

Es muss darauf hingewiesen werden, dass ^{die} ~~die~~ Gleichungen ein Minimum von Willkür aufweist. Denn es gilt unser $B_{\mu\nu}$ keinen Tensor zweiten Ranges, der aus den $g_{\mu\nu}$ und ~~den~~ Ableitungen gebildet ist, keine höhere als zweite Ableitungen enthält, und die letzten linear ist.

Dass diese ~~aus~~ aus der Forderung der allgemeinen Relativität auf rein mathematischem Wege fließenden Gleichungen mit dem Bewegungsgesetz (86) in erster Näherung das Newton'sche Attraktionsgesetz, in zweiter Näherung die Erklärung der vom Leverrier entdeckten (nach Anbringung der Störungs-Korrekturen übrig bleibenden) Perihelbewegung des Merkur liefern, muss nicht unserer Ansicht von der physikalischen Richtigkeit der Theorie abhelfen.

§ 15. ~~Die~~ ~~einige~~ ~~einige~~ Hamilton'sche Funktionen für das Gravitationsfeld. Poincaré-Gesetz.

Um zu zeigen, dass die Feldgleichungen dem Poincaré-Gesetz entsprechen, ist es am bequemsten, sie in folgender Hamilton'scher Form zu schreiben

$$\left. \begin{aligned} \delta \int \mathcal{H} dx^0 &= 0 \\ \mathcal{H} &= g^{\mu\nu} T_{\mu\beta} T_{\nu\alpha}^{\alpha} \end{aligned} \right\} (42a)$$

wobei δ die Variationen an den $g_{\mu\nu}$ des betrachteten begrenzten vierdimensionalen Integrationsraumes, $T_{\mu\nu}$ ist zu verstehen, dass die Form (32a) der Gleichungen (31) äquivalent ist. Zu diesem Zweck betrachten wir \mathcal{H} als Funktion der $g^{\mu\nu}$ und $T_{\mu\nu}^{\alpha}$ ($= \frac{\partial \mathcal{L}}{\partial x^{\alpha}}$). Dann ist zunächst

$$\begin{aligned} \delta \mathcal{H} &= T_{\mu\beta}^{\alpha} T_{\nu\alpha}^{\beta} \delta g^{\mu\nu} + 2 g^{\mu\nu} T_{\mu\beta}^{\alpha} \delta T_{\nu\alpha}^{\beta} \\ &= - T_{\mu\beta}^{\alpha} T_{\nu\alpha}^{\beta} \delta g^{\mu\nu} + 2 T_{\mu\beta}^{\alpha} \delta (g^{\mu\nu} T_{\nu\alpha}^{\beta}) \end{aligned}$$

Nun ist aber

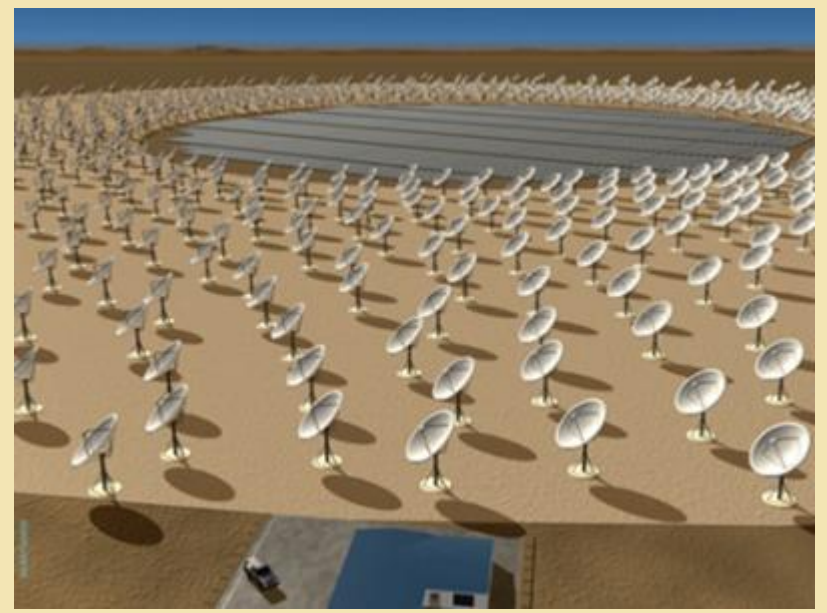
$$\delta (g^{\mu\nu} T_{\nu\alpha}^{\beta}) = -\frac{1}{2} \delta \left[g^{\mu\nu} g^{\beta\lambda} \left(\frac{\partial g_{\nu\lambda}}{\partial x^{\alpha}} + \frac{\partial g_{\nu\alpha}}{\partial x^{\lambda}} - \frac{\partial g_{\alpha\lambda}}{\partial x^{\nu}} \right) \right]$$

Die aus den beiden letzten Termen der runden Klammer hervorgehenden Terme unterscheiden sich durch ihr β -Symbol, welches durch sind von verschiedenen β bezeichnet und gehen auseinander hervor (da die Permutation der Summationsindizes belanglos ist) durch Vertauschung der Indizes μ und β hervor. Sie haben einander ein Ausdruck für $\delta \mathcal{H}$ weg, weil $\delta \mathcal{H}$ mit die β -Indizes der Indizes μ und β symmetrischen Größen $T_{\mu\nu}^{\alpha}$ multipliziert werden. Es bleibt also nur das erste Glied der runden Klammer zu berücksichtigen, sodass man mit Rücksicht auf (31) erhält

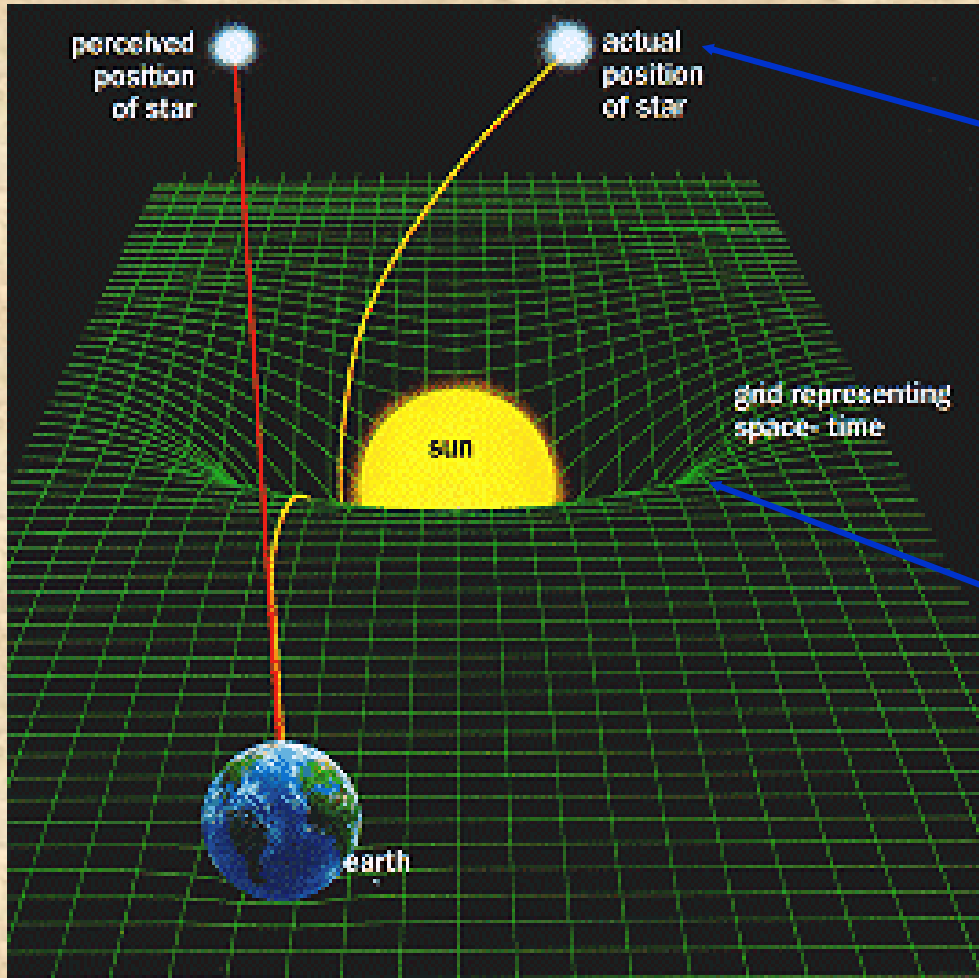
* Äquivalent lässt sich dies nun von dem Tensor $B_{\mu\nu} = A_{\mu\nu} (g^{\alpha\beta} T_{\alpha\beta}^{\gamma})$ behaupten, wobei $A_{\mu\nu}$ eine Konstante ist. Tritt man jedoch dieses Glied mit, so kommt man wieder zu den Gleichungen $B_{\mu\nu} = 0$.

