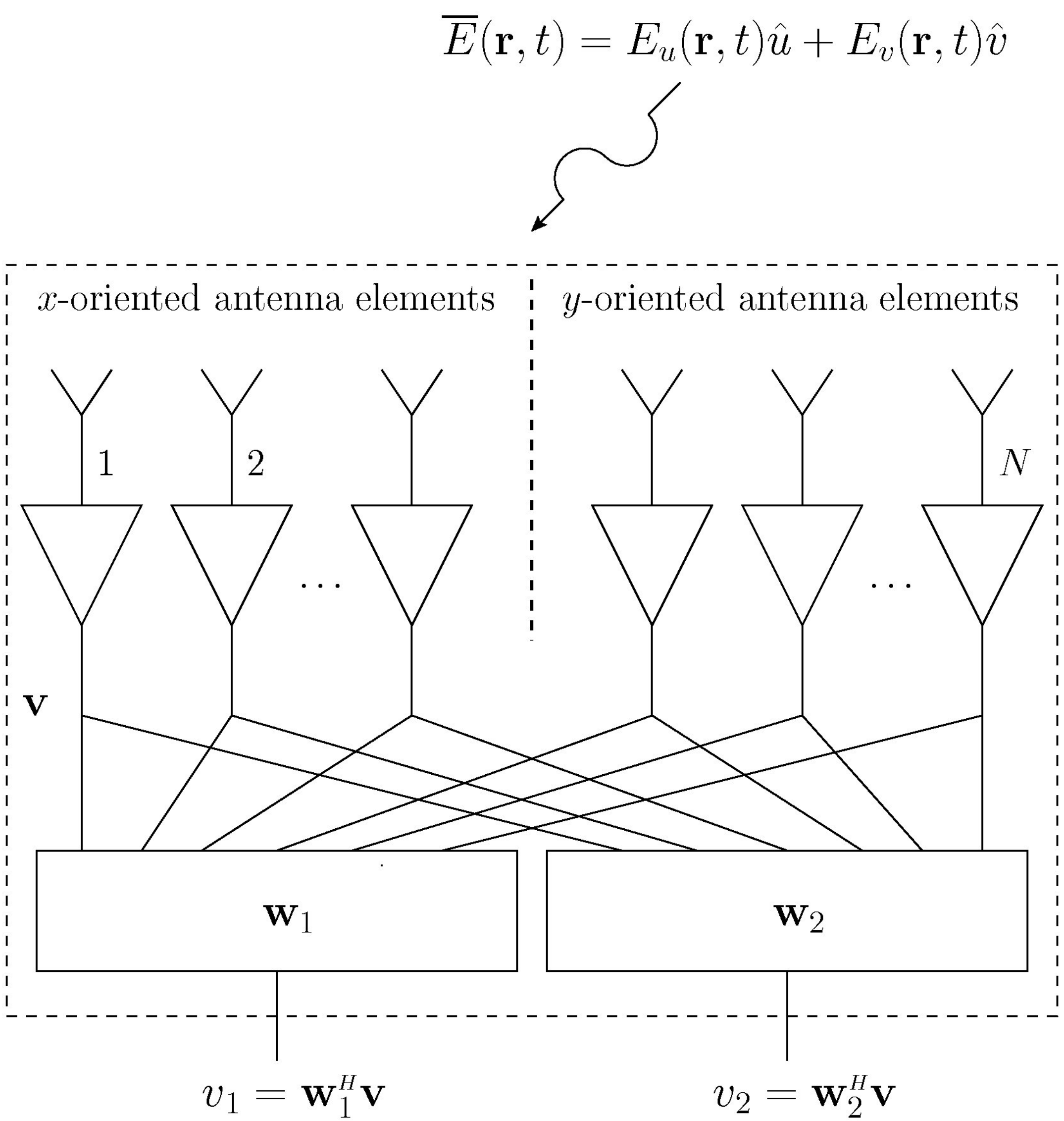
$\mathbf{E}(\mathbf{r}, t)$	incident field
E_u, E_v	*u*- and *v*-component
$\mathbf{v}$	output voltage vector
$\mathbf{w}_1$, $\mathbf{w}_2$	BF weights
v_1, v_2	BF output voltages



Optimal polarimetric calibration (1)

Warnick et al., IEEE TAP, accepted 2011

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$\mathbf{v}_u$, $\mathbf{v}_v$ voltage response to pure *u*- or *v*-polarized signal

Assume: $\mathbf{V} = [\mathbf{v}_u, \mathbf{v}_v]$ is known

BF output covariance matrix: $\mathbf{W}^H (\mathbf{R}_s + \mathbf{R}_n)\, \mathbf{W}$

where $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2]$

$\mathbf{R}_s$ is the signal covariance matrix

$\mathbf{R}_n$ is the noise covariance matrix

We want to:

1. minimize the noise: $\mathrm{argmin}_{\mathbf{w}}\, \mathrm{tr}(\mathbf{W}^H \mathbf{R}_n \mathbf{W})$
2. preserve polarization: $\mathbf{W}^H \mathbf{V} = \mathbf{I}$

Optimal polarimetric calibration (2)

Warnick et al., IEEE TAP, accepted 2011

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Steps to solution

- Reformulate using Lagrange multipliers
- Take derivatives and set them to zero
- Use contraint to find Lagrange multipliers

Solution

$$\mathbf{W} = \mathbf{R}_n^{-1} \mathbf{V} (\mathbf{V}^H \mathbf{R}_n^{-1} \mathbf{V})^{-1}$$

