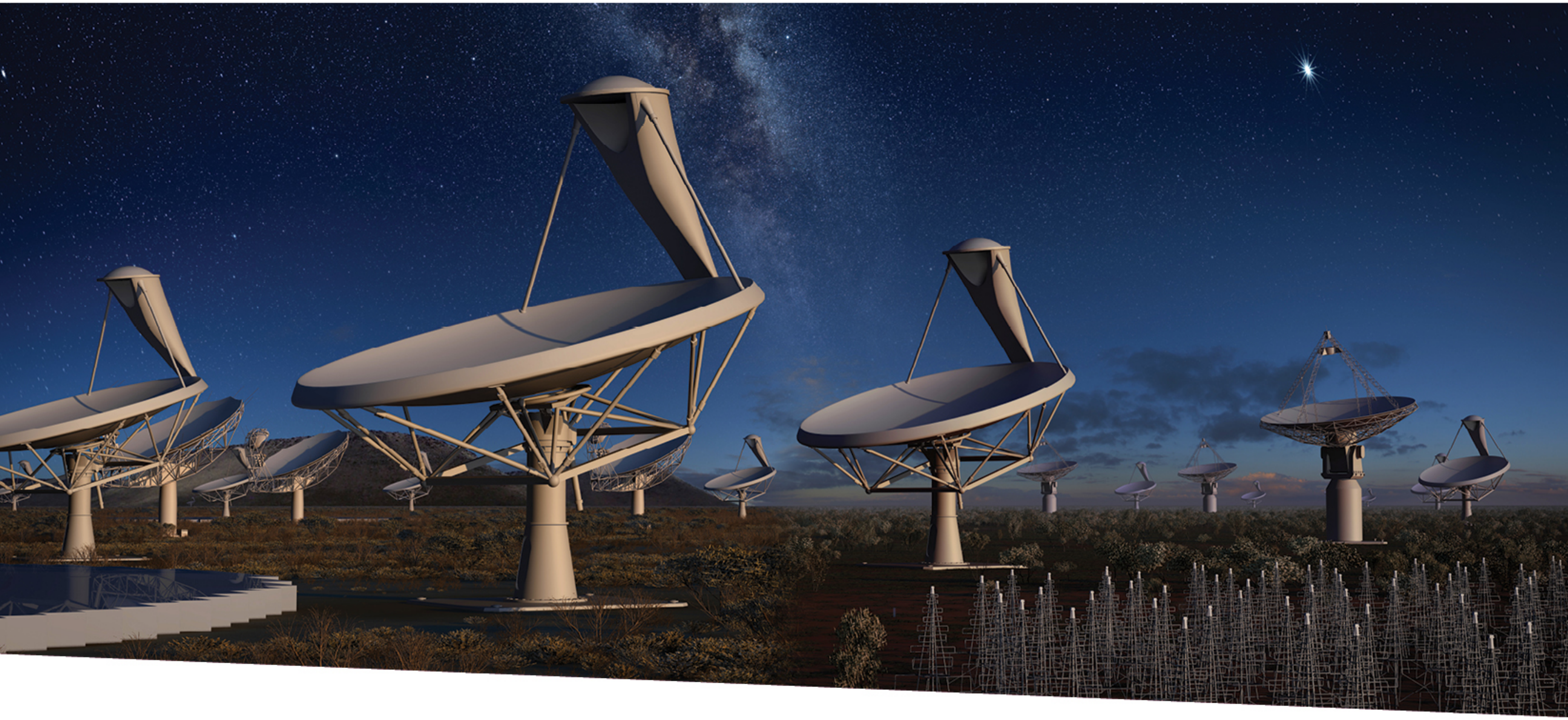
SKA1-Low Error Analysis



SQUARE KILOMETRE ARRAY

Exploring the Universe with the world's largest radio telescope

Robert Braun, Science Director

25 February 2016

SKA1-Low Configuration

Scientific Constraints:

- The highest possible filling factor of both individual stations and the core configuration over the key frequency interval of 100 – 200 MHz.
- Instantaneous field-of-view that exceeds about 4 deg^2 for EoR imaging and 16 deg^2 for EoR power spectra (both apply to the frequency range 50 – 200 MHz).
- Ability to provide excellent quality of ionospheric calibration: enough high sensitivity pierce points.
- Ability to provide excellent quality of direction dependent gain calibration: extremely low far sidelobes of station beam.
- High sensitivity and good visibility sampling to angular scales of about 10 to 1000 arcsec.

Practical constraints:

- Site-specific and maintenance constraints.
- Infrastructure Cost.

SKA1-Low Configuration

Desired solution:

- Highest possible filling factor of antennas in station tied to a nominal frequency (the λ/2 antenna spacing) of no lower than about 100 MHz.
- Tightest practical packing of stations within core consistent with maintenance requirements.
- Logarithmic decline of collecting area beyond core: radii of about 350m to 35km.
- Smallest total number of extra-core sites plus minimum spanning tree with adequate aperture sampling and instantaneous visibility coverage.
- Hierarchical station definition allowing “tuneable” choice of beam-forming scales (discrete or continuous) about 10 – 90 m.
- Identical station definition both inside and outside core.

