Radio interferometer calibratability and limits to polarimetry

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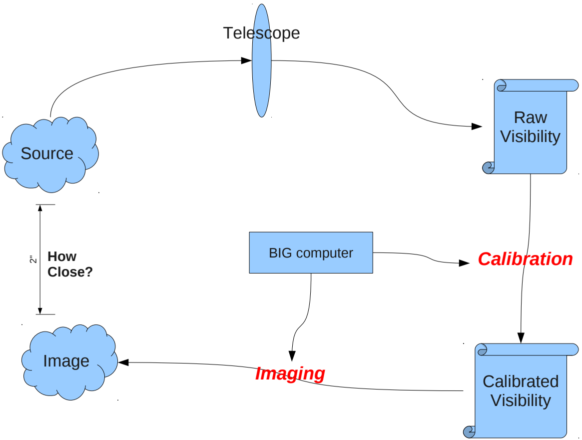
Calibratability



Measurement Equation

Radio interferometric measurement equation RIME is a linear relationship between Visibility and Brightness via Gains

$$\begin{bmatrix} \vdots \\ V_{pq} \\ \vdots \end{bmatrix} \longleftrightarrow \begin{bmatrix} . & . & . \\ . & G_{pqs} & . \\ . & . & . \end{bmatrix}, \begin{bmatrix} \vdots \\ B_s \\ \vdots \end{bmatrix}$$

Calibration is the process of determining **G** and applying it in the RIME above enabling *Imaging*, which is the inversion problem

$$\begin{bmatrix} \vdots \\ B_s \\ \vdots \end{bmatrix} \longleftrightarrow \mathrm{Inv} \begin{bmatrix} . & . & . \\ . & G_{pqs} & . \\ . & . & . \end{bmatrix}, \begin{bmatrix} \vdots \\ V_{pq} \\ \vdots \end{bmatrix}$$

Chimera of calibration

Conjecture

If I know my gains completely, then I can image perfectly :-)

Corollary

Interferometer performance is not important, so long as I know its gains (I can calibrate away deficiencies!)

Counter Example

Along beam-null, NO amount of calibration will produce a sensible image :-(

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Fundamental theorem of Calibration

Definition

Calibratability is the degree to which the gains in a RIME are invertible

Conjecture

In general conditioning of gains sets the limits of calibratability

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Why Calibratability is Important

- As an interferometer design tool: construction and observation scheduling
- It's the calibratability, stupid!
  - Computational muscle is not the end all of CalIm: it's applying it where/when it makes a difference
- Working out whether your existing image (using your favorite algorithm) can be improved upon
  - Sets ultimate limits of imaging, like CRB but without assuming a particular source

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Calibratability Microcosm: polarimetry!

Basic (Jones) Measurement Equation for interferometer element is

$$\mathbf{V} = \mathbf{J}\mathbf{e}$$

where $\mathbf{V}$ is measured voltages, $\mathbf{e}$ is Jones vector and $\mathbf{J}$ is "Jones" matrix.
Full polarimetric calibration is the inversion

$$\hat{\mathbf{e}} = \mathbf{J}^{-1}\mathbf{V}$$

This seems to give perfect solutions...
But there's always noise and errors, so the inversion is prone to errors...
Mathematically the condition number (of the Jones matrix) determines the inversions sensitivity to error propagation, i.e. calibratability.
But instead of matrix condition for calibratability (obscure to many radio astronomers due to lack of physical meaning) I suggest a related parameter to do with feed "leakiness"

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What's leaky and what's "bad" calibration

There's leakiness and then there's "proper" leakiness:

Figure: Is this a leaky crossed dipole feed? (ans: Yes, leaky)

Figure: Is this also a leaky feed? (ans: No, it's calibratable via coord sys transformation)

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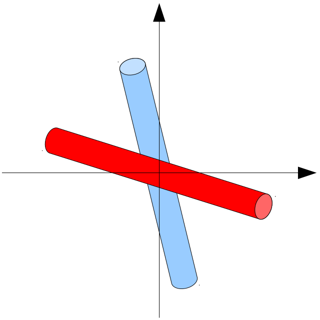

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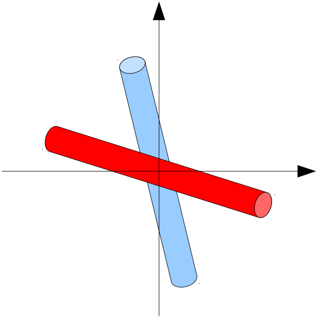

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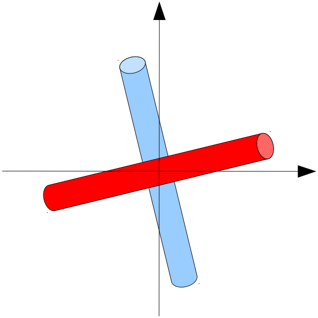

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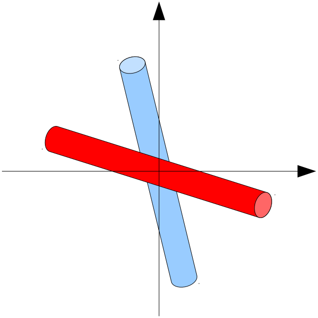

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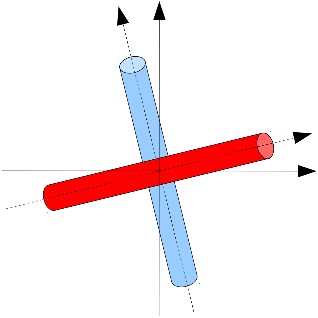

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Cross polarization ratio (XPR)...