Interpretation

- Maximum sensitivity beam former
- Correction for optimal polarimetric fidelity

Beamforming in practice

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Optimal method requires

- generally unavailable orthogonally polarized ref. sources
- polarimetric processing (incl. 2N frontend correlator)

→ **practical systems exploit bi-scalar processing**

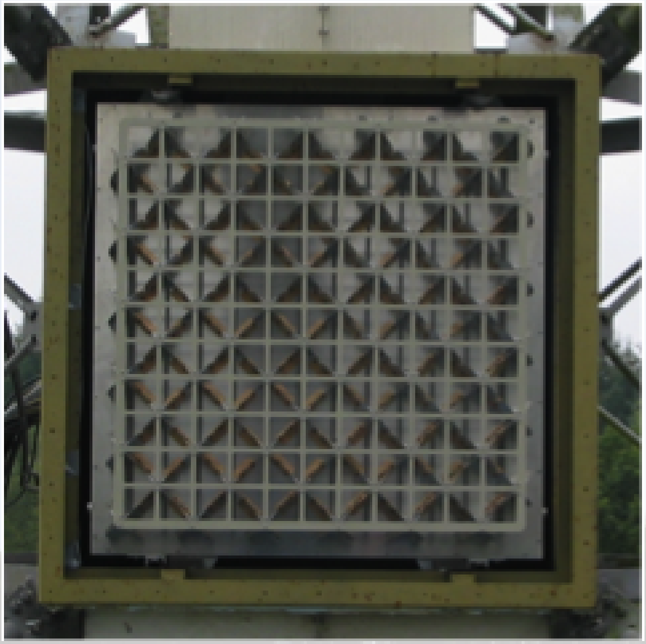
- separate treatment of both polarizations
- reduces complexity of processing system
- relies on intrinsic polarimetric quality of antennas
- possibly sacrifices some sensitivity

Question: how bad is this?

Example: Aperture Tile in Focus

- PAF for WSRT, increases survey speed 25x
- **key specs**

Frequency range	1000 – 1750 MHz
Instantaneous bandwidth	300 MHz
System temperature	< 55 K
Aperture efficiency	75%
Polarization	dual linear
Simultaneous beams	37 dual pol
Field of view	8 deg^2
Reflectors	12 x 25 m