SKA1-Low Configuration

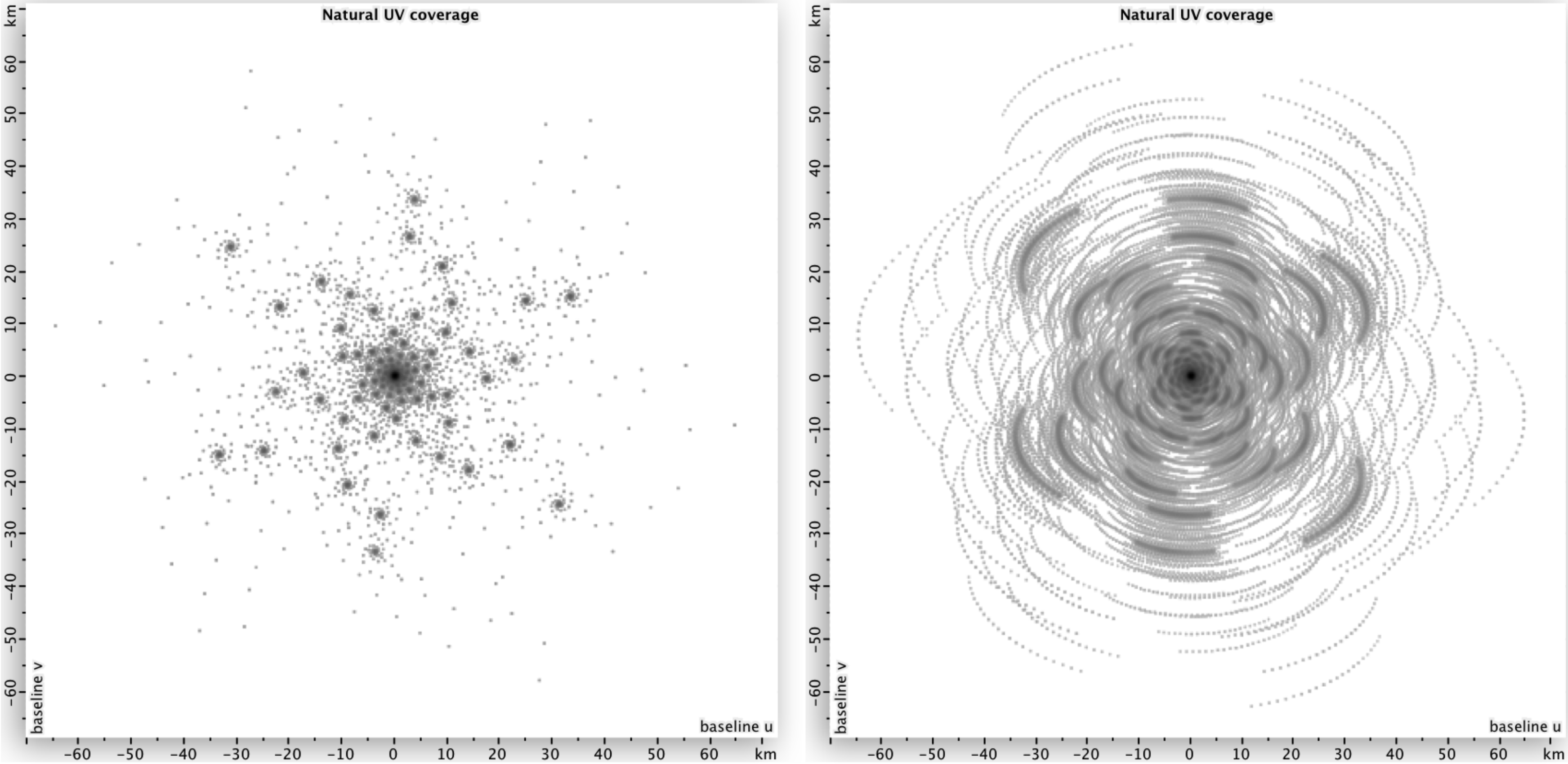


Figure 1: The SKA1-low snap-shot (left) and 4-hour tracking (right) visibility coverage for a monochromatic observation at a nominal declination of -30.

SKA1-Low Configuration

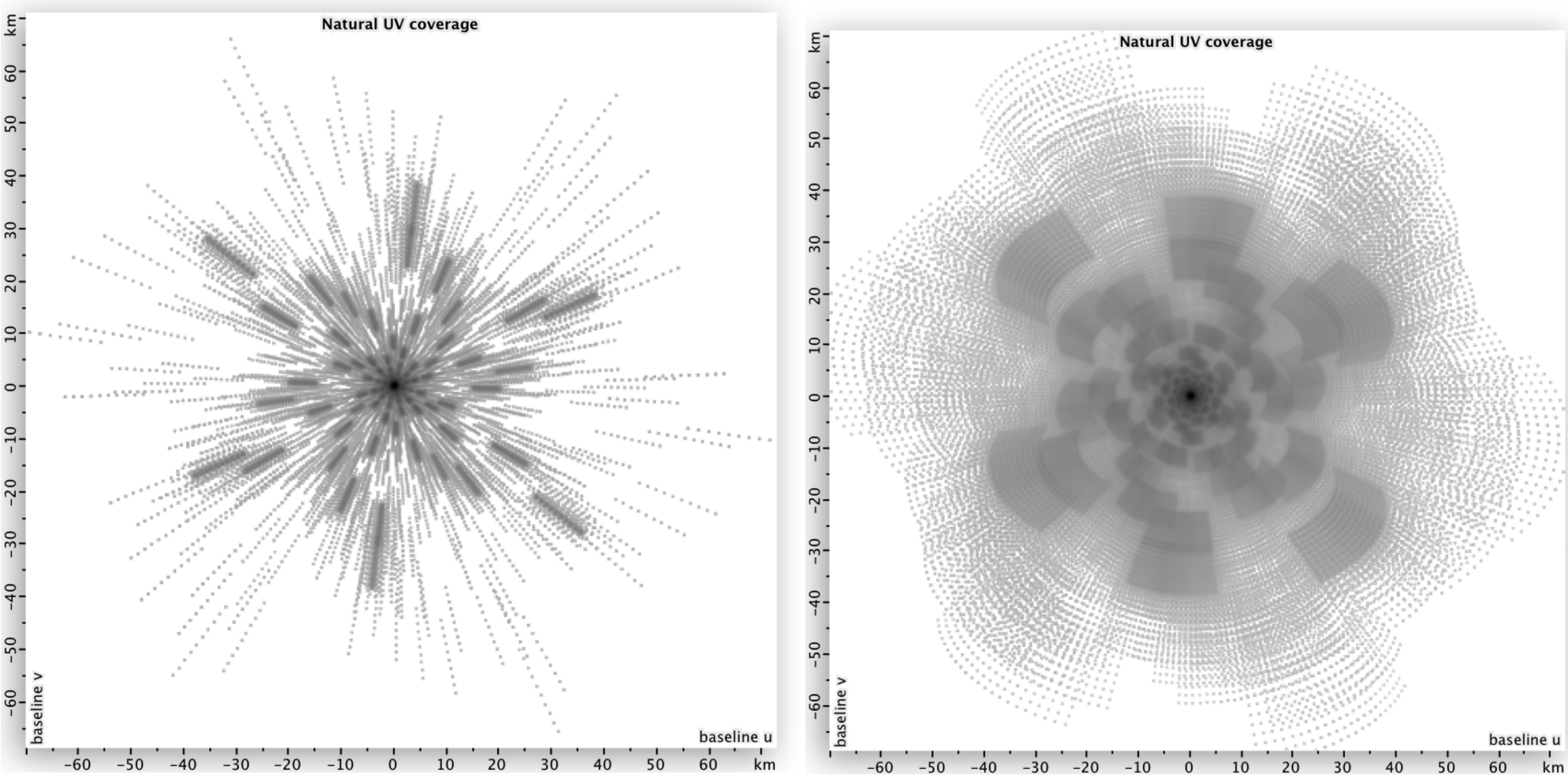


Figure 1: The SKA1-low snap-shot (left) and 4-hour tracking (right) visibility coverage for a broad-band continuum observation (with 30% fractional bandwidth as example) at a nominal declination of -30.