Tests of General Relativity with the SKA

DIMITRIOS PSALTIS
University of Arizona



GENERAL RELATIVITY HAS TWO INGREDIENTS



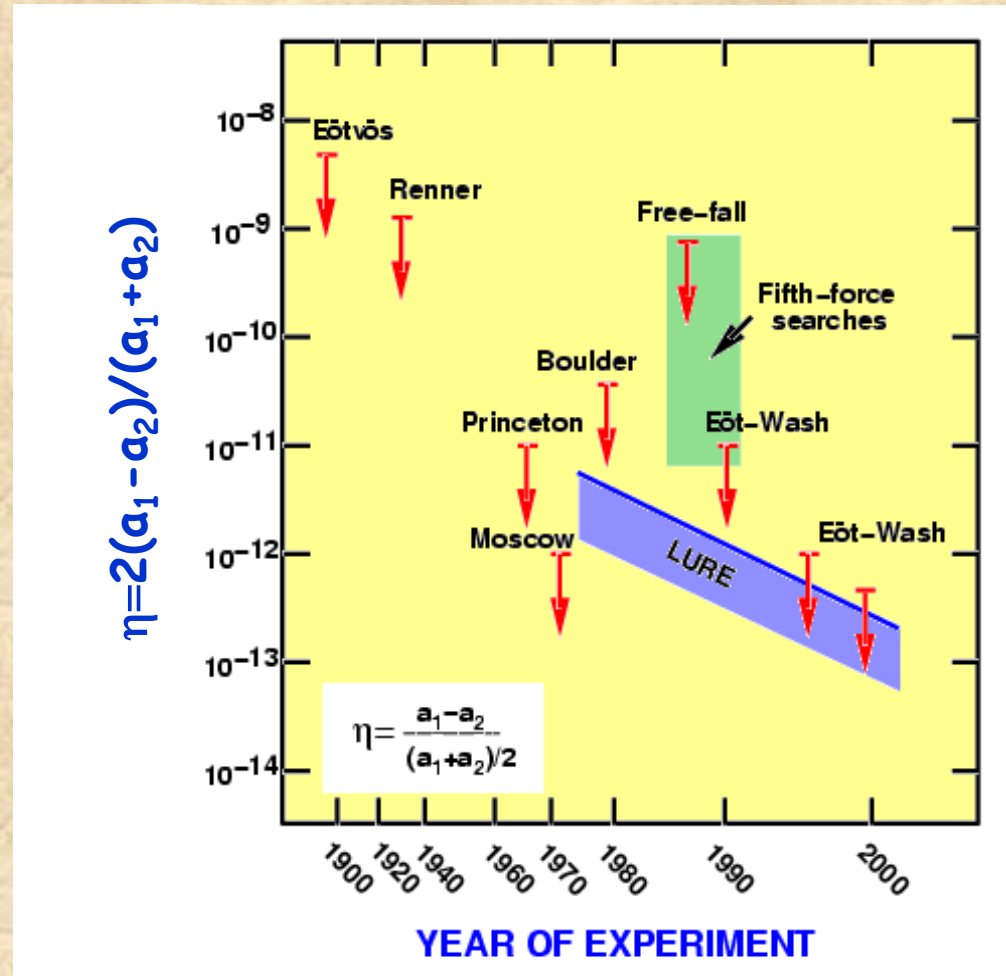
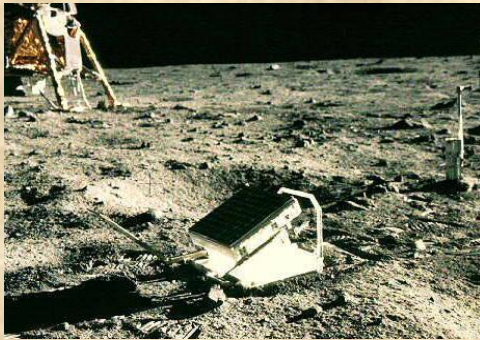
The Equivalence Principle

The Einstein Field Equation

The Equivalence Principle encompasses Einstein's two revolutions in our understanding of gravity:

- ◆ **Gravity is geometry (particles and photons follow geodesics)**
- ◆ **Gravity is relativistic (obeys the Lorentz symmetry)**

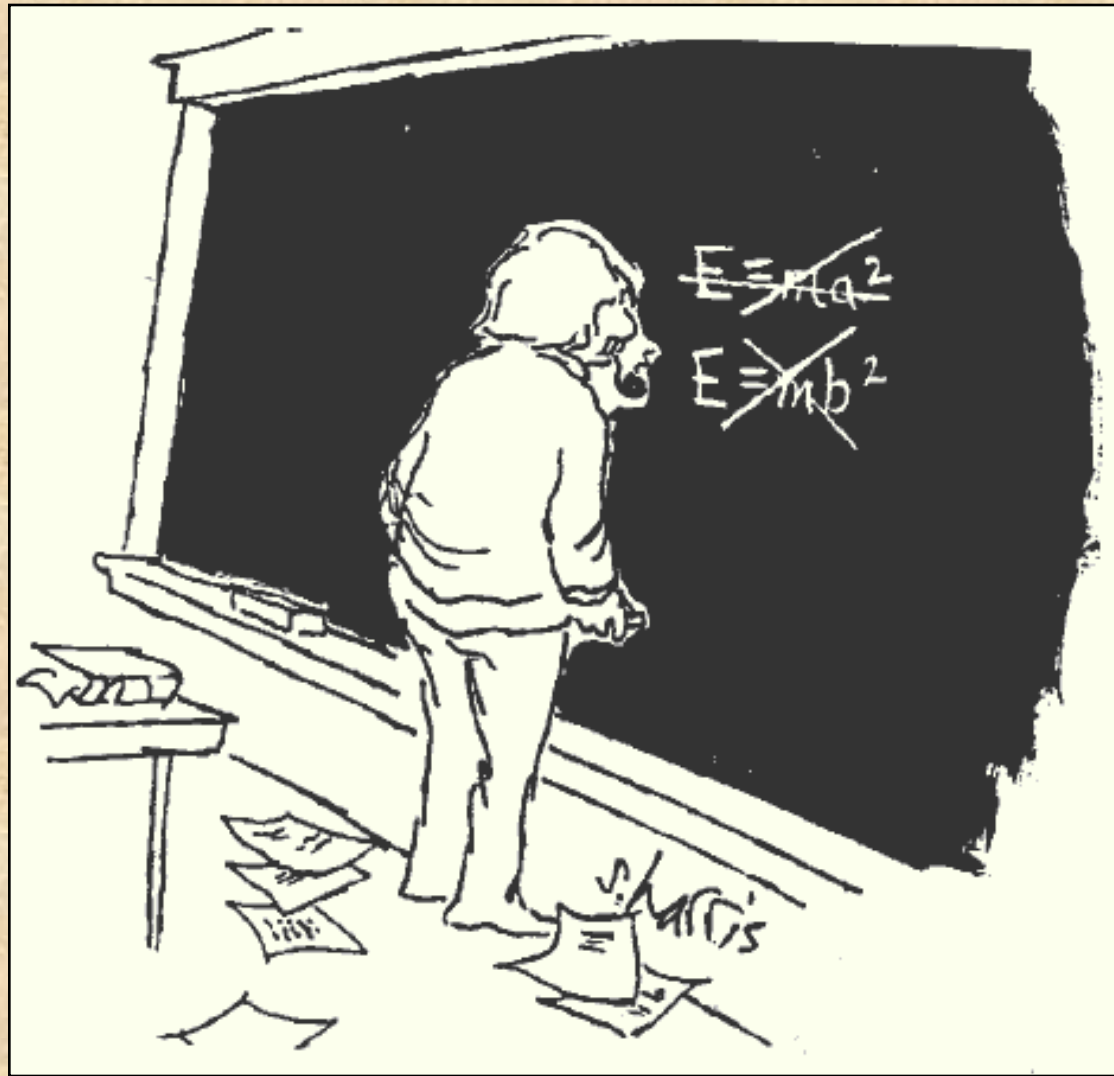
➤ The Equivalence Principle Has Been Tested to a Very High Degree



Will
2001

Double Neutron Star observations with the SKA will improve limits on potential violation of the Strong Equivalence Principle

The Field Equation for Gravity is no revolution...