- **Beam spec: 1% error at HPBW rel. to main beam**

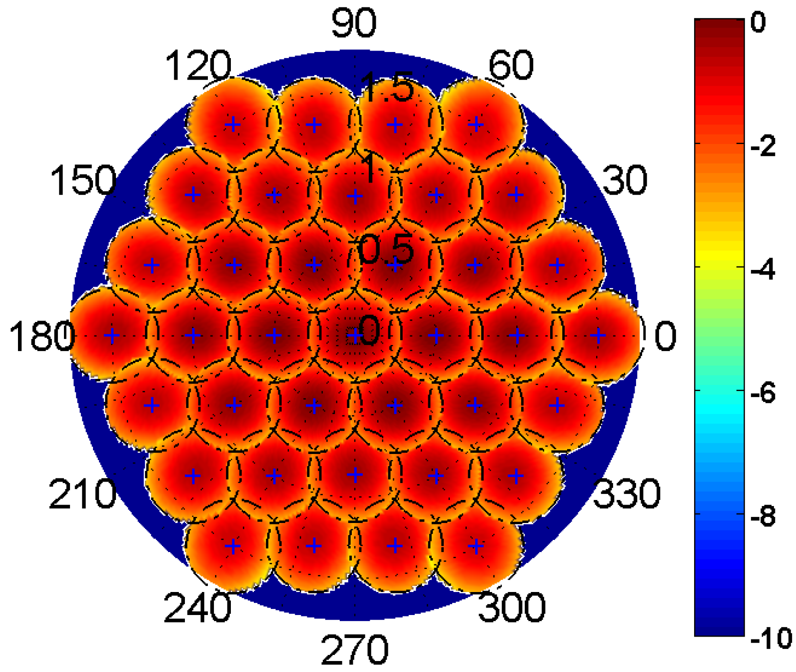
Filling the FoV

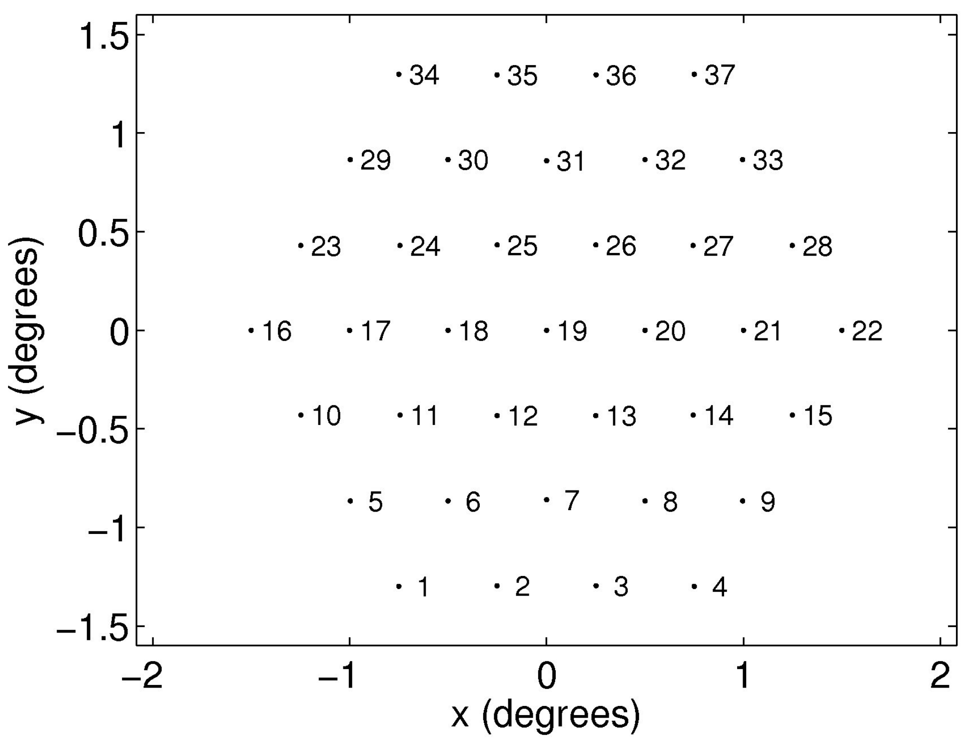
Ivashina et al., URSI GASS, Aug 2011

EM-simulation of APERTIF prototype for 37 beams

left: compound beams in x-polarization

