Interferometric Imaging in Radio Astronomy with the Sparsity-Promoting Frank-Wolfe Algorithm

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joint work with Matthieu Simeoni, Julien Fageot, Martin Vetterli

SKA Days 2021
INTRODUCTION

Measurement → Data Processing → Sky Image

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Adrian Jarret
Data Model and Assumptions
From interferometric measurements to Inverse Problem

- Fourier-type measurements$^1$:
  \[ V(u,v) = \mathcal{F}\{I\}(u,v) = \iint I(l,m)e^{-2\pi i (ul+vm)} \, dl \, dm \]

- Linear inverse problem:
  \[ V = \Phi(I) \in \mathbb{C}^L \]

Van Cittert-Zernike theorem

Visibility
Sky Image
Data Model and Assumptions

Optimization Problem

- Discretization:
  \[ I \approx \beta \in \mathbb{R}^N \implies V = G\beta \]

- Our strategy, LASSO\[^2\]:
  \[
  \text{Minimize : } \frac{1}{2} \| V - G\beta \|_2^2 + \lambda \| \beta \|_1
  \]

- Classical solvers: PDS\[^3\], APGD\[^4\], FISTA\[^5\]
Numerical Strategy
Our contribution: Reweighted Frank-Wolfe

Algorithm 1 Reweighted Frank-Wolfe (RFW)

Candidate locations: $S_k \leftarrow \emptyset$

for $k = 1, \ldots, k_{\text{max}}$ do

1. Estimate new location(s): $i_k \in \arg \max_{i \in \{1, \ldots, N\}} |G^* (V - G\beta_k)|_i$

1.(bis) Update locations: $S_{k+1} \leftarrow S_k \cup \{i_k\}$

2. Complete best reweighting: $\beta_{k+1} \leftarrow \arg \min_{\text{Supp}(\beta) \subset S_{k+1}} \frac{1}{2} \|V - G\beta\|_2^2 + \lambda \|\beta\|_1$

end for
Numerical Strategy

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\begin{algorithmic}
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  \For{$k = 1, \ldots, k_{\text{max}}$}
    \State 1. Estimate new location(s): $i_k \in \arg \max_{i \in \{1, \ldots, N\}} |G^* (V - G \beta_k) |_i$
    \State 1.(bis) Update locations: $S_{k+1} \leftarrow S_k \cup \{i_k\}$
  \EndFor
\end{algorithmic}
Numerical Strategy

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\begin{algorithm}
\textbf{Algorithm 1} Reweighted Frank-Wolfe (RFW)

Candidate locations: \( S_k \leftarrow \emptyset \)
\begin{algorithmic}
\For{\( k = 1, \cdots, k_{\text{max}} \)}
\State 1. Estimate new location(s): \( i_k \in \arg \max_{i \in \{1, \ldots, N\}} |\mathbf{G}^* (\mathbf{V} - \mathbf{G}\beta_k)|_i \)
\State 1.\( \text{(bis)} \) Update locations: \( S_{k+1} \leftarrow S_k \cup \{i_k\} \)
\EndFor
\end{algorithmic}
\end{algorithm}

CLEAN-like
Numerical Strategy
Our contribution: Reweighted Frank-Wolfe

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end for
Numerical Strategy

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end for
Numerical Strategy

Our contribution: Reweighted Frank-Wolfe

**Algorithm 1** Reweighted Frank-Wolfe (RFW)

Candidate locations: $S_k \leftarrow \emptyset$

for $k = 1, \cdots, k_{\text{max}}$ do

1. Estimate new location(s): $i_k \in \arg\max_{i \in \{1, \ldots, N\}} |G^* (V - G \beta_k)_i| < \lambda$

1.(bis) Update locations: $S_{k+1} \leftarrow S_k \cup \{i_k\}$

2. Complete best reweighting: $\beta_{k+1} \leftarrow \arg\min_{\text{Supp}(\beta) \subseteq S_{k+1}} \frac{1}{2} \|V - G \beta\|_2^2 + \lambda \|\beta\|_1$

end for

Natural Stopping Criterion
Performances on Simulated Data

Data Simulation

Source Sky Image

Convoluted Sources

Dirty Image
Performances on Simulated Data
Reweighted FW

Source Sky Image

Lambda factor: 0.05 - Sparsity: 162 - Time: 23.503 - Residual: 0.224
Performances on Simulated Data

Reweighted FW

Convoluted Sources

FW convoluted
Performances on Simulated Data

Effect of $\lambda$

Minimize: $\frac{1}{2} \| V - G\beta \|_2^2 + \lambda \| \beta \|_1$
Performances on Simulated Data

Effect of $\lambda$

\[
\text{Minimize : } \frac{1}{2} \| V - G\beta \|_2^2 + \lambda \| \beta \|_1
\]
Performances on Simulated Data

Comparison with CLEAN

FW Reconstruction

Time: 22.949 - Residual: 0.224

CLEAN Reconstruction

Time: 42.482 - Residual: 0.115
Performances on Simulated Data
Comparison with CLEAN

FW convoluted

RRMSE: 0.746

CLEAN Convoluted

RRMSE: 0.985
Conclusion

Is Reweighted Frank-Wolfe a decent contender for CLEAN?

- Competitive running time
- Improved reconstruction accuracy
- Natural stopping criterion
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- Natural stopping criterion

- Well suited for RA
  - Greedy = adapted to sparse problems
  - Parametric reconstruction (penalization parameter) = adjustable
Conclusion

Is Reweighted Frank-Wolfe a decent contender for CLEAN?

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- Improved reconstruction accuracy
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- Positivity constraint
- Natural extension to continuous data
  - Beyond-the-grid precision
  - Principled theoretical framework
References


Appendice
If there is enough time
Performances on Simulated Data

Improved CLEAN

FW Reconstruction

Time: 22.949 - Residual: 0.224

CLEAN Reconstruction

Time: 3.017 - Residual: 0.197
Performances on Simulated Data

Improved CLEAN

FW convoluted

CLEAN Convoluted

RRMSE: 0.746

RRMSE: 0.936