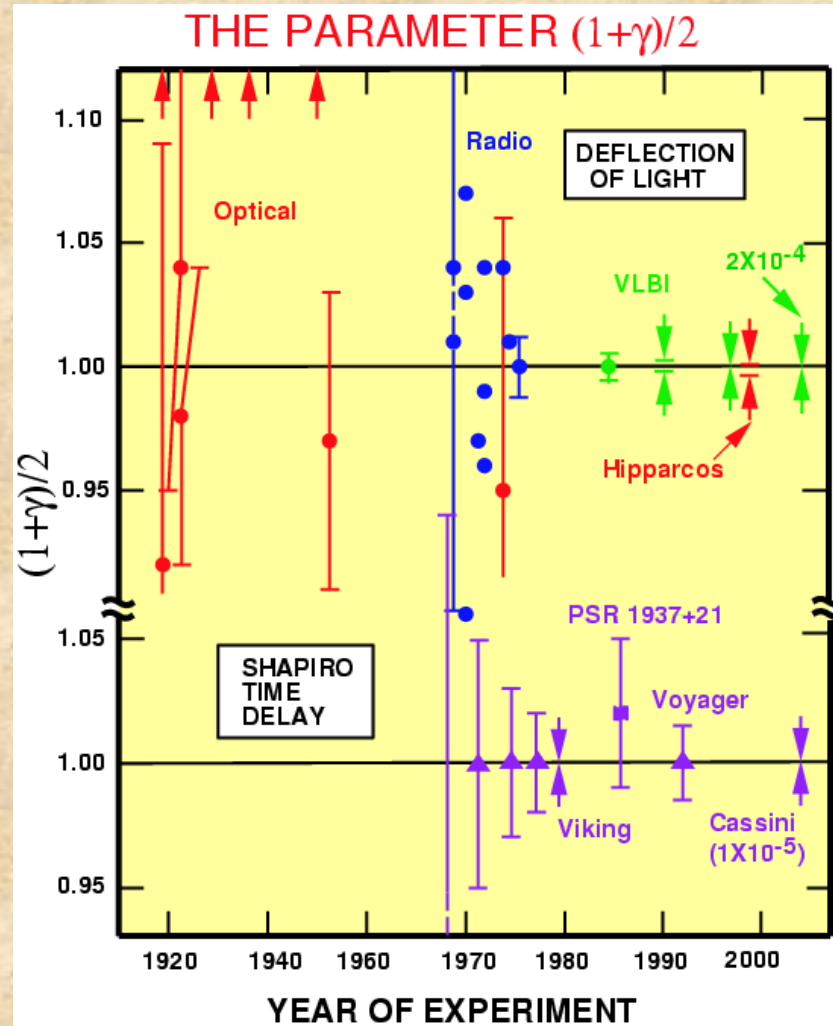
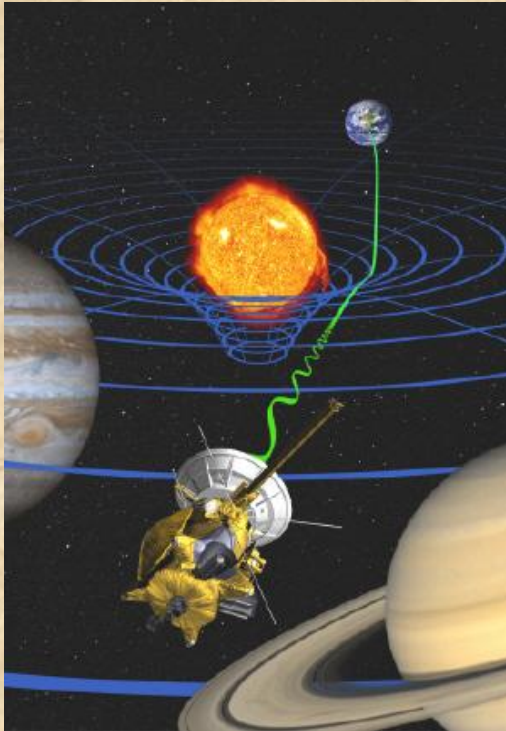
The Field Equation for Gravity follows general Newtonian ideas

Ricci curvature Matter+Radiation

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + S_m$$

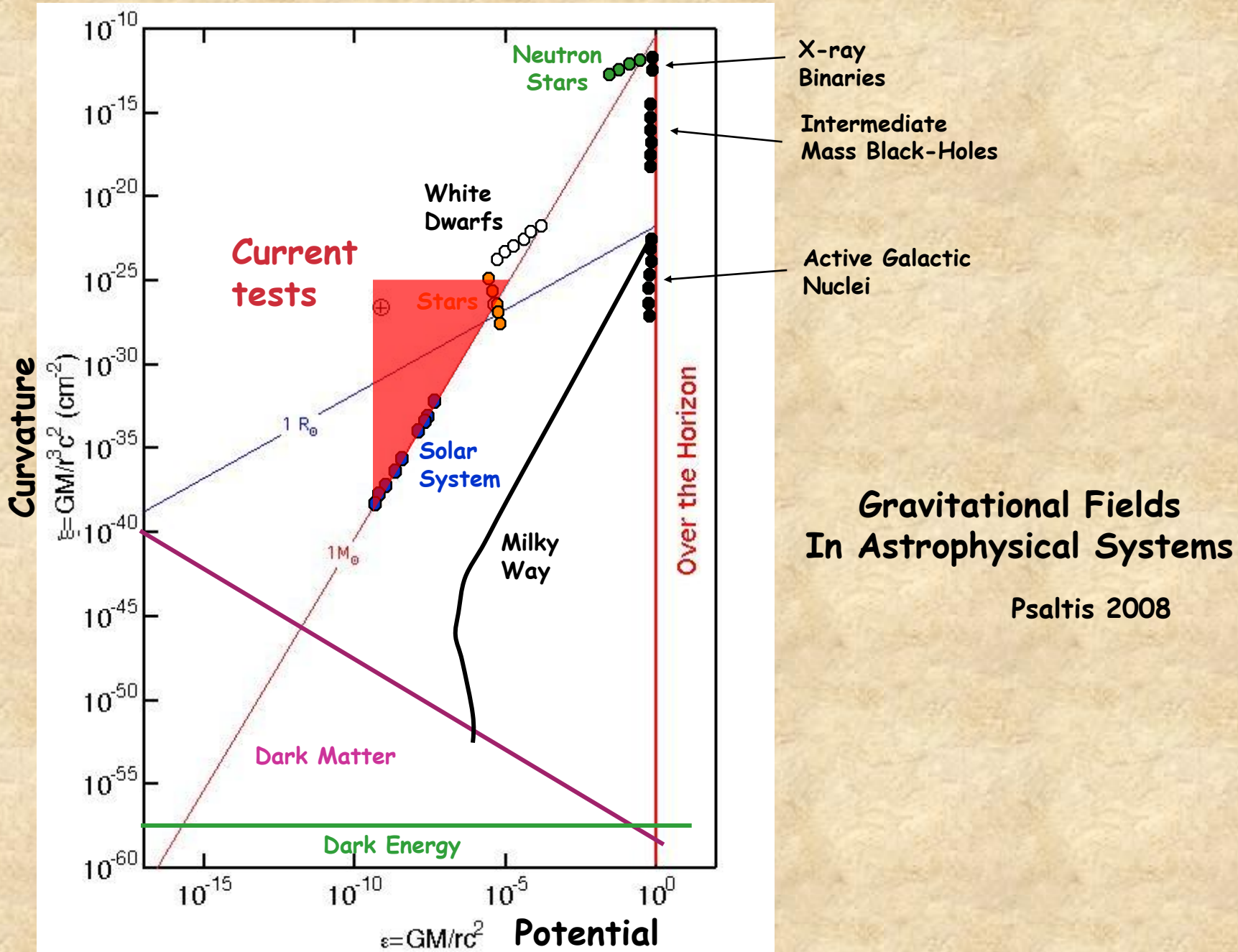
- ◆ 3+1 Dimensions of Spacetime
- ◆ Two gravitational degrees of freedom
- ◆ Only matter and radiation source the gravitational field
- ◆ No parity violating terms
- ◆ ...

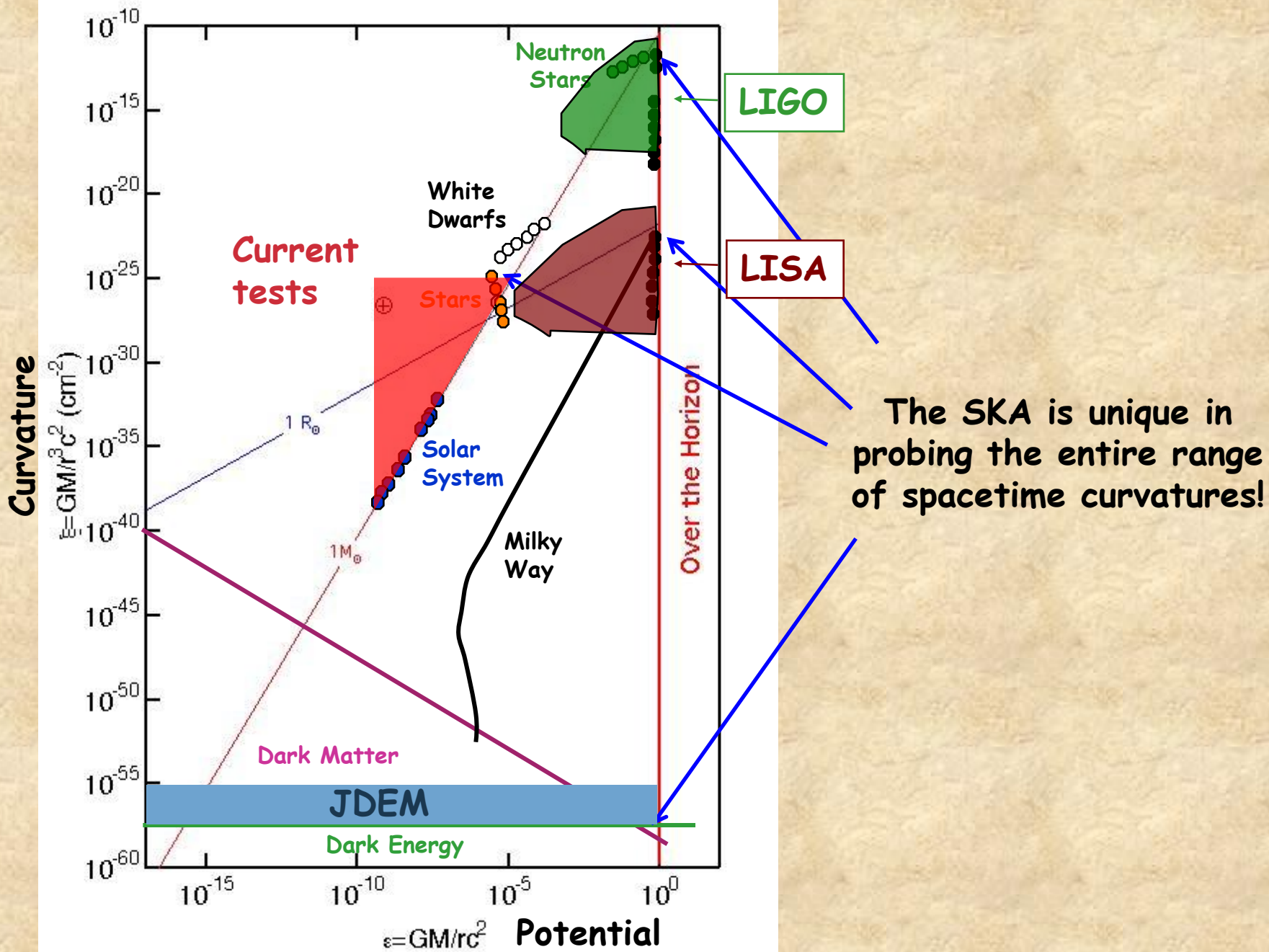
➤ The Einstein Field Equations have been tested to $\sim 10^{-5}$