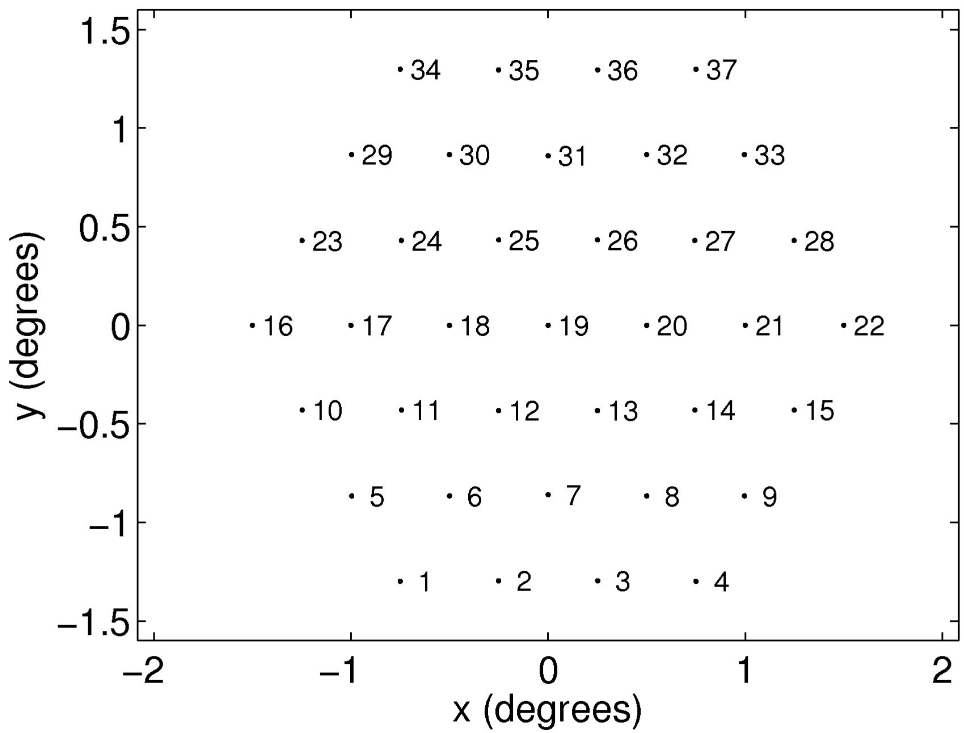
right: beam center locations with indices

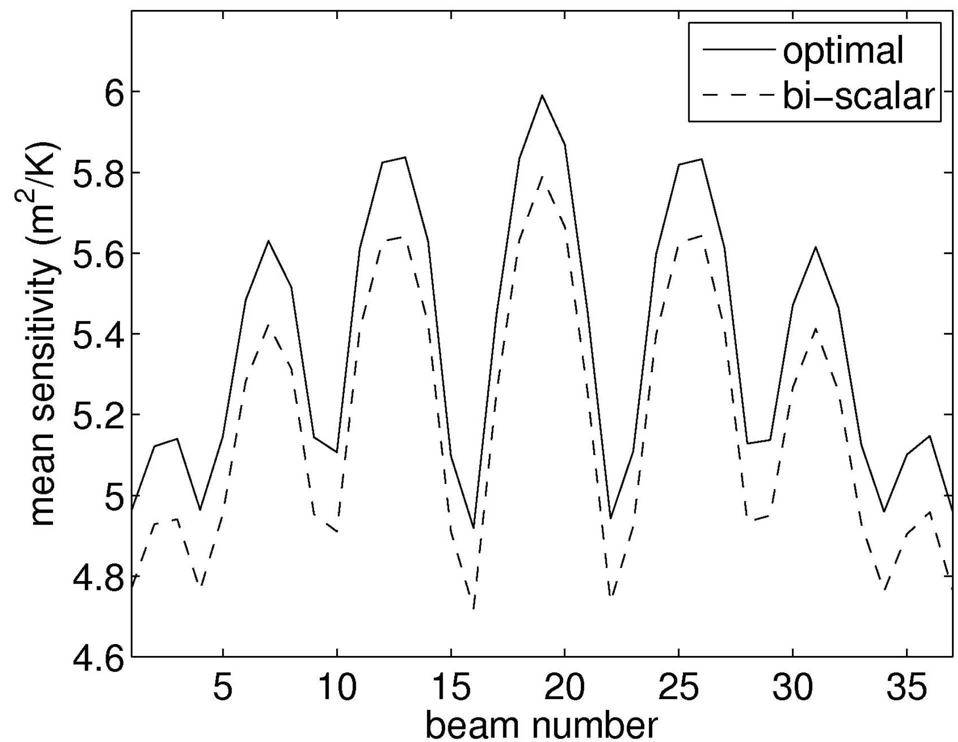




Sensitivity comparison
Wijnholds et al., URSI GASS, Aug 2011

- EM-simulation of APERTIF prototype for 37 pointings
- Sensitivity loss only 4%
- Recoverable at cost of half the bandwidth