SKA1-Low Configuration

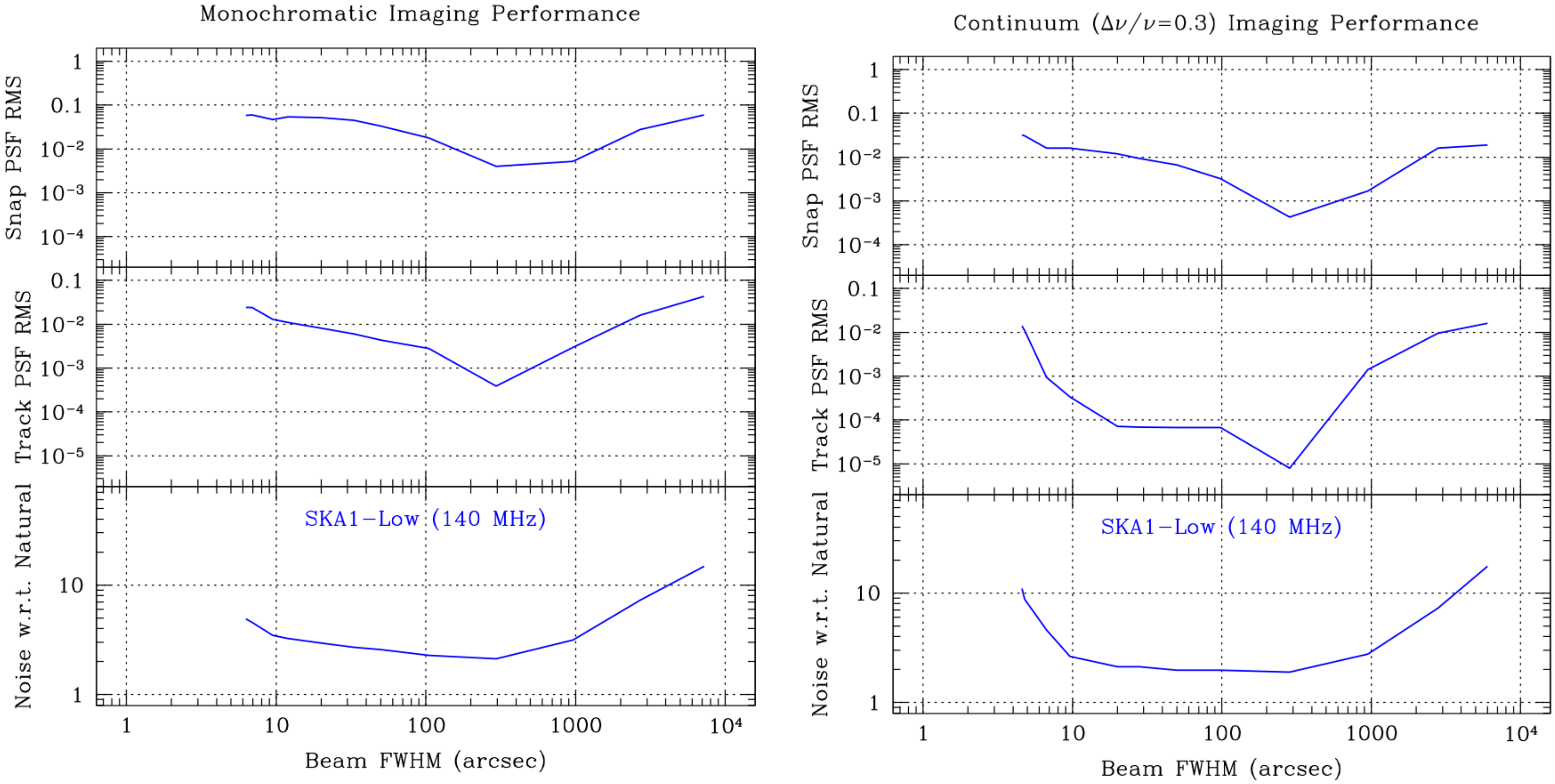


Figure 1: Monochromatic (left hand panel) and broad-band continuum (right hand panel) image noise relative to the total array SEFD (bottom) as well as PSF near-in sidelobe levels for snap-shot (top) and 4-hour track (middle) observations for the SKA1-low configuration as function of required Gaussian beam size at a nominal frequency of 140 MHz.

SKA1-Low Instrument/Calibration parms.

- Parametric model relating residual calibration errors to effective image noise (Braun, 2013, A&A 551, 91)
- Each effect described by both intrinsic magnitude as well as correlation timescale and frequency bandwidth:

$$\sigma_{\mathrm{Vis}},\ \tau_{\mathrm{T}},\ \Delta\nu_{\mathrm{F}}$$

- Basic unit of observation is an n-hour tracking observation (eg. HA = -4 – +4^{h} or -2 – +2^{h})

SKA1-Low Instrument/Calibration parms.

- Distinction between effects due to sources within the image field or outside
  - Inside image: standard radiometer equation

$$\sigma_{Map} = \sigma_{Vis}/[M_T M_F N(N-1)/2]^{0.5}$$

  - Outside image: via PSF sidelobes and via self-cal noise propagation

    PSF noise scales as N^{-2}, self-cal noise as $N^{-1.5}$,

    so **self-cal noise** dominates for large N (dish/station number)

$$\sigma_{Map} = \sigma_{Vis} (S_{Max}/S_{Tot}) \{N_C / [M_T M_F N^2(N-3)]\}^{0.5}$$

- Outcome of multi-track observing campaign depends on nature of each error
  - Errors associated with random processes average down as √number tracks
  - Errors in source model of sky or description of the stationary instrumental response do not average down

SKA1-Low Instrument/Calibration parms.

Parameter	Definition
φ_C	Main beam “external” gain calibration error
η_F	Far sidelobe suppression factor
ε_F	Far sidelobe attenuation relative to on-axis
ε_S	Near-in sidelobe attenuation relative to on-axis
ε_M	Discrete source modelling error
P (arcs)	Mechanical slowly varying systematic pointing error
τ_P (min)	Timescale for slowly varying pointing error
ε'_P	Rapidly varying random pointing induced gain error
τ'_P (sec)	Timescale for rapid pointing errors
ε_Q	Main beam shape asymmetry
ε_B	Main beam shape modulation with frequency
l_C (m)	Effective “cavity” dimension for frequency modulations of main beam
τ^*	Nominal self-cal solution timescale (10% PSF smearing at first null)
$\Delta\nu^*$	Nominal self-cal solution bandwidth (10% PSF smearing at first null)
σ_{Sol}	Self-cal solution noise per visibility required for convergence
σ_{Cfn}	Source confusion noise
σ_{Cal}	“External” gain calibration noise
σ_T	Thermal noise
σ_N	Nighttime far sidelobe noise term
σ_D	Daytime (includes Sun) far sidelobe noise term
σ_S	Near-in sidelobe noise term
σ_P	Main beam slow pointing induced noise term
σ'_P	Main beam rapid pointing induced noise term
σ_Q	Main beam asymmetry induced noise term
σ_B	Main beam frequency modulation induced noise term
σ_M	Source modelling error induced noise term

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SKA1-Low assumed instrumental parameters