Will 2006

PPN Metric:
$$ds^2 = -\left(1 - 2\frac{GM}{rc^2}\right)dt^2 + \left(1 + \gamma\frac{2GM}{rc^2}\right)dr^2 + r^2d\Omega$$

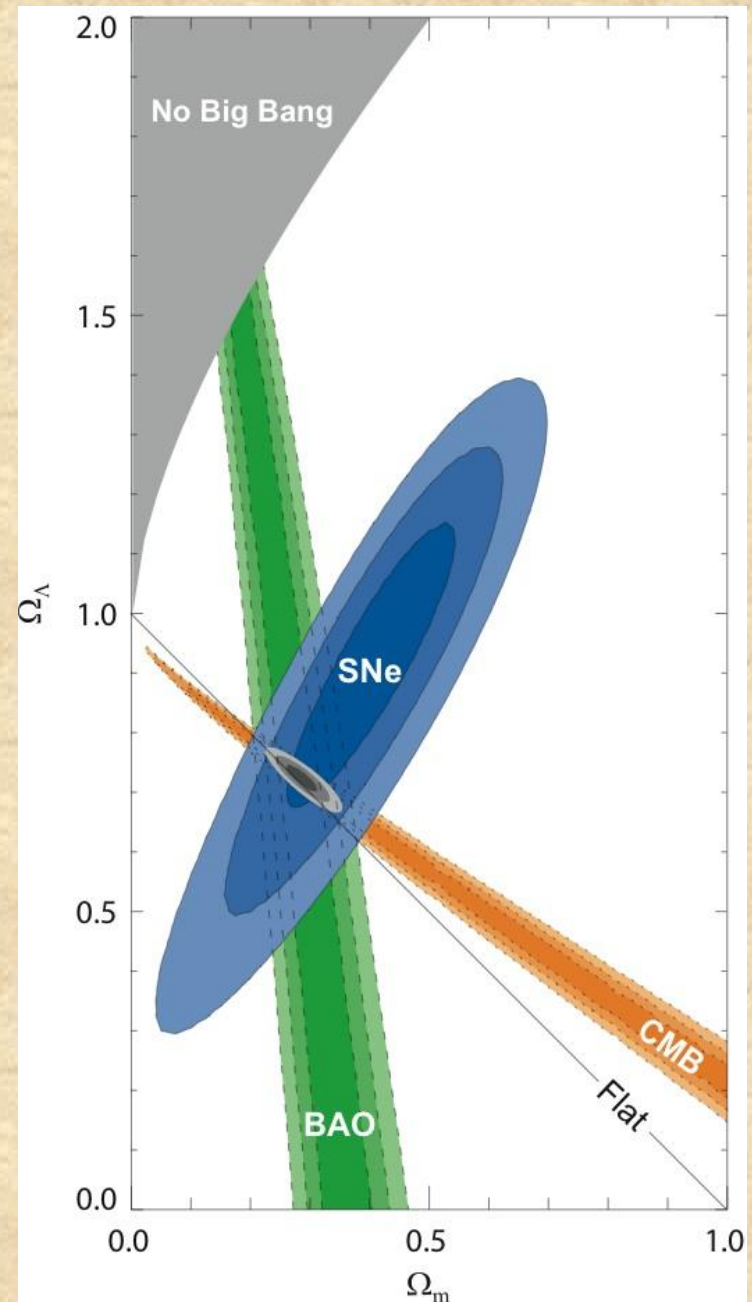




Are Matter and Radiation the Only Sources of Gravity?

(i) Dark Energy

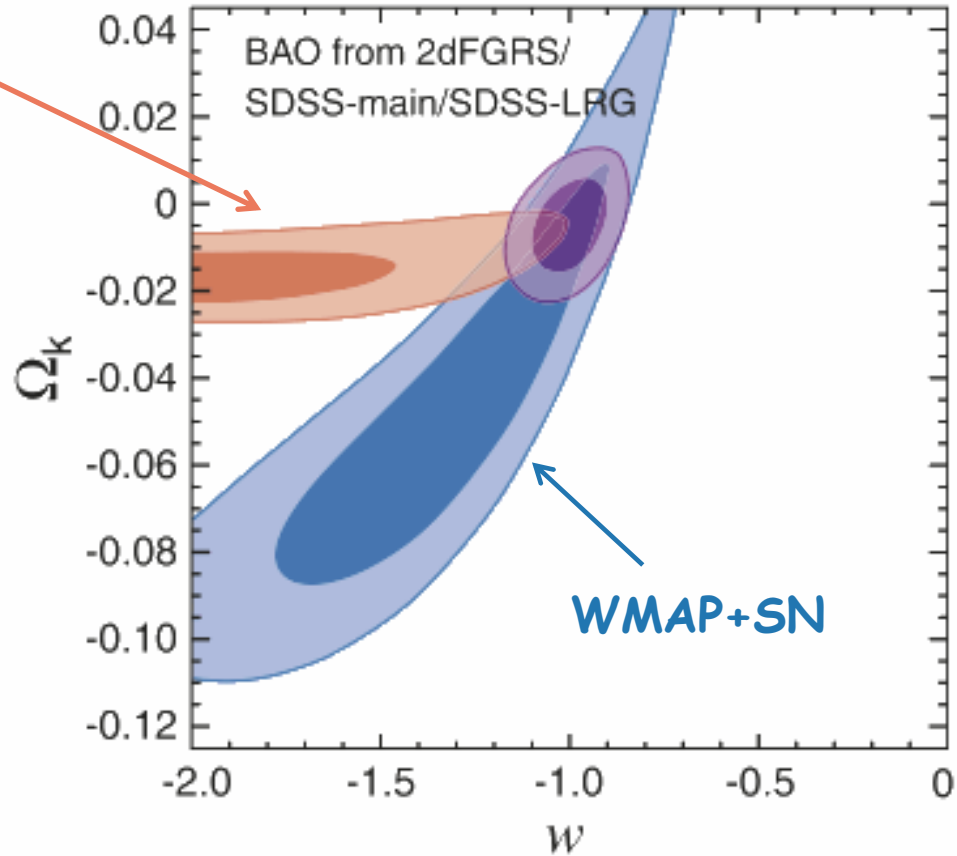
Is it a cosmological constant, or a new gravitating field?



Are Matter and Radiation the Only Sources of Gravity?

(i) Dark Energy?

