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Keeping SKA1-LOW station beamforming errors acceptable

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Abstract

The quality of the station beams produced by the digital station beamformers of the low-frequency telescope of the Square Kilometre Array (SKA) will be crucial for its performance. The various factors affecting the quality and stability of the station beams, such as electronic stability, calibration errors and the element beam patterns of the individual antennas, have therefore been studied extensively. This contribution aims to provide an overview of this work and synthesise this in a recommendation for further studies.

1 Introduction

The Square Kilometre Array (SKA) will be a cutting-edge radio telescope to transform our understanding of the Universe[1]. The first phase of its low-frequency telescope, SKA1-LOW, will be built in Australia. It will operate as a synthesis array consisting of 512 aperture array stations [1]. These aperture array stations are subarrays consisting of 256 antennas whose signals are combined in a digital beamformer to form the station output signal. The quality of the station beams is crucial for the performance of the SKA1-LOW telescope. The various factors affecting the station beamformer, such as the stability of the electronic components, the calibration of the individual receive paths within the station and the electromagnetic behaviour of the antennas, have therefore been studied in detail over the last years. The aim of this contribution is to provide an overview of the relevant outcomes of these endeavours and to synthesise this into a recommendation on further work towards a calibration strategy for the SKA1-LOW stations.

This paper starts with an overview of the errors that have been identified and studied and a practical tolerance on the beamforming errors. A key insight from these studies is that a common gain change in all receive paths does not affect beamformer performance and can (under reasonable circumstances) be dealt with by direction-independent calibration at array level, i.e., the *differential* receive path gain errors are the ones that matter. A physics-based receive path gain model that can be driven by outside temperature and solar irradiance measurements from a weather station, was developed to assess the impact of electronic drift on station beam performance. This model is briefly described in Sec. 3. This work puts us in a good position to further develop station calibration as discussed in Sec. 4.

[1] see skao.int

2 Beamformer error budget

2.1 Overview of errors

The station beamformer makes a weighted superposition of the output signals of the individual receive paths. The weights are chosen such that the geometric delays across the station are compensated for the station beam pointing direction for each frequency channel and such that gain differences between receive paths are corrected. As a result, the station beam is a weighted superposition of the beams of the individual antennas. Imperfect corrections will therefore result in some station beam degradation. Below, various sources of error are briefly discussed.

Embedded Element Patterns (EEPs)
Station calibration is done using a known external signal to probe the complete analog receive path. This makes the gain of the individual antennas towards the calibration source a constituent of the receive path gain. As the SKA1-LOW stations have a random layout, the electromagnetic environment of each antenna is slightly different. This causes their EEPs to differ slightly. A significant effort therefore went into developing a validated electromagnetic model [2, 3]. If sky model based calibration is used while ignoring these EEP differences, calibration may cause a relative error in amplitude between 5% and 13% and a phase error between 1.5 and 3.5 degrees depending on sidereal time [4]. The simulations also indicate that the RMS gain variation between the EEPs can be as high as 10% (ignoring low-elevation effects) [4], which is an indication of the gain calibration errors to expect when using a single source for calibration at a reasonable elevation. This source of calibration errors produces a systematic error that depends on sidereal time. As the extend of spatial structures introduced by electromagnetic coupling is intrinsically bounded by the electromagnetic size of the station, these variations are as smooth as the station beam. Continuous calibration including these errors will thus produce gradients that are likely tractable by direction-dependent calibration at array level. This conjecture was confirmed by simulations [5].

EEP variations also affect the station beam by their gain differences in the station pointing direction if these gain differences are not corrected. Fortunately, simulations indicate that the station beam predicted by combining the average EEP with the array factor provides a sufficiently accurate prediction of the station beam [6].

Sky model
As the apparent flux of celestial sources as seen by an antenna is the product of its gain towards the source and the actual flux of the source, the accuracy of the calibration source model is equally important as the accuracy of the EEP model for getting good calibration results. This is confirmed by LOFAR station calibration results showing systematic effects with a period of a sidereal day that did not clearly improve by including an EEP model [4, 5]. This is likely caused by residual flux from the diffuse emission from the Galactic plane that remains after applying the baseline restriction. Unfortunately, this baseline restriction cannot be further extended due to the limited size of the stations. As improving the diffuse emission model is extremely challenging, in particular if the aim is to make it at least as accurate as the EEP model, different routes are being explored, such as using the Sun as dominant source on the sky, either in model based calibration [7] or in self-holography [8], and holography with other stations [9]. In all these approaches, the aim is to set up the calibration measurement in such a way that the calibration signal is sufficiently dominant to make the bias due to unmodelled other signals sufficiently small, which works if the Signal-to-Interference ratio of the calibrator can be made large enough [10].

Another option that has been considered, is calibration of a cluster of stations as one superstation allowing for more rigorous filtering of large spatial structures. This poses its own challenges. Due to the higher resolution and sensitivity, ionospheric phenomena start to affect the calibration results: source flux ratios may be affected by ionospheric scintillation and apparent source positions may shift due to ionospheric density gradients. Fortunately, the random variation of these effects may be exploited to reduce the associated calibrations errors if the system is sufficiently stable to combine multiple measurements.

Noise
All calibration measurements have a finite SNR which causes a random error on the calibration results.