Telescope	VLA B-Cfg	SKA1-Mid	LOFAR-NL	SKA1-Low
N	27	197	62	512
d (m)	25	15	31	35
B_{Max} (km)	11	150	80	65
B_{Med} (km)	3.5	2.6	6.6	4.0
φ_C	0.1	0.1	0.2	0.2
τ_C (min)	15	15	15	15
η_F	0.1	0.2	0.5	0.5
ε_S	0.02	0.01	0.1	0.1
P (arcs)	10	10		
τ_P (min)	15	15		
ε'_P	0.01	0.01	0.01	0.01
τ'_P (sec)	5	5	60	60
ε_Q	0.055	0.04	0.01	0.01
ε_B	0.05	0.01	0.01	0.01
l_C (m)	8.2	7	10	10

LOFAR-NL Configuration

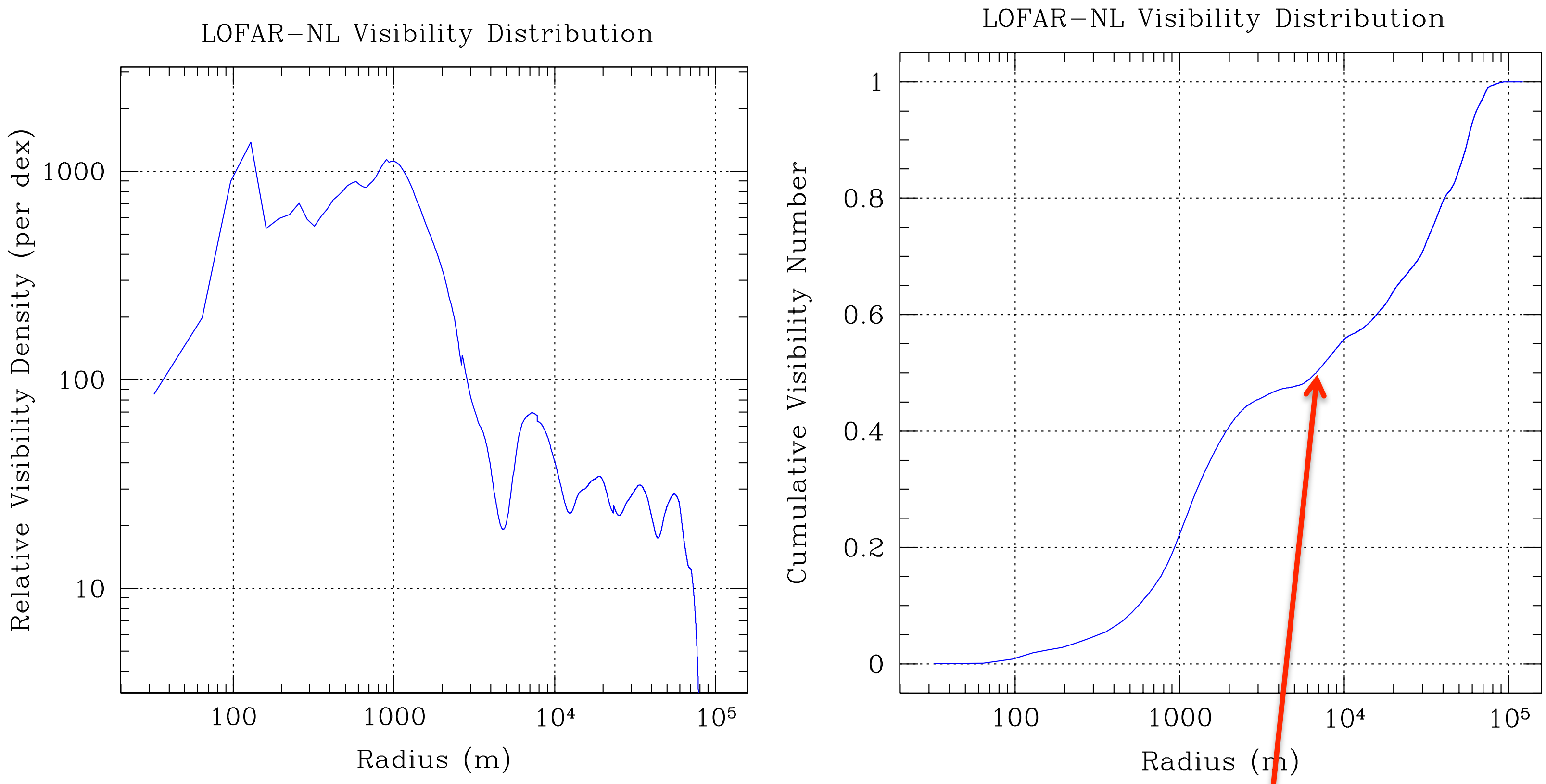
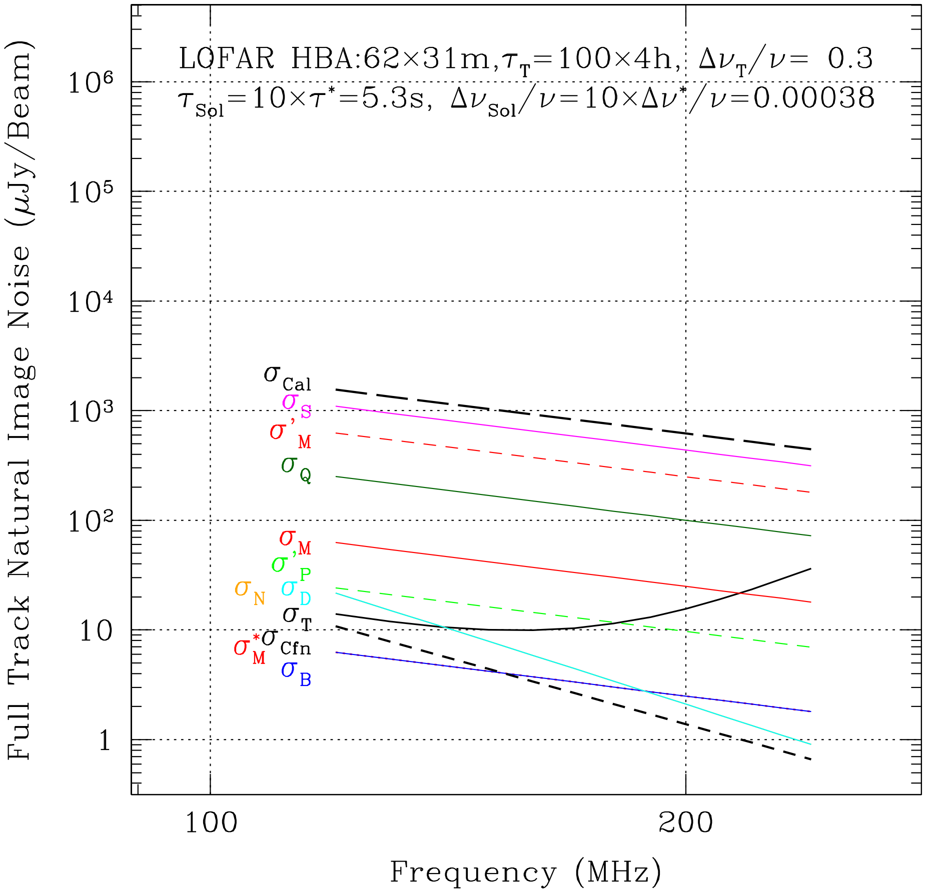
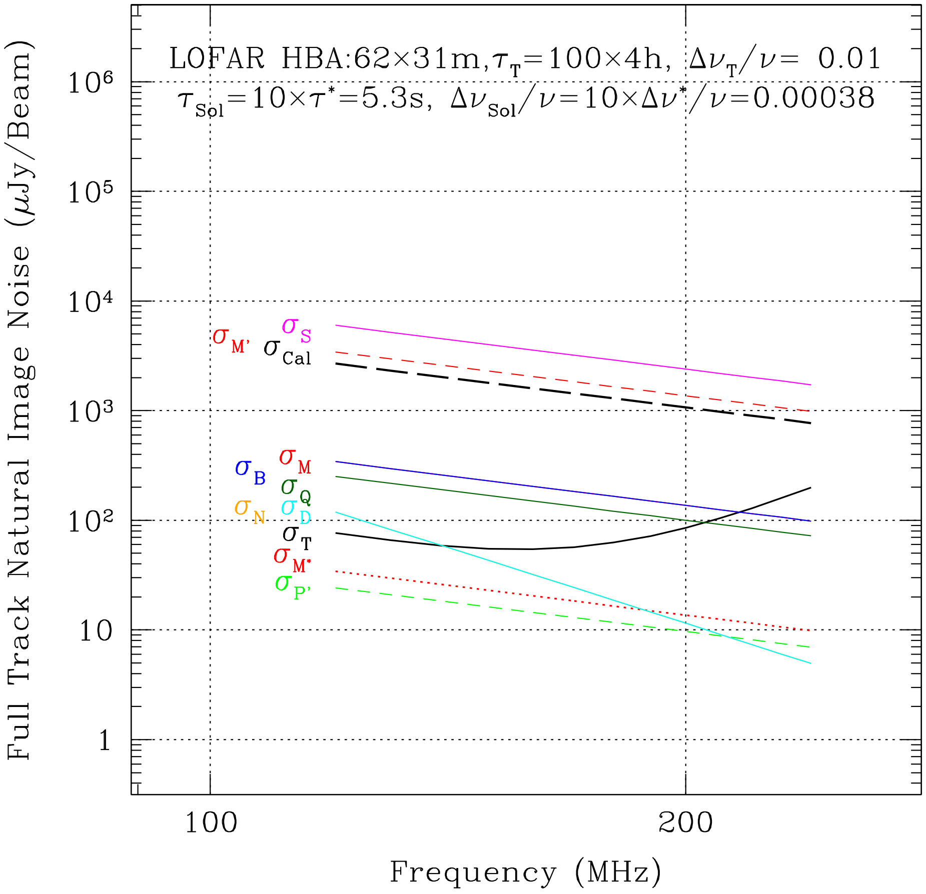