WMAP+BAO



Komatsu et al. 2008

$$P = w\rho$$

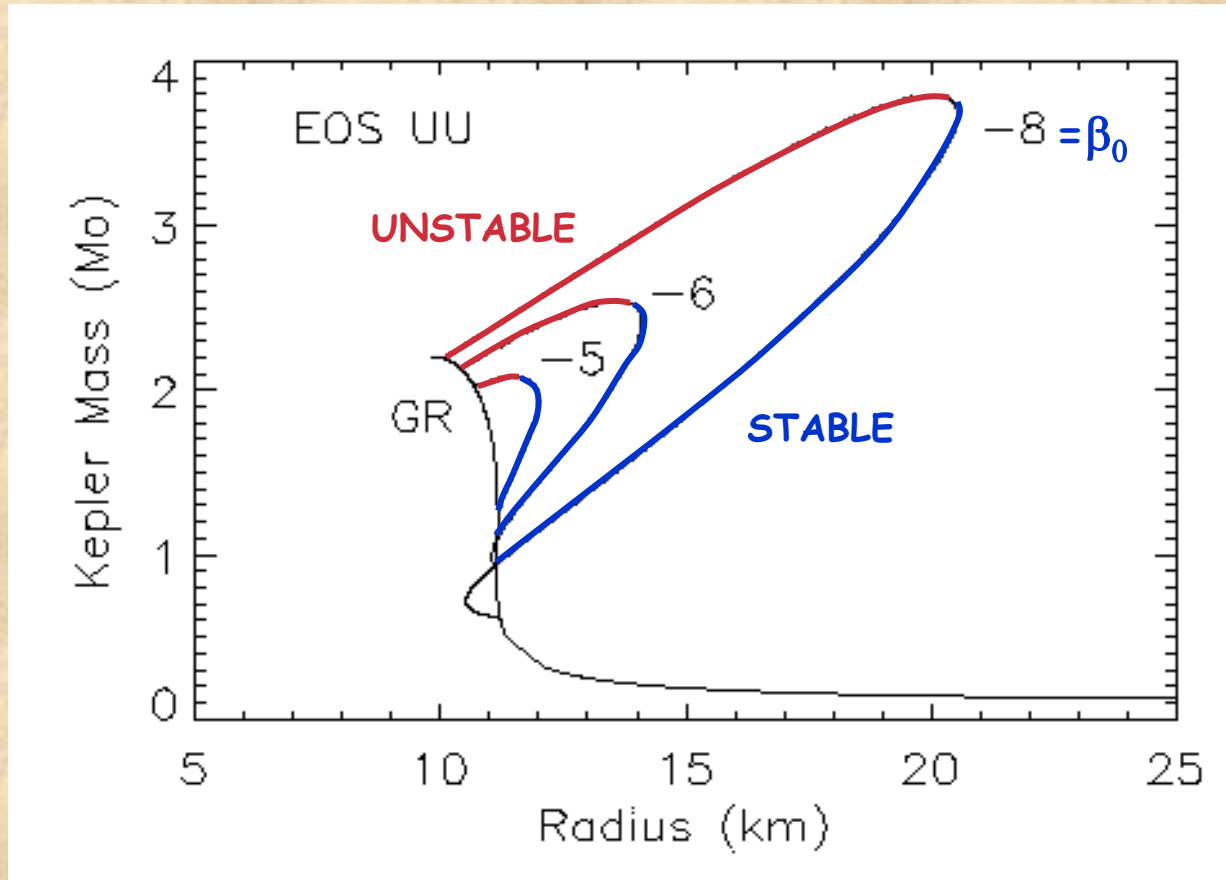
Dark Energy Equation of State

A 21cm Baryon-Acoustic-Oscillation survey with SKA will significantly improve these limits

Are Matter and Radiation the Only Sources of Gravity?

(ii) A scalar field that couples to gravity

$$A(\phi) = 1 + a\phi + \frac{1}{2}\beta\phi^2 + \dots$$



Are Matter and Radiation the Only Sources of Gravity?

(ii) A scalar field that couples to gravity

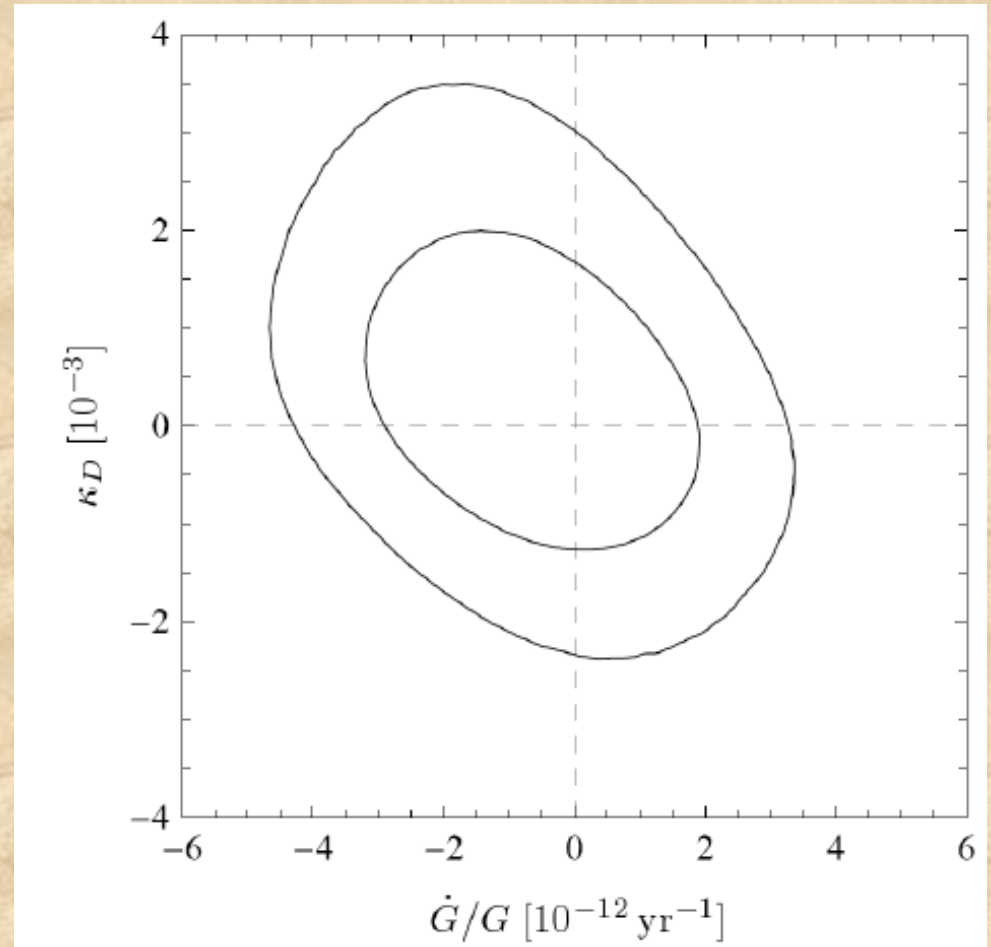
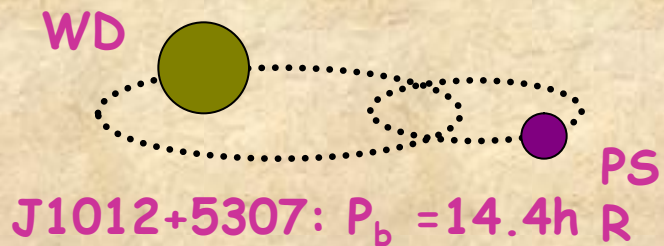
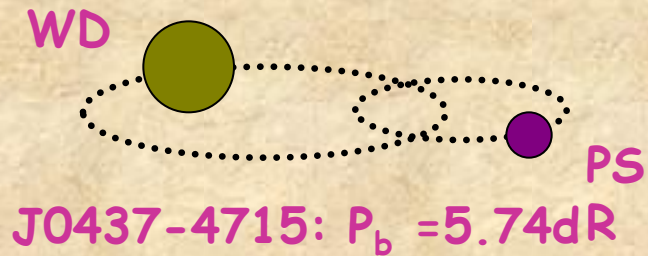
The presence of a scalar field leads to dipole gravitational radiation!

Eardley 1975

$$\dot{P}_{dipole} = -\frac{4\pi GM_{\bullet}}{c^3 P} \frac{M_{NS}}{M_{NS} + M_C} \left[\kappa_D \cdot s^2 \right]_{\alpha, \beta}$$

Are Matter and Radiation the Only Sources of Gravity?

(iii) A scalar field that couples to gravity



Lazaridis et al. 2009

How many Degrees of Freedom are there in the Gravitational Field?