So in the latter case, Jones matrix is factorizable as follows

$$\mathbf{J} = g \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} = g\cos\alpha \begin{pmatrix} 1 & \tan\alpha \\ \tan\alpha & 1 \end{pmatrix} = g\cos\alpha \begin{pmatrix} 1 & d \\ d & 1 \end{pmatrix}$$

where $d \neq 0$ is the "raw" leakage term (a.k.a d-term).
But a change of coordinates to rotated frame (i.e. calibration of alignment) gives

$$\mathbf{J}' = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \mathbf{J} = g \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

which has $d = 0$! Thus, "raw" leakage may be possible to calibrate away

...and Intrinsic cross polarization ratio (IXR)

On the other hand, the SVD factorization is invariant to coordinate transformation: Jones matrix can always be written

$$\mathbf{J} = g\mathbf{U}\begin{pmatrix} 1 & d_{\text{intrinsic}} \\ d_{\text{intrinsic}} & 1 \end{pmatrix}\mathbf{V}^{\dagger}, \quad \mathbf{U},\mathbf{V} \text{ unitary}$$

so there is a choice of sky and feeds coord-sys for which the Jones matrix is

$$\mathbf{J}' = g\begin{pmatrix} 1 & d_{\text{intrinsic}} \\ d_{\text{intrinsic}} & 1 \end{pmatrix}\mathbf{V}^{\dagger}$$

where $d_{\text{intrinsic}}$ is related to the maximum and minimum amplitude gains g_{max}, g_{min} of the polarimeter.
Thus "proper, uncalibratable" leakage is given by the Intrinsic cross polarization ratio

$$\text{IXR} = \frac{1}{|d_{\text{intrinsic}}|^2} = \frac{g_{\text{max}} + g_{\text{min}}}{g_{\text{max}} - g_{\text{min}}} = \frac{g_{\text{max}}/g_{\text{min}} + 1}{g_{\text{max}}/g_{\text{min}} - 1} = \frac{\text{cond}(\mathbf{J}) + 1}{\text{cond}(\mathbf{J}) - 1}$$

where $\text{cond}(\mathbf{J})$ is the Jones condition number

Limits of Calibratability

Ultimately the relationship between calibratability and IXR come from the provable relationship

$$\mathrm{rel.RMS}(\hat{\mathbf{e}}) \equiv \frac{\|\Delta\mathbf{e}\|}{\|\mathbf{e}\|} \lessapprox \left(1 + \frac{2}{\sqrt{\mathrm{IXR}}} + \dots\right)\left(\frac{\|\Delta\mathbf{J}\|}{\|\mathbf{J}\|} + \frac{\|\Delta\mathbf{V}\|}{\|\mathbf{V}\|}\right),$$

where $\Delta\mathbf{V}$ is thermal noise in data and $\Delta\mathbf{J}$ is the imprecision in the Jones matrix

(These results are given in *Carozzi, Woan* IEEE TAP special issue “Future radio telescopes” June 2011)

Calibratability and Antenna Sensitivity

Calibratability is link to antenna sensitvity. Sensitivity can be extended polarimetrically

$$\frac{A_{\text{eff}}}{T} \Longrightarrow \frac{\|\mathbf{M}\|}{\|\mathbf{T}\|}$$

where **M** is the effective Mueller matrix, **T** is the Stokes antenna temperature and $\|\cdot\|$ is a matrix/vector norm.
A related parameter is SNR of the Stokes estimate from the telescope

$$\frac{\|\mathbf{S}\|}{\|\Delta\mathbf{S}\|} \gtrsim \left(1 - \frac{2}{\text{IXR}}\right)\left(\frac{\|\mathbf{M}\|}{\|\mathbf{T}\|}\|\mathbf{S}\| - \frac{\|\Delta\mathbf{M}\|}{\|\mathbf{M}\|}\right)$$

Mueller IXR

Equivalently in the Mueller formalism, the calibratability of

$$\mathbf{S}' = \mathbf{M}\mathbf{S}$$

where $\mathbf{S}$, $\mathbf{S}'$is the true and measured Stokes parameters and $\mathbf{M}$ is the telescopes Mueller matrix, is ultimately determined by

$$\mathrm{IXR}_M = \frac{G_{\mathrm{max}} + G_{\mathrm{min}}}{G_{\mathrm{max}} - G_{\mathrm{min}}}$$

the intrinsic Mueller cross-polarization ratio. Revealingly,IXR_M is identical to what is known as *instrumental polarization*