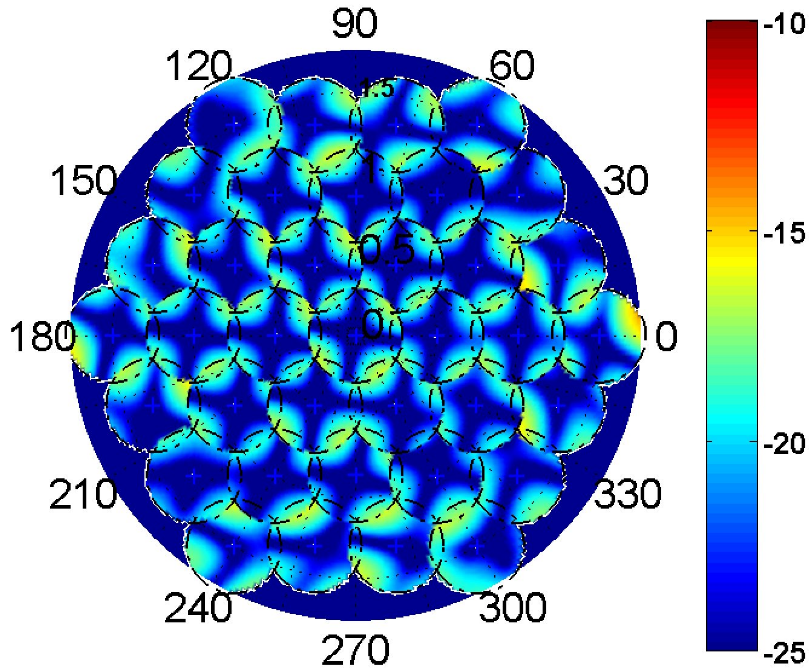


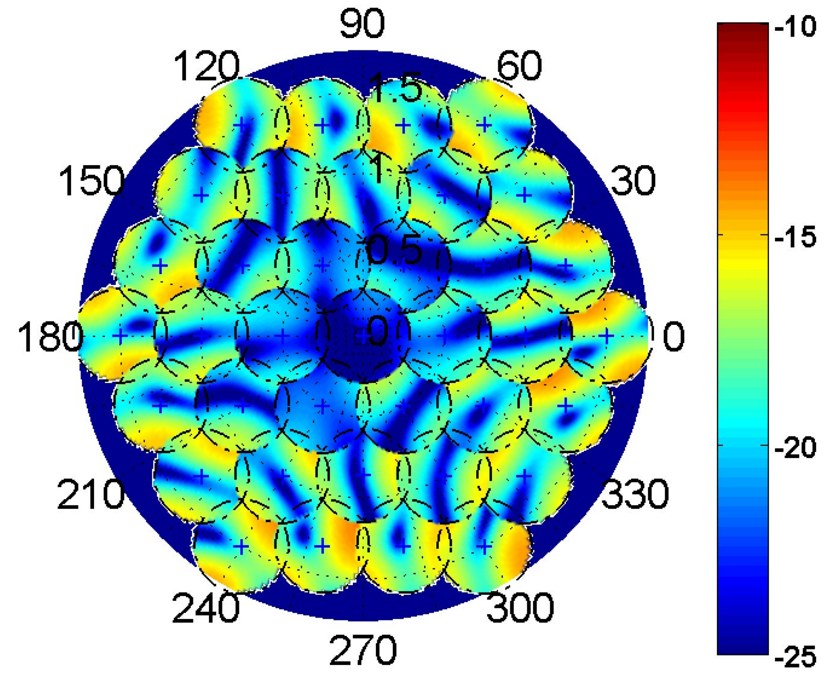
Polarimetric comparison
Ivashina et al., URSI GASS, Aug 2011