Newtonian gravity

$$\nabla^2\Phi = 4\pi G\rho \Rightarrow \Phi = -\frac{GM}{r} + \Phi_0$$

and General Relativity

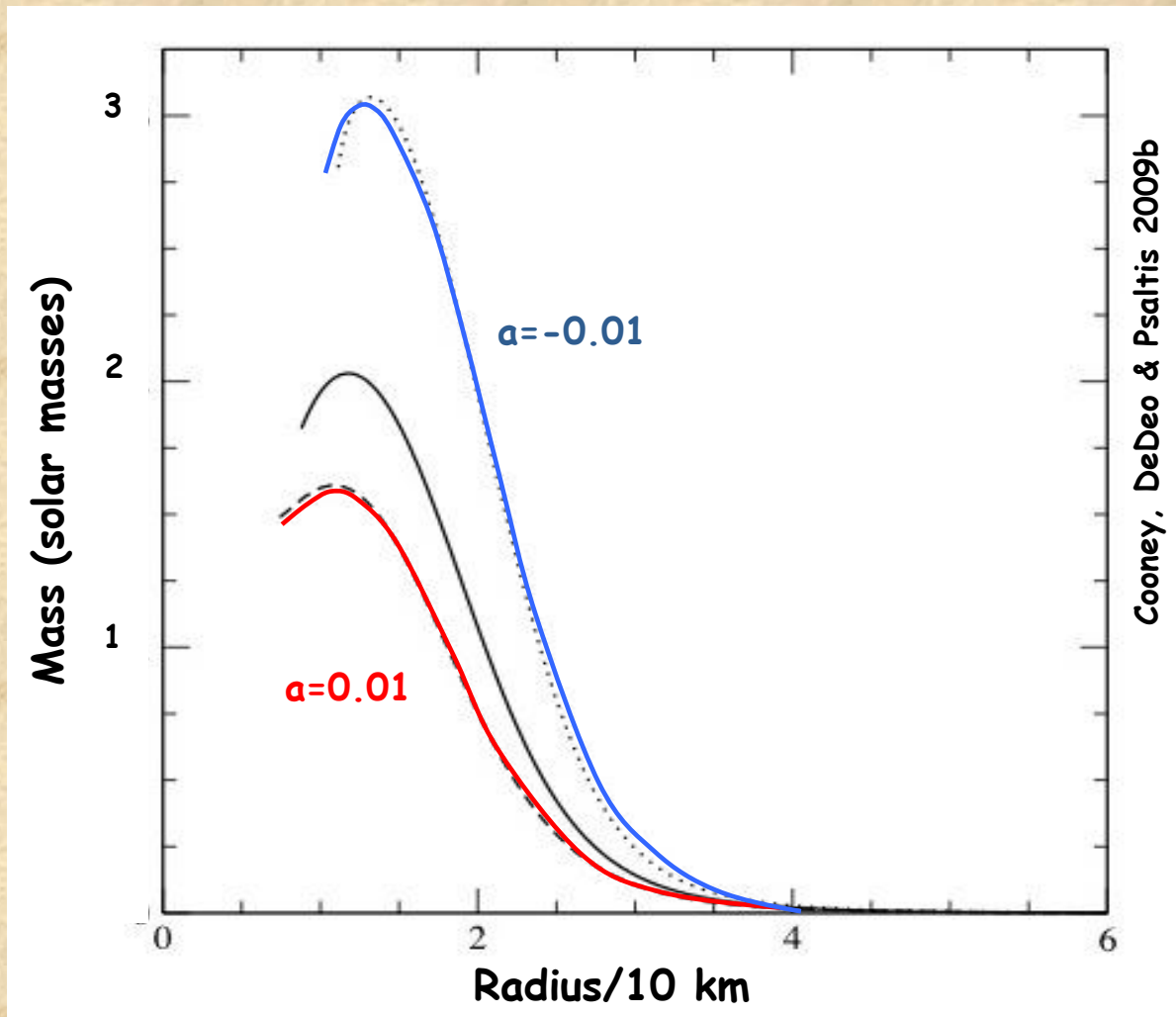
$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R$$

have only two, but what if, e.g.,

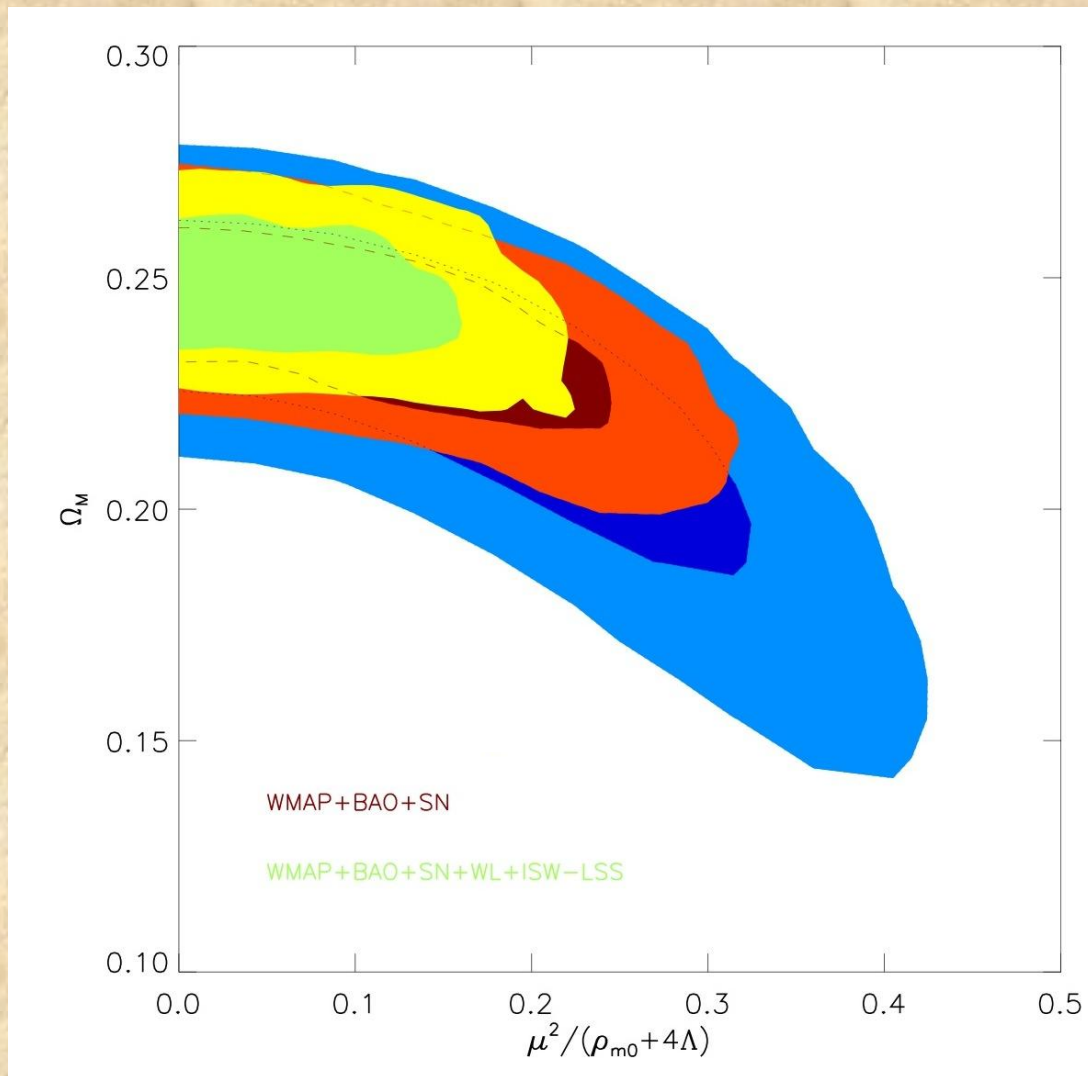
$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[\alpha R^2 + R - \Lambda + \frac{\mu^4}{R} + \frac{\eta^6}{R^2} \right]$$

How many Degrees of Freedom are there in the Gravitational Field?

Neutron stars in $f(R) = aR^2 + R$



Cosmology with $f(R) = R - 2\Lambda + \frac{\mu^4}{R}$



Bean, Cooney, DeDeo & Psaltis 2010

Parity violation in Gravity?

The moment of inertia of a slowly spinning neutron star is

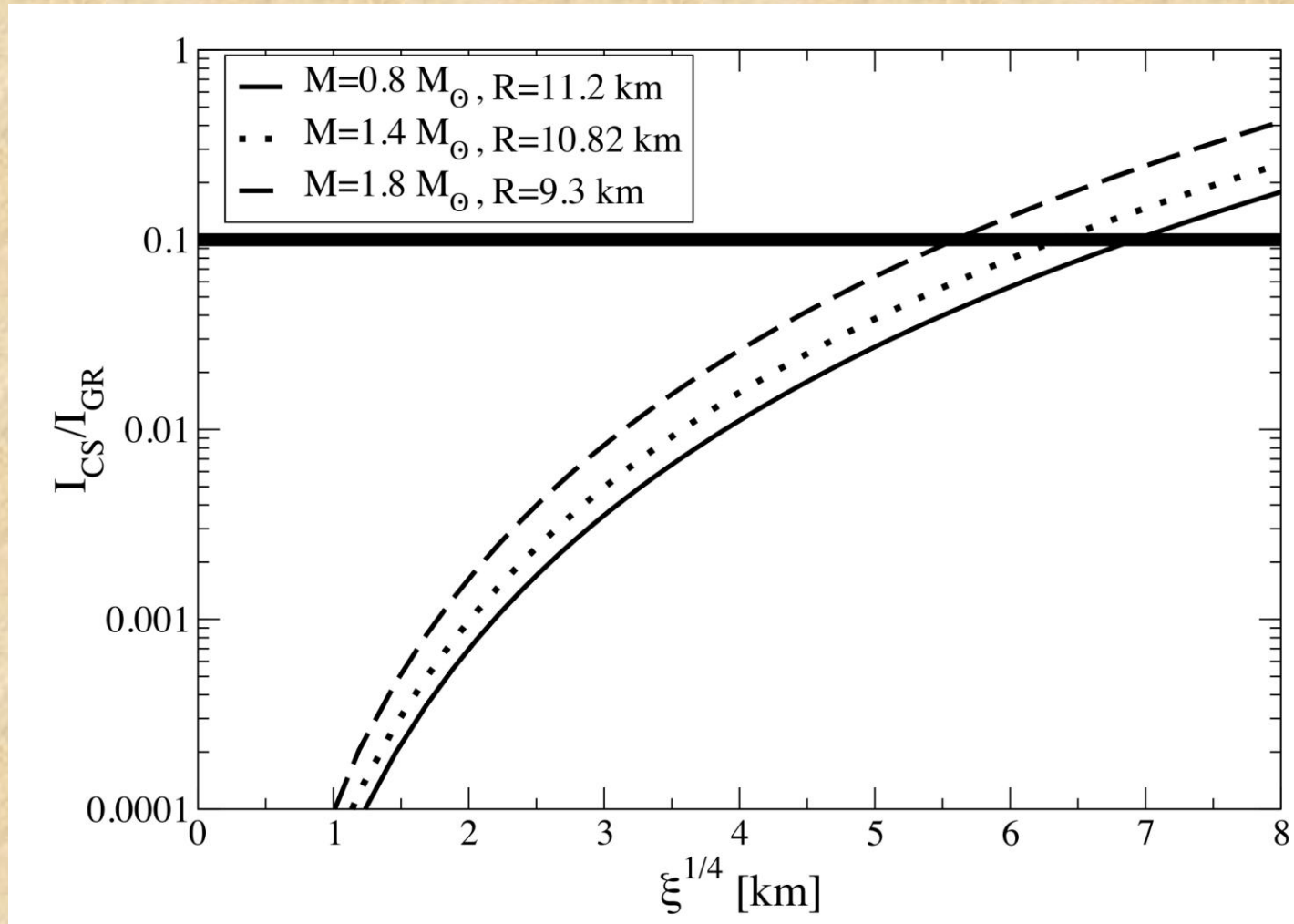
$$I = \frac{8\pi}{3} \int r^4 (\rho + P) \sqrt{\frac{g_{rr}}{g_{tt}}} \left(1 - \frac{g_{t\phi}}{\Omega} \right) dr$$