Interferometer IXR

Continuing the preceding treatment of polarimetric calibratability to interferometry, we have

$$\mathbf{S}_{pq} = \mathbf{M}_{pq}\mathbf{S}^{\mathrm{bri}}$$

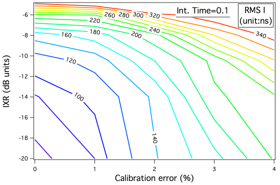
where $\mathbf{S}_{pq}$ is the Stokes visibility (complex), $\mathbf{S}^{\mathrm{bri}}$ is the Stokes brightness (real), and $\mathbf{M}_{pq}$ is the interferometer Mueller matrix (complex, not real!). Again an intrinsic value can be analogously assigned

$$\mathrm{IXR}_I = \frac{G^{pq}_{\mathrm{max}} + G^{pq}_{\mathrm{min}}}{G^{pq}_{\mathrm{max}} - G^{pq}_{\mathrm{min}}}$$

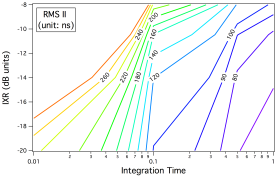
Polarization purity effects on Pulsar timing

Results of end-to-end Monte-Carlo simulations using different timing methods versus IXR

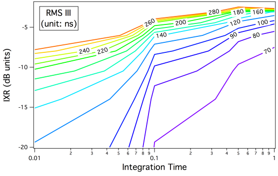
Method I: *Intensity timing* (raw)



Method II: *Invariant interval* (raw)



Method III: *Matrix template matching* (calibrated)



Figures by R. Paulin in *Karastergiou et al.* (to be submitted 2010)

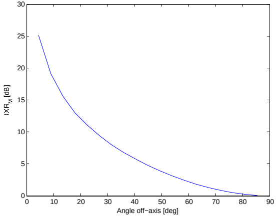
EoR & polarization leakage in AA-lo

At low frequencies SKA is approximated by short dipoles in a plane. Its Jones matrix is

$$\mathbf{J} = \frac{1}{\sqrt{1-m^2}} \begin{pmatrix} n & -lm \\ 0 & 1-m^2 \end{pmatrix}$$

for which

$$\mathrm{IXR_M} = \frac{1+n^2}{1-n^2}$$



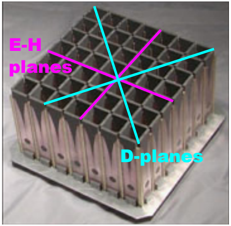
EoR contamination

Off-zenith, polarized galactic foreground leaks into intensity. *Jelić et al.* (2010)

Leaked polarized to total power ratio is $1/\mathrm{IXR_M}$

Dense AA Widefield Polarimetry Issues

Differences in E&H-plane versus D-plane



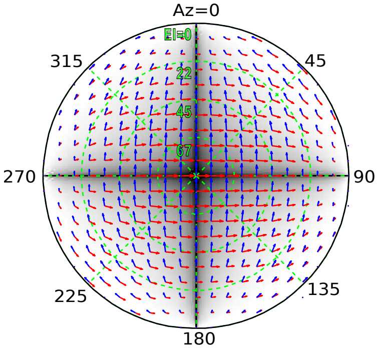
This makes widefield polarimetry difficult and impacts on for surveys...

Suggested solution

Polarization diversity by varying tile orientations *Ref. Carozzi, Maaskant (2010) Proceedings of Wide field Science for SKA*

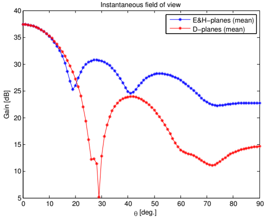
Widefield Calibratability

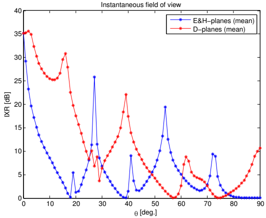
IXR over the entire hemisphere:



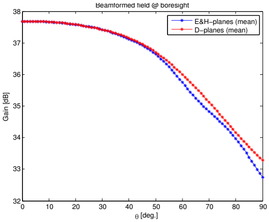
Dense AA Polarimetry Performance

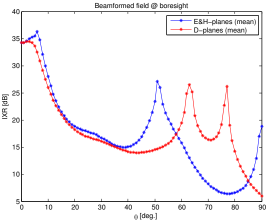
Instantaneous FoV





Beamforming





Conclusions

- Neither computational muscle nor algorthimic might is all there is to Cal & Im in future "software telescopes"
  - Bad telescope design can never be replaced by clever software
  - Some things can never be "calibrated away"
- IXR characterizes polarimetric calibratability
- Analogous IXR for full RIME under developement