Figure 9. Relative visibility density (left) and cumulative visibility distribution (right) for LOFAR-NL based on a 4-hour track at δ = +30°. The median baseline length for such an observation is 6.6km.

LOFAR-NL deep integrations





- Noise budget for deep integrations

LOFAR-NL deep integrations

- A very high modelling precision of ε_M =0.002 must be achieved.
  - 20,0000 – 50,000 source components (mostly main beam and near-in sidelobes) being used for the most demanding apps
  - Current models based on wavelets, Gaussians, delta functions
  - Must take account of time and bandwidth smearing for data comparison
  - Scope for improved source representation
- Post-calibration frequency modulation of the main beam gain must be less than ε_B = 0.002.
- Post-calibration residual main beam azimuthal asymmetries must be less than ε_Q = 0.0005.
  - SageCal approach uses 100's of clusters of nearby source components to determine direction dependent gain solutions: combination of ionospheric phase and station beam shape amplitude
  - Good station beam model would make this much easier/better

LOFAR-NL deep integrations

- Random electronic gain variations ($\tau \approx 1^{m}$) that induce station “pointing” offsets must be kept below $\underline{\varepsilon'_{P}} = 0.006$.
- The brightest 1.0 dex [$= \log_{10}(\varepsilon_{S}/\underline{\varepsilon_{S}}) = \log_{10}(0.01/0.001)$] of random sources occurring within the main beam near-in sidelobes must be included in the self-cal model.
  - Need to include 2000 – 3000 sources brighter than about 35 mJy
- The brightest 0.2 dex [$= \log_{10}(\eta_{F}/\underline{\eta_{F}}) = \log_{10}(0.5/0.3)$] of sources occurring over the entire visible sky must be included in the self-cal model and subtracted.
  - Need to include all sources brighter than about $S_{1.4GHz} \approx 520$ Jy: only Cygnus A and Cas A (and Sun!)
  - (Also depends on $B_{Med} = 6.6$km!)