and depends on the parity violating potential of the gravitational theory

Parity violation in Gravity?

Test case: Neutron Stars in Chern-Simons gravity

Yunes, Psaltis, Ozel, & Loeb, 2010, PRD



Testing the General Relativistic No-Hair Theorem

- expand Kerr metric in multipoles
- use observations to measure at least 3 multipole coefficients
- investigate whether

$$q = -a^2$$

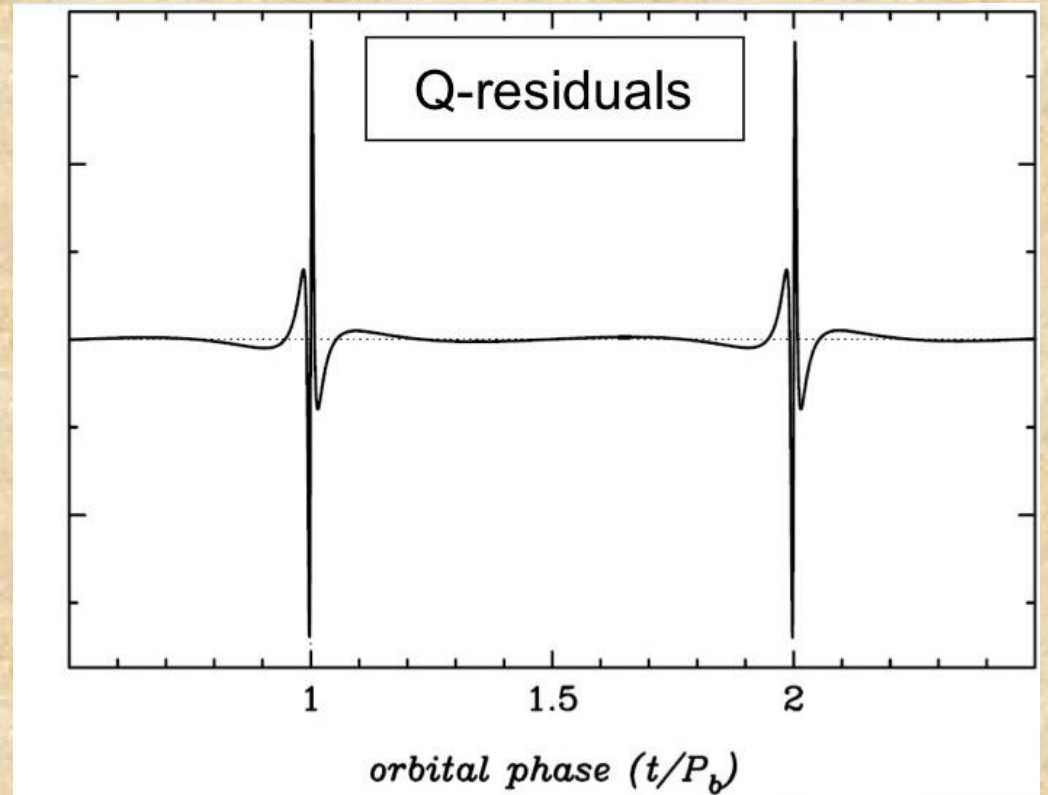
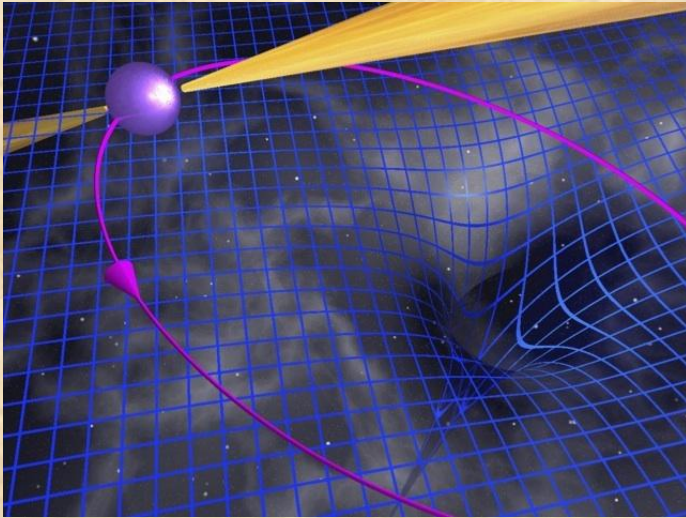
by writing a general spacetime that satisfies $R_{\mu\nu}=0$
but with

$$q = -(a^2 + \varepsilon)$$

and measuring the deviation parameter ' ε '

Ryan 1995; Wex & Kopeikin 1999; Collins & Hughes 2004; Glampedakis & Babak 2006
Will 2008; Vigeland & Hughes 2009; Johannsen & Psaltis 2009, 2010

A BH-PSR binary

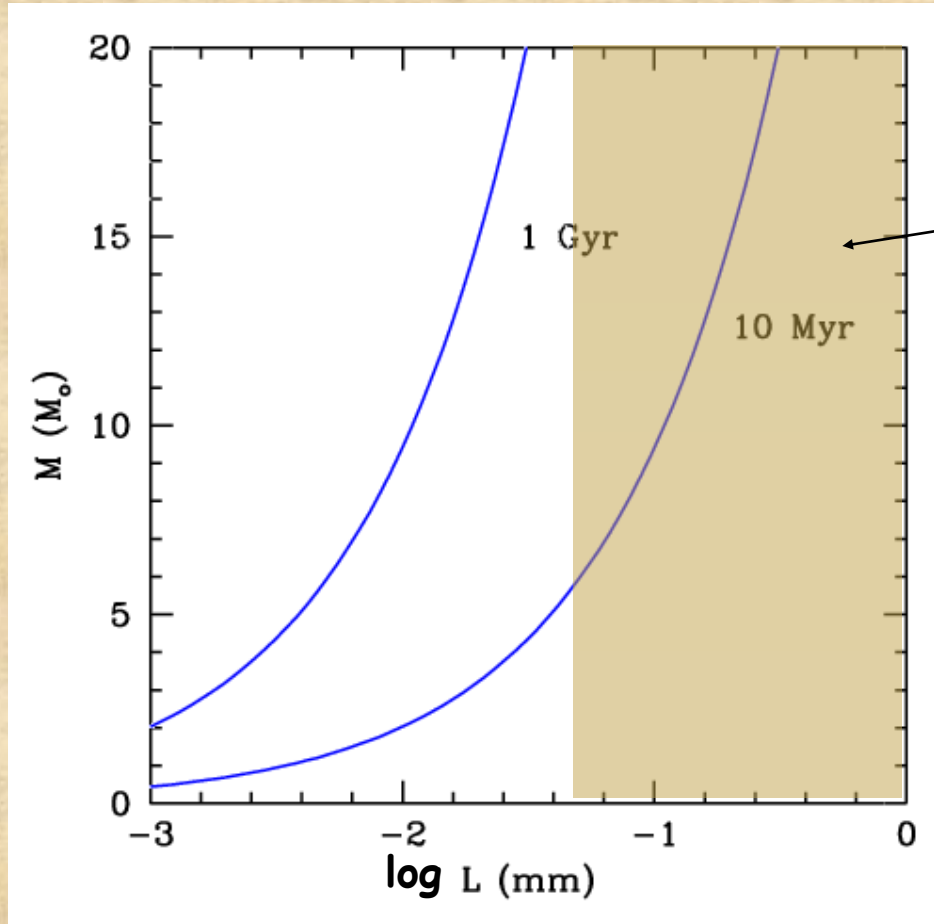


Wex & Kopeikin 1999

How many are the dimensions of the spacetime?

In a universe with large 'RS' extra dimensions, black holes evaporate FAST due to emission of gravitons in the bulk

EMPARAN et al. 2002



Tabletop experiments:

$$L < 0.05 \text{ mm}$$

Large Extra Dimensions?

In a BH-PSR binary, the rate of change of the orbital period due to the evaporation of the black hole may dominate that due to the emission of gravitational waves.