left: correlation BF output signals for optimal BF

right: correlation BF output signals for bi-scalar BF

Bi-scalar method relies on polarimetric quality of antennas





Measured dominant eigenvectors
Wijnholds et al., URSI GASS, Aug 2011

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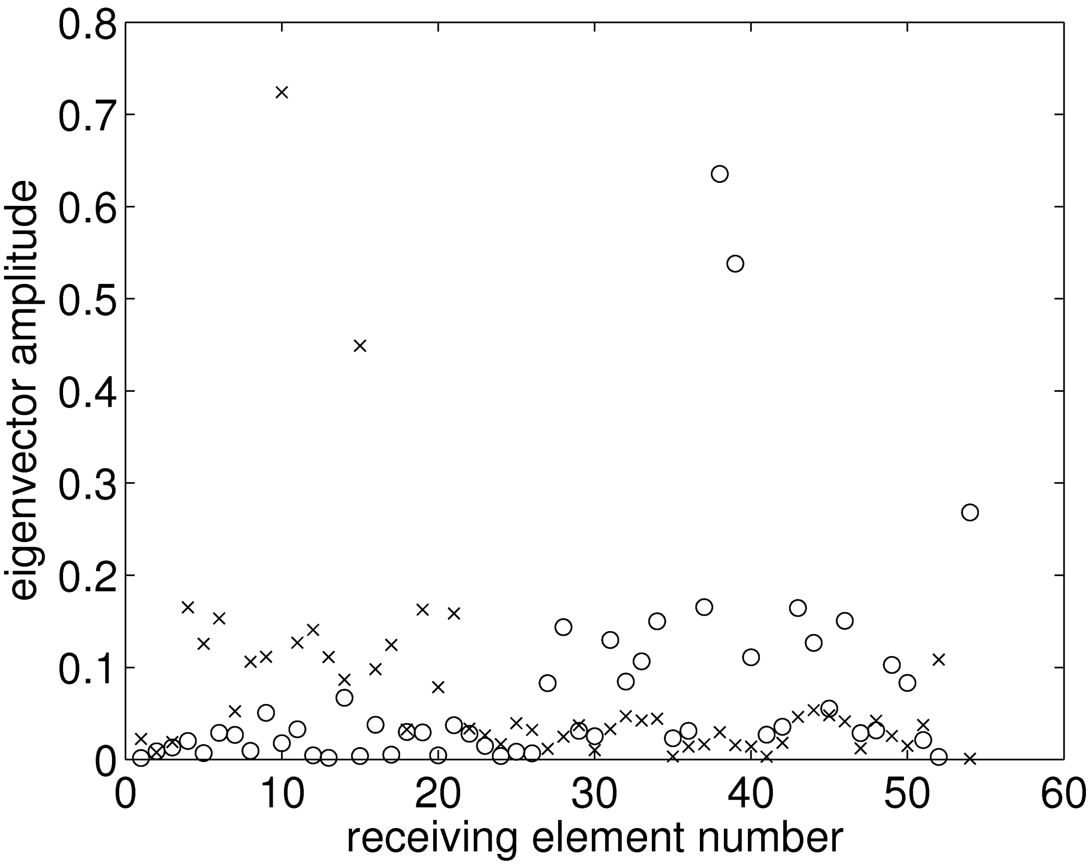
- measurement on unpolarized source
- amplitudes of elements of two dominant eigenvectors
- 2% sensitivity loss due to ignoring cross-pol (4% in sims)
- -28 dB cross-pol level (sims typically -45 dB)
- acceptable for actual system



Error analysis

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- Beamformer equation: $y(t) = \mathbf{w}^{H}(\boldsymbol{\theta})\ \mathbf{v}(t)$

  $\mathbf{w}^{H}(\boldsymbol{\theta})$ weight vectors parameterized by $\boldsymbol{\theta}$

  $\mathbf{v}(t)$ receiving element output voltages

  $y(t)$ beamformer output voltage

- $\boldsymbol{\theta}$ depends on element response and noise covariance

- assumed parameter covariance models:

  - for calibration: Cramer-Rao bound

  - for drift: independent parameter variation

- standard error propagation formula

$$\mathrm{var}(y) = (\partial y/\partial \boldsymbol{\theta}^{T})\ \mathrm{cov}(\boldsymbol{\theta})\ (\partial y/\partial \boldsymbol{\theta}^{T})^{T}$$

Propagation of calibration errors

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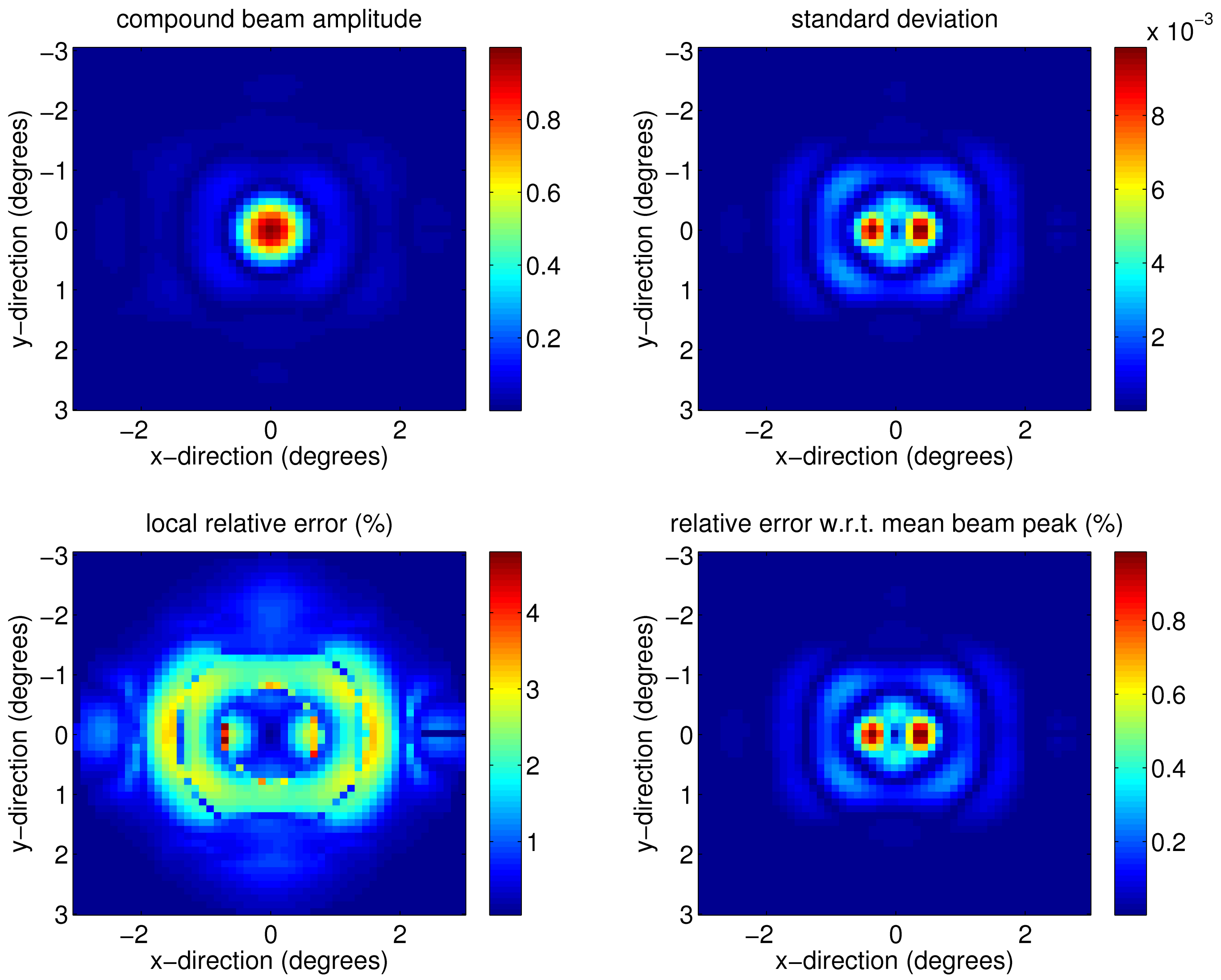
- SNR = 200
- bi-scalar BF
- constraint: beam peak fixed (selfcal)
- **SNR of 200 needed to satisfy beam requirement for APERTIF**