SKA1-Low Configuration

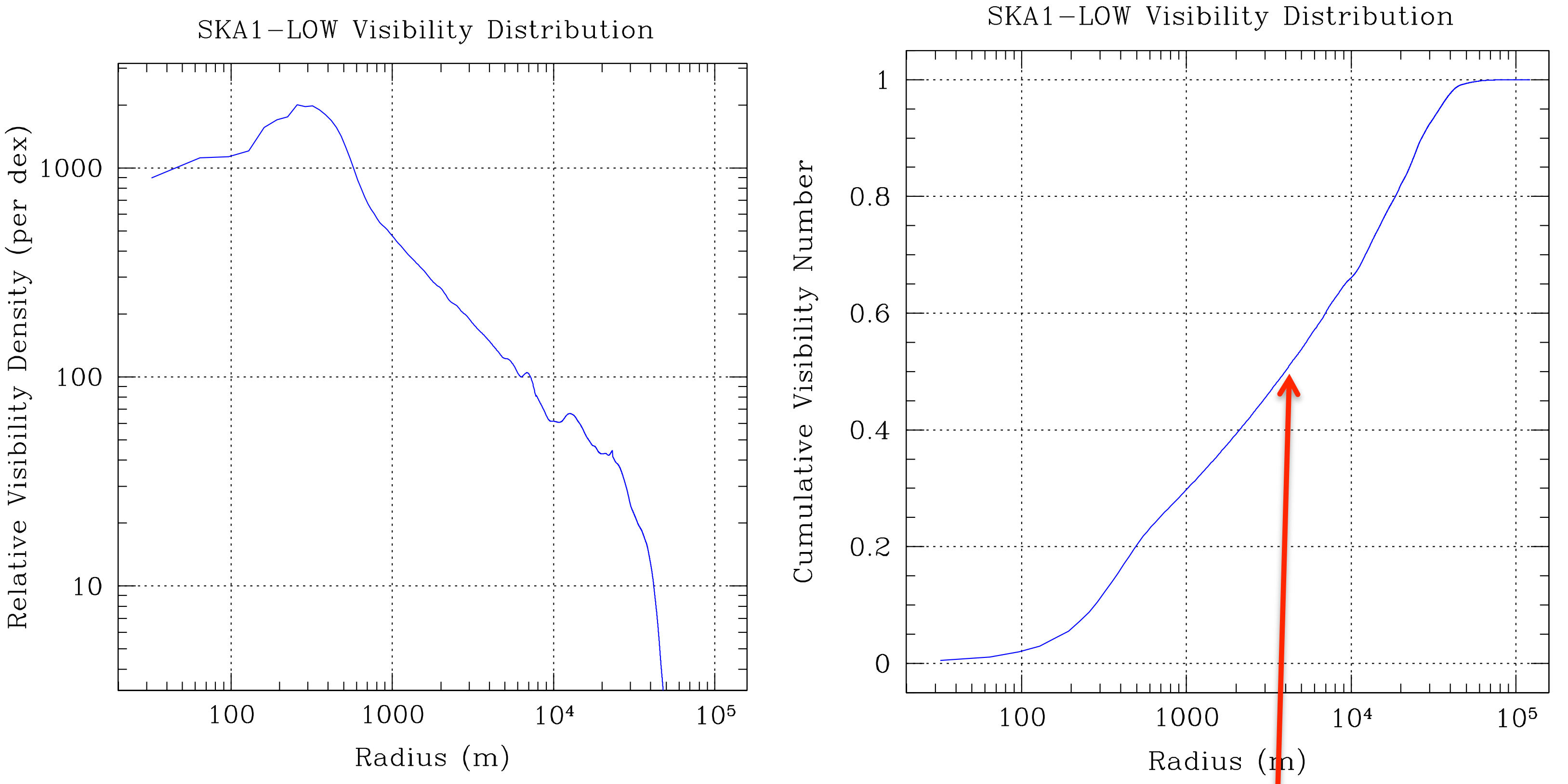
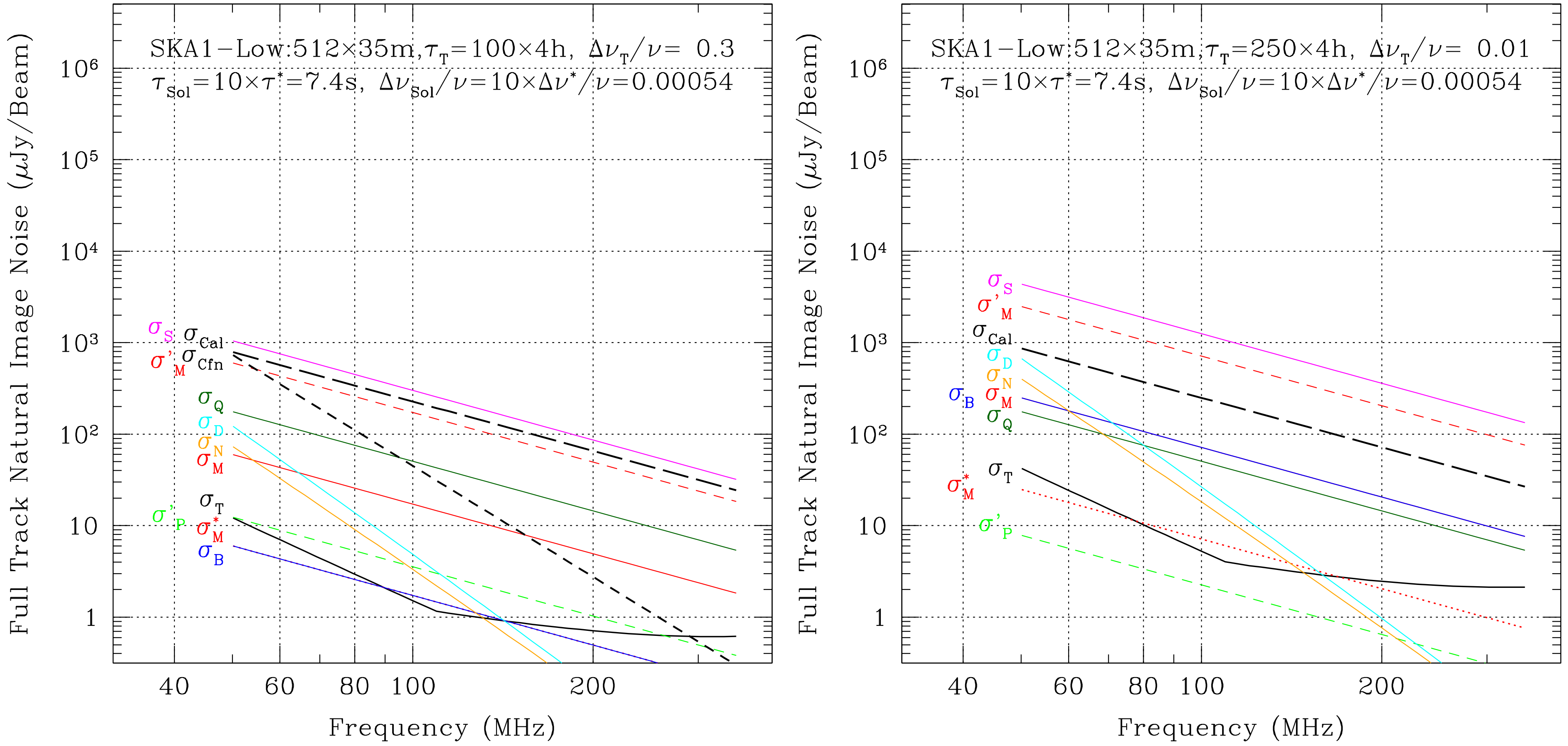


Figure 13. Relative visibility density (left) and cumulative visibility distribution (right) for SKA1-Low based on a 4-hour track at δ = -30°. The median baseline length for such an observation is 4.0km.