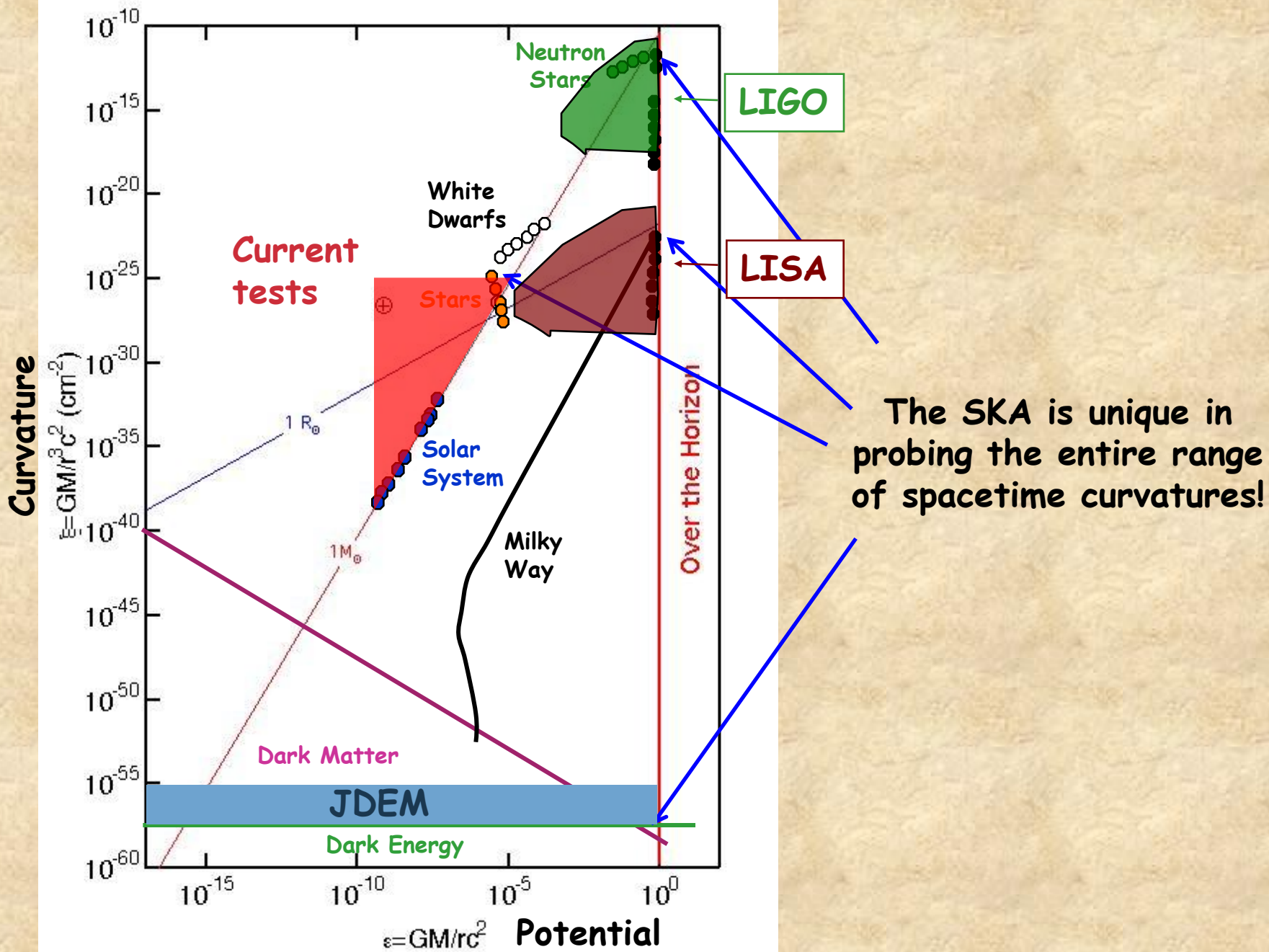
Psaltis & Johannsen 2010

If we find no evidence for black-hole evaporation, then the size of the extra dimensions has to be

$$L \leq 0.1 \left(\frac{P}{2\text{hr}} \right)^{4/3} \left(\frac{M_{BH}}{10M_{\odot}} \right) \mu m$$

Conclusions

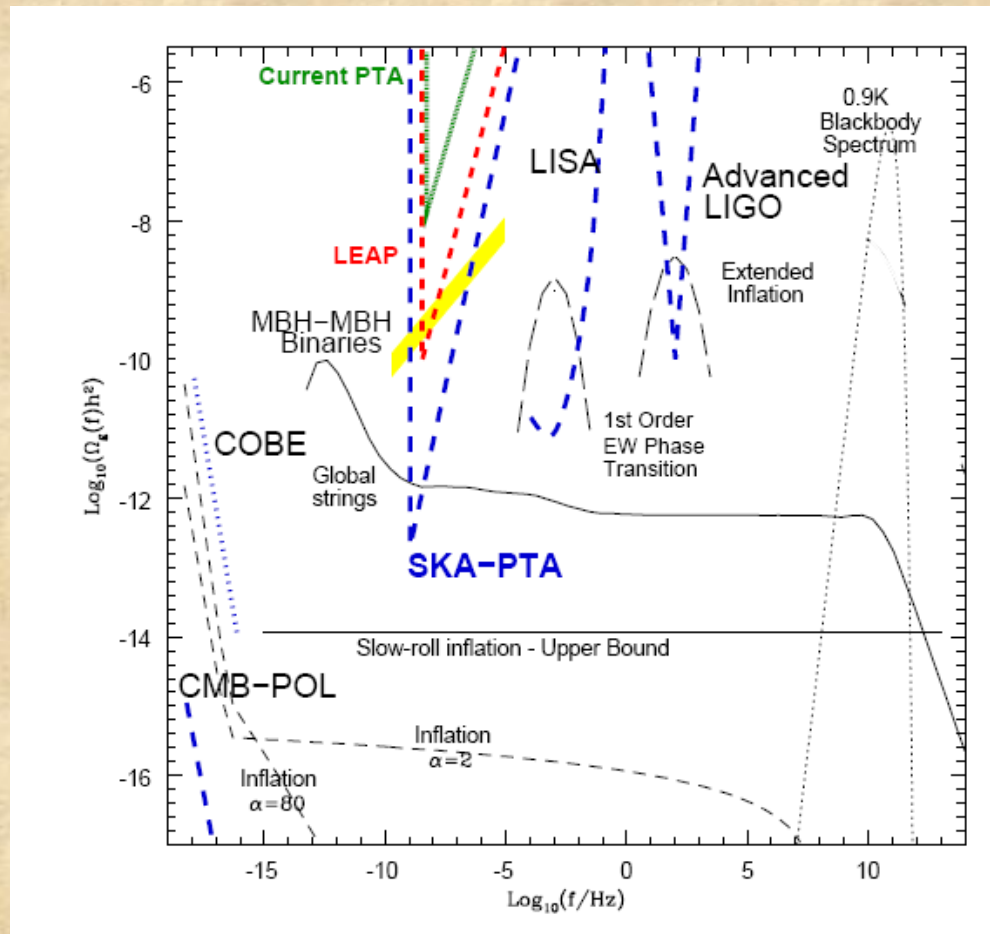
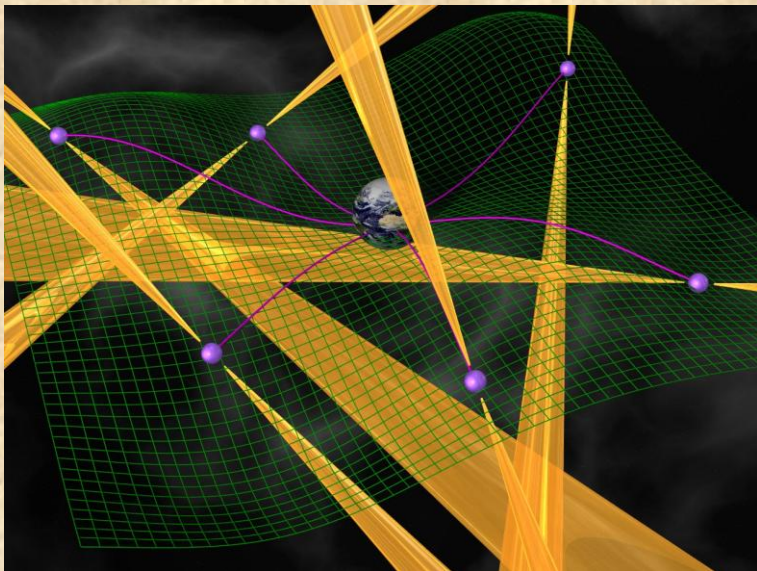
- ✧ General Relativity has been very accurately tested only in the weak-limit regime
- ✧ Many possibilities open regarding the fundamental properties of the gravitational field, e.g., sources, degrees of freedom, dimensionality, parity.
- ✧ The SKA is unique in probing a wide range of scales and a large number of phenomena that will help testing our understanding of gravity



Are Matter and Radiation the Only Sources of Gravity?

(i) The scalar field responsible for inflation (the inflaton)

A Pulsar Timing Array



Kramer et al. 2004