Propagation of drift errors (on axis)

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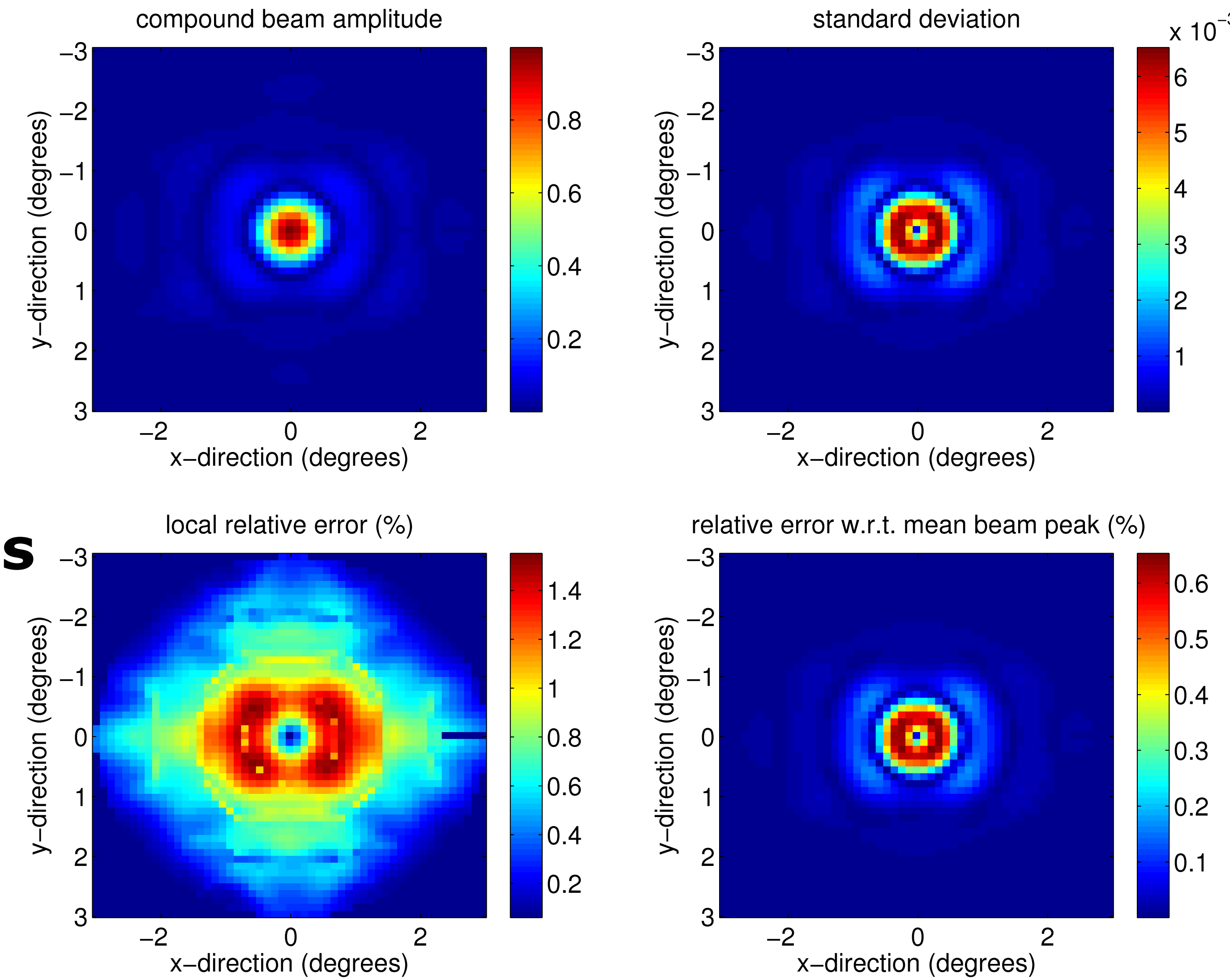
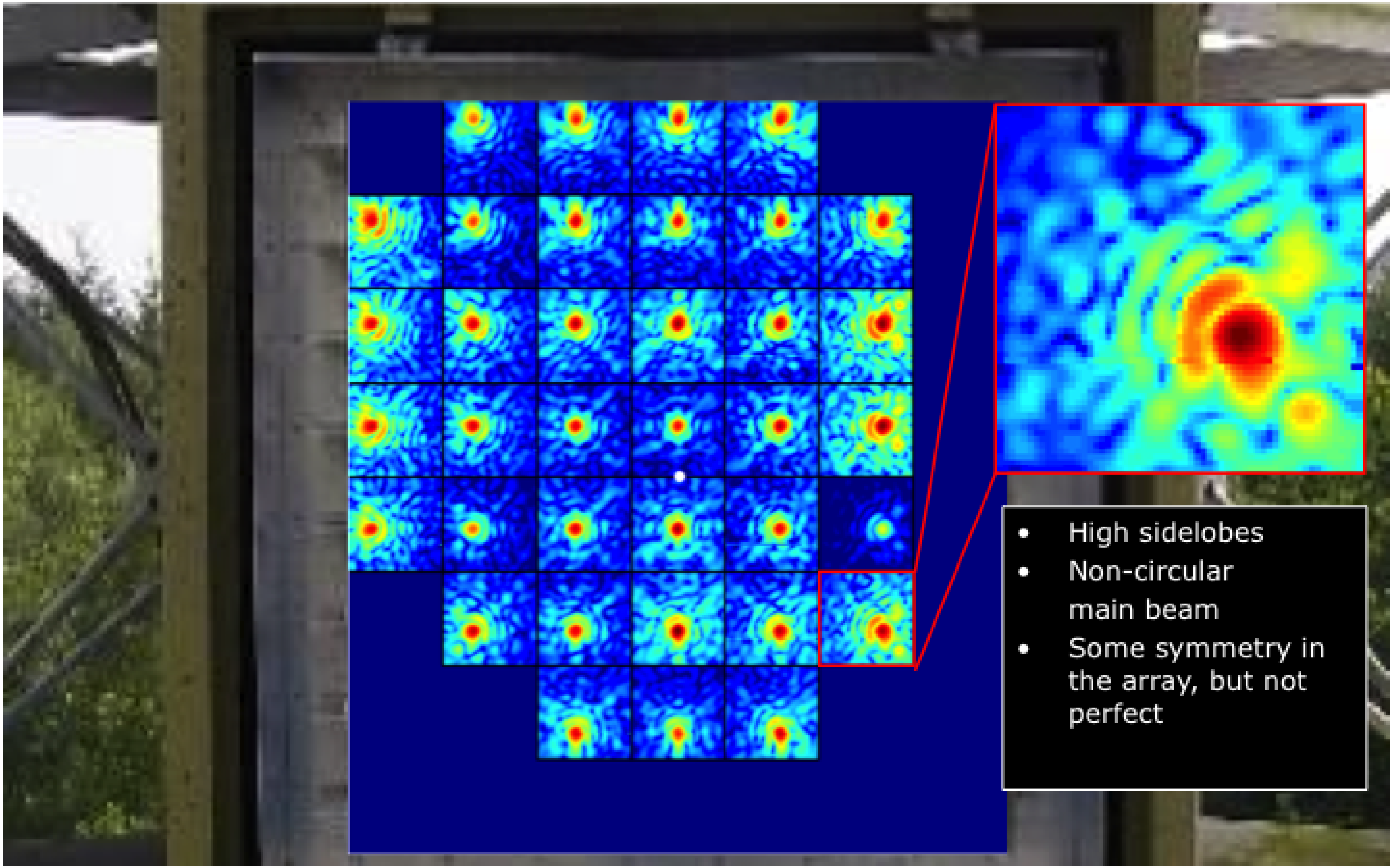
- 2% rel. error
- bi-scalar BF
- constraint: beam peak fixed (selfcal)
- **2% variations well within acceptable tolerances**