SKA1-Low deep integrations



- 512x35m station correlations noise budget

SKA1-Low deep integrations

- Extremely high modelling precision of ε_M =0.001 must be achieved.
  - 100,000's of source components
  - Will almost certainly require new source representation methods
  - Must take account of time and bandwidth smearing for data comparison
- Post-calibration frequency modulation of the main beam gain must be less than ε_B = 0.002.
- Post-calibration residual main beam azimuthal asymmetries must be less than ε_Q = 0.0004.
  - Very high quality station beam model probably vital in guiding choice of suitable "clusters" to use in self-cal

SKA1-Low deep integrations

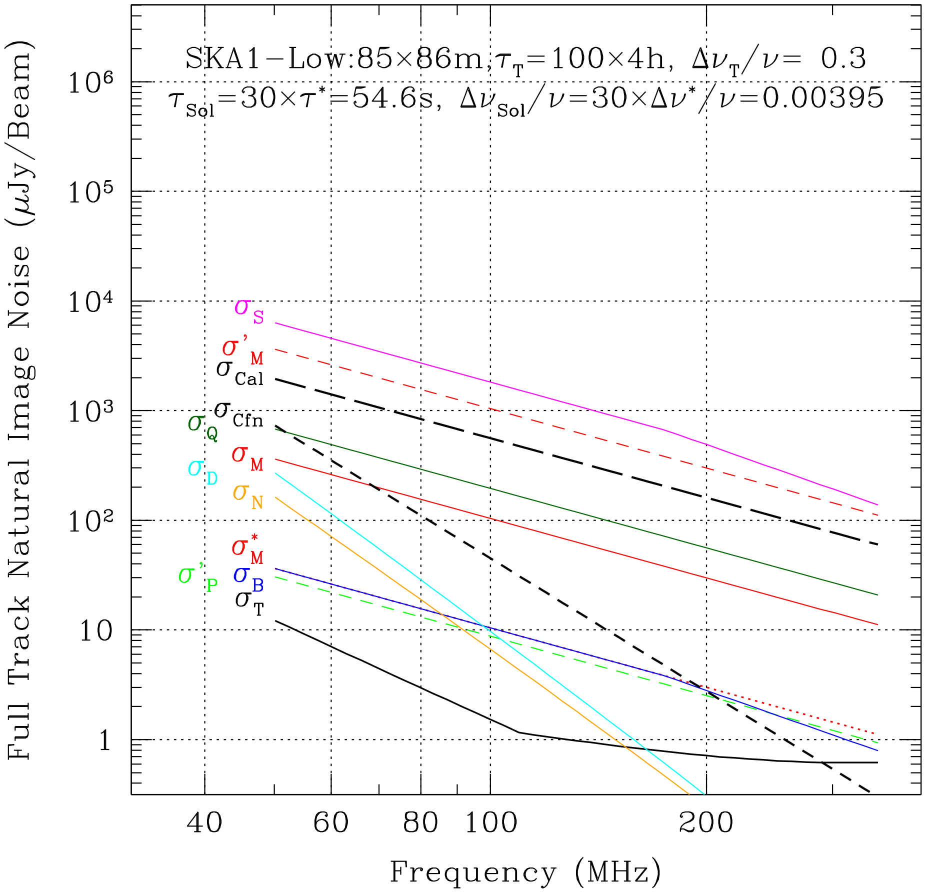
- Random electronic gain variations ($\tau \approx 1^{m}$) that induce station “pointing” offsets must be kept below $\underline{\varepsilon'_{P}} = 0.004$.
- The brightest 1.3 dex [$= \log_{10}(\varepsilon_{S}/\underline{\varepsilon_{S}}) = \log_{10}(0.01/0.001)$] of random sources occurring within the main beam near-in sidelobes must be included in the self-cal model.
  - Need to include 3000 – 4000 sources brighter than about 15 mJy
- The brightest 1.0 dex [$= \log_{10}(\eta_{F}/\underline{\eta_{F}}) = \log_{10}(0.5/0.05)$] of sources occurring over the entire visible sky must be included in the self-cal model and subtracted.
  - Need to include 5 – 10 sources brighter than about $S_{1.4\mathrm{GHz}} \approx 85$ Jy

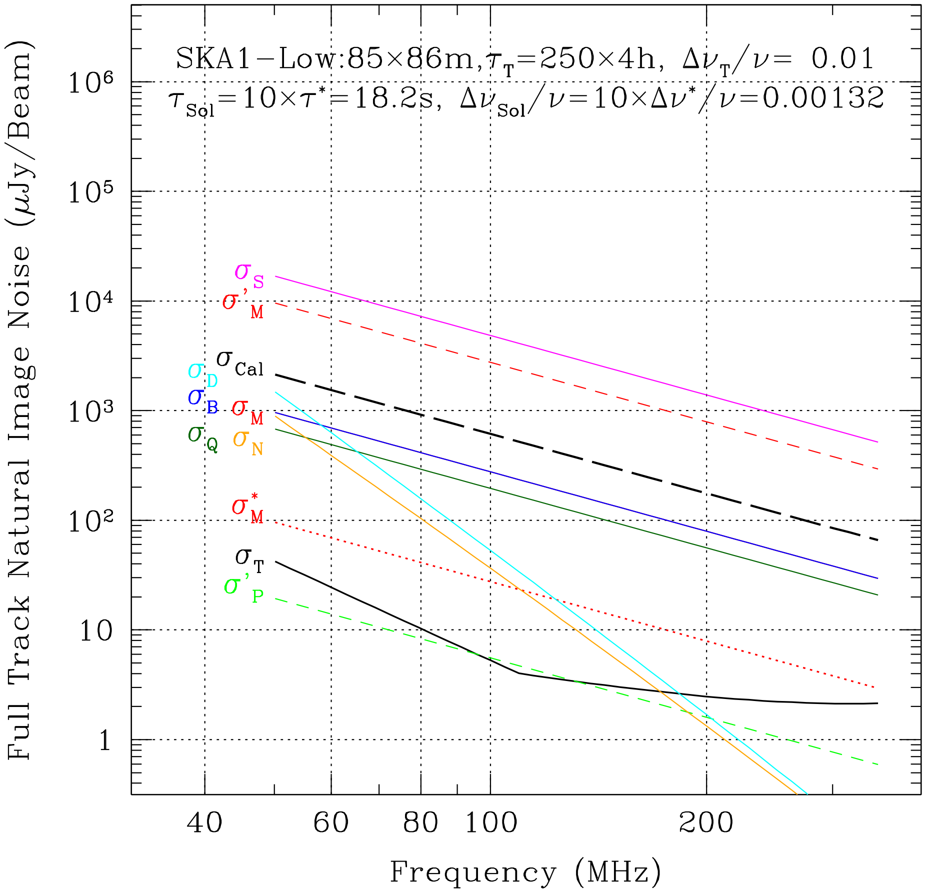
SKA1-Low / LOFAR-NL calibration challenge

Telescope	Application	η_F	ε_S	P	ε'_P	ε_Q	ε_B	ε_M
VLA B-Cfg	Self-cal Sol	-	-	-	-	-	-	0.1
	Spectral	-	0.004	8	0.03	0.01	0.006	0.01
	Continuum	-	0.001	0.6	0.002	0.0007	0.003	0.002
SKA1-Mid	Self-cal Sol	-	-	-	-	-	-	-
	Spectral	-	0.0007	6	0.06	0.001	0.001	0.001
	Continuum	-	0.0006	1	0.01	0.0003	0.001	0.001
LOFAR-NL	Self-cal Sol	-	-	-	-	-	-	0.1
	Spectral	0.3	0.001	-	0.03	0.003	0.002	0.002
	Continuum	0.3	0.001	-	0.006	0.0005	0.02	0.002
SKA1-Low	Self-cal Sol	0.15	-	-	-	-	-	0.1
	Spectral	0.05	0.0005	-	0.02	0.003	0.002	0.001
	Continuum	0.08	0.0006	-	0.004	0.0004	0.01	0.001

- For most calibration parameters, improvement of 2× relative to LOFAR is enough
- Largest increment, 6×, in realm of “all-sky” source modeling at 50 – 100 MHz

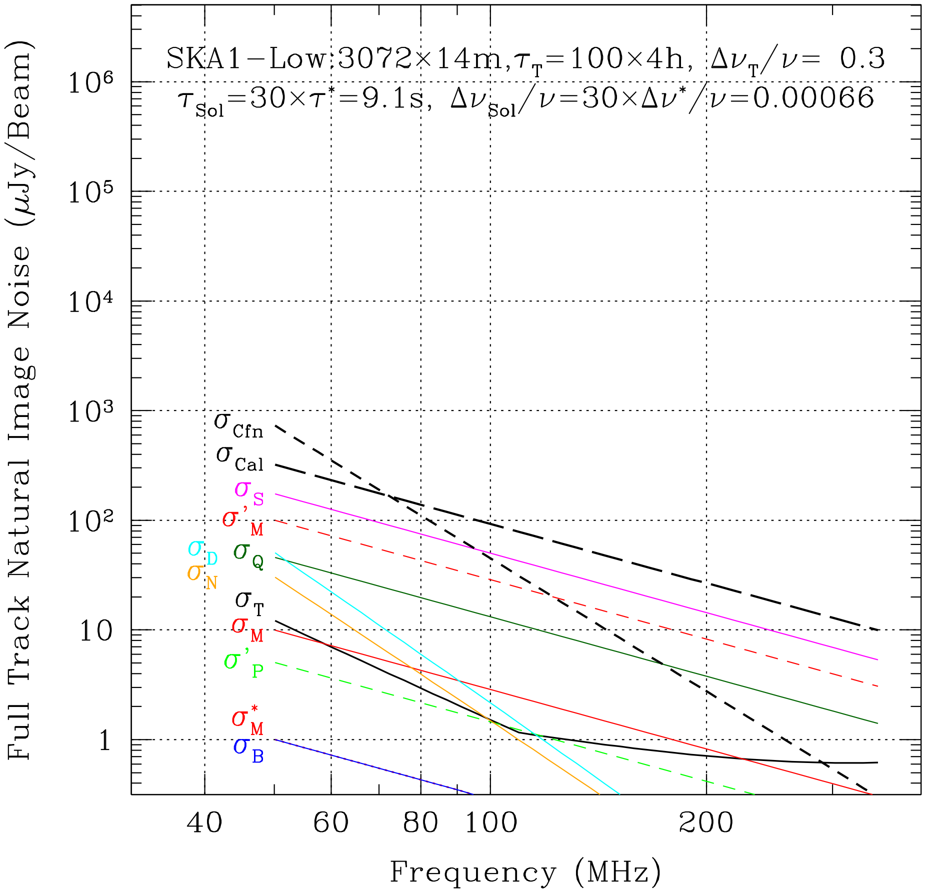
SKA1-Low deep integrations

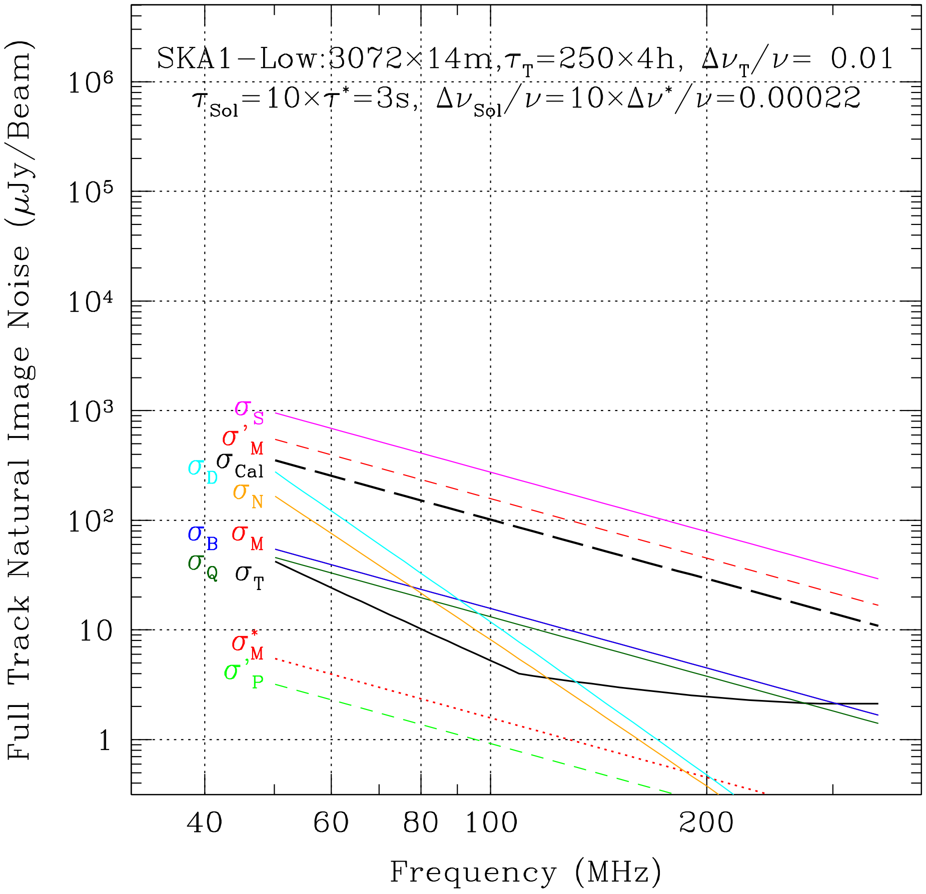




- 85x86m super-station correlations noise budget
- Calibration challenge exacerbated by factor ≈ 4

SKA1-Low deep integrations





- 3072x14m sub-station correlations noise budget
- Calibration challenge relaxed by factor ≈ 4

SKA1-Low implications

- Median baseline length of configuration is vital factor in determining magnitude of calibration challenge
  - Keep B_{Med} as large as possible: must keep ≥ 50% stations B ≥ 4km
  - Only viable method of keeping calibration tractable
  - Required precision scales as $B_{Med}^{-1.5}$
- Effective station number has major implications for calibration and HPC requirements (in opposite sense)
  - Standard "station": cal. challenge about 2x LOFAR @ HPC = 1
  - "Super-station": cal. challenge about 8x LOFAR @ HPC = 1/36
  - "Sub-station": cal. challenge about 0.5x LOFAR @ HPC = 36
  - Required precision scales as $N^{-1,}$ but HPC scales as N^2
- Keeping option of all three beam-forming modes ("sub-" and "super-" as well as "station") could be vital for both science and calibration

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