Element patterns on the sky

Van Cappellen, AJDI, 27 Mar 2008



- High sidelobes
- Non-circular main beam
- Some symmetry in the array, but not perfect

Propagation of drift errors (off axis)

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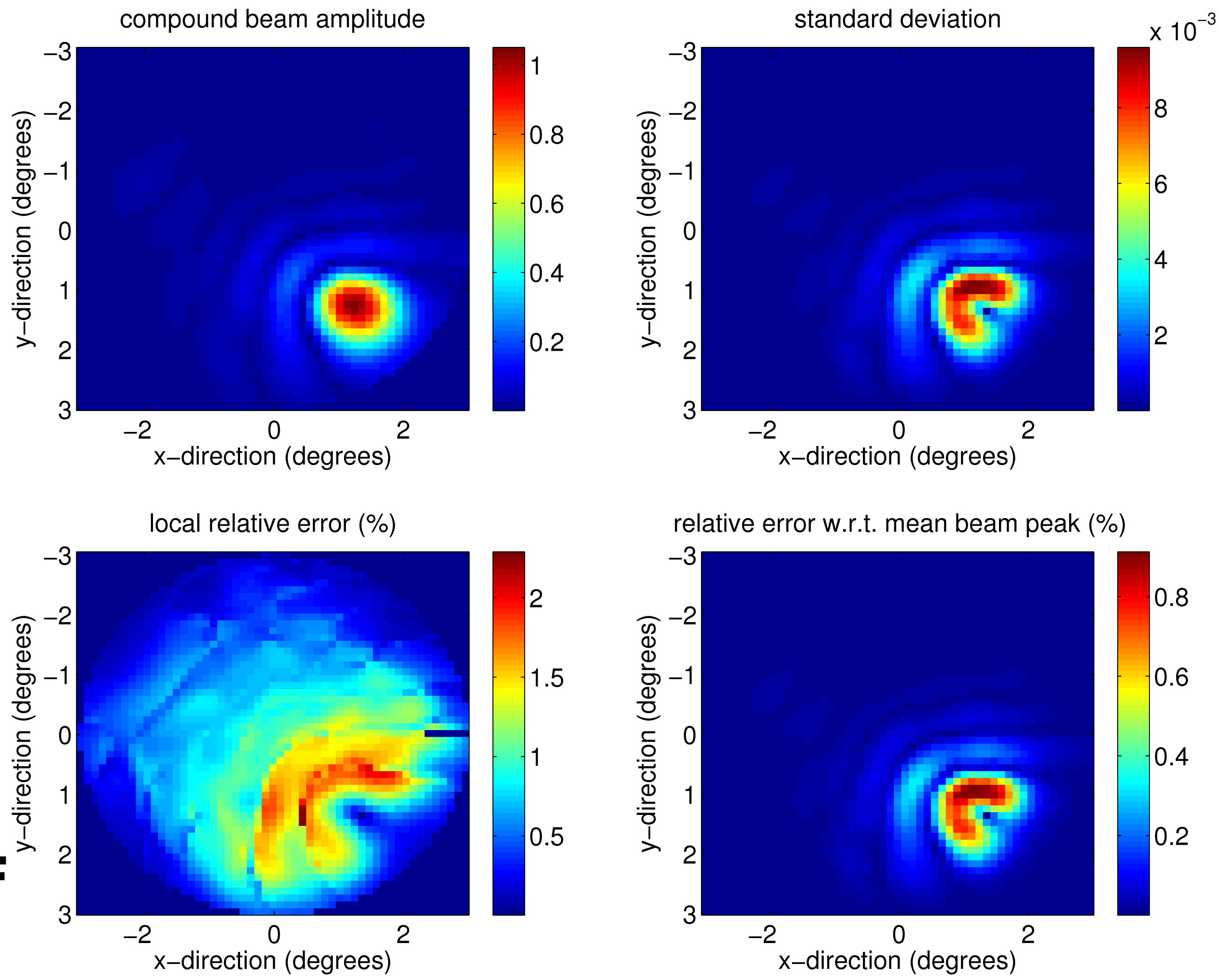
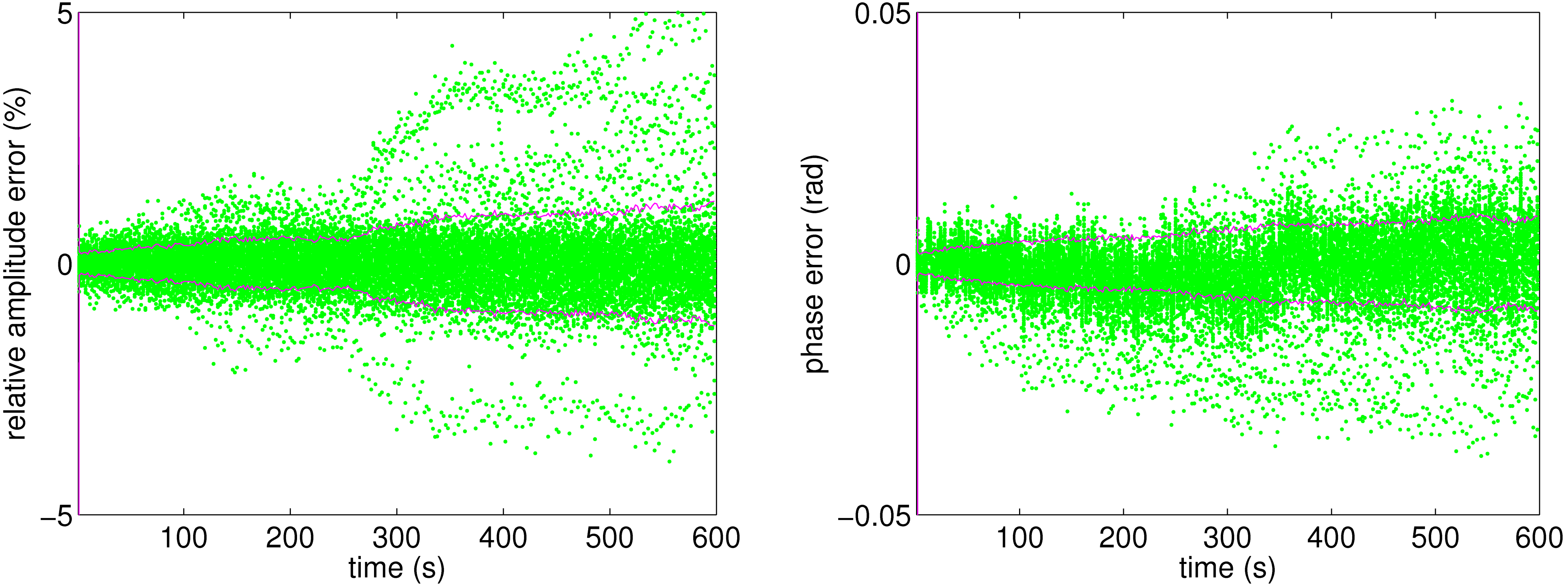
- 2% rel. error
- bi-scalar BF
- constraint: beam peak fixed (selfcal)
- **max 2% variation acceptable to satisfy beam spec APERTIF**



Measured drift using apex-source

- 5 min observation at 1441.5 MHz
- gain calibrated using first 10 s
- < 1% variation after 5 min → 10 – 15 min update rate?



Conclusions

- Good progress on PAF analysis
  - sims and measurements give similar results
  - wide range of calibration methods available
  - comparison between methods possible
  - error propagation analysis available
- Application to APERTIF system
  - only 2% (sims: 4%) sensitivity loss bi-scalar BF
  - -28 dB cross-pol level bi-scalar BF acceptable
  - calibration measurement should have SNR of 200
  - 10 – 15 min calibration update interval seems ok