Receive path stability
The impact of the effects described above changes with time, either in a systematic or a random way. For example, the effect of errors or simplifying assumptions in the EEP model will change when the source and array geometry changes. This implies that many of the errors described above can be reduced by combining multiple measurements over frequency and time. LOFAR, for example, works with station calibration tables that are considered valid for several months. This is enabled by the stability of the individual receive paths achieved by burying the coaxial cables connecting the antennas to the station backend and by controllable cooling of the station backend. For a new system, like SKA1-LOW, the behaviour of the receive path gains over time and frequency need to be assessed to see which trade-offs are possible between letting the receive path gains drift over time before recalibrating the station and the behaviour of calibration errors when recalibrating the station at a certain cadence.

Table 1. Summary of error sources.

error	type	time and freq. behaviour
calibration		
EEP model	systematic	variable
source model	systematic	variable
ionosphere	random	variable
noise	random	stable in magnitude
electronic drift	systematic	variable

In this analysis, beamforming errors due to quantisation noise in the digital beamformer and inaccuracies in the antenna positions are assumed to be negligible. Table 1 summarises the errors and their characteristics considered in this contribution.

2.2 Tolerance on beamforming errors

Beamforming errors cause loss of coherence (and hence sensitivity) during the beamforming stage and may cause changes in the beam shape [11]. If the beam shape changes slowly enough, they can be tracked by direction-dependent calibration at array level [11]. Coherence loss, however, cannot be recovered. If the relative error n the beamformer weights is ε, the beamformer efficiency is [11]

$$\eta_{\mathrm{BF}} \approx 1 - 2\varepsilon^2. \tag{1}$$

This result implies that a beamformer efficiency of 99%, i.e., 1% sensitivity loss, requires $\varepsilon \leq 7.1\%$ while a beamformer efficiency of 98% requires $\varepsilon \leq 10\%$.

Since the tolerable relative errors on the beamformer weights are limited by the acceptable level of coherence loss, the beam shape variation will also be limited. Achieving the rate of change allowable by direction-dependent calibration [11] would imply some dynamic process that causes rapid variations within the bounds dictated by the required beamformer efficiency. This would then cause the beam to be dithering on the sky, which could be considered as another form of measurement noise. It would be nice to validate this conjecture with a model like the one described in the next section.

3 Receive path gain model

To assess the impact of the stability of the electronic components in the analog receive path and to explore various

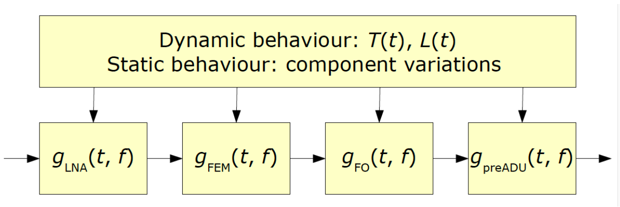

Figure 1. Overview of the receive path gain model.

ideas regarding station calibration to keep the station beamformer errors within the desired bounds, a physics-based receive path gain model was developed. This model was validated by component measurements in the field where possible. For some components, measurements were used as lookup table to characterise the behaviour of that component with frequency and/or temperature or to inform the production tolerances specified in the model.

Figure 1 provides an overview of the model. The antenna is connected to a low noise amplifier (LNA) that amplifies the signal so that it can be transmitted to the frontend module (FEM) by a short coaxial cable (effect assumed negligible). The FEM conditions the signal to the needs of a laser diode driving the fibre optic (FO) link used to transmit the analog signal to a central processing facility. In this climate controlled facility, the pre-ADU boards receive the signals from the RF-over-fiber links and condition it for the needs of the analog-to-digital converter units (ADUs). This analog signal chain was broken down into four components that each exhibit dynamic behaviour driven by changes in temperature and solar irradiation and static behaviour due to production tolerances.

The LNA model was informed by measurements in a climate chamber of its transfer function over frequency at various temperatures. These data were used to model the LNA gain as function of frequency by means of linear interpolation as well as to establish its rate of change with component temperature. The component temperature in the field was modelled by adding heating terms due to LNA power dissipation inside its enclosure and solar irradiation on the enclosure to the ambient temperature.

The gain variations of the FEM were assumed to be dominated by the effect of FEM temperature on the laser diode driving the FO link. The performance of this laser was measured extensively in the lab resulting in a table of gains as function of frequency and temperature, which was used by the model as lookup table. The FEM temperature was modelled in a similar way as the LNA temperature. This model was validated against measurements from a temperature logger inside the box containing the FEMs for 16 antennas.

The FO link was modelled using a physical model of the behaviour of a FO cable, in particular its thermal expansion. This model was based on extensive measurements on FO links, both in the lab (climate chamber) and in the field. As the model assumes that the FO cable lies on the ground or is only covered with a thin layer, it is driven by both the ambient temperature and solar irradiance.

As the pre-ADUs are located in a temperature-controlled environment, we assume their gains to be constant over time, ignoring the very slow changes due to small temperature changes and component degradation that are negligible in comparison with the dynamic behaviour of the other components. The contribution of the pre-ADUs to gain differences between receive paths is therefore due to production tolerances.

4 Towards a station calibration strategy

The aim of station calibration is to keep the beamforming errors below an acceptable limit. It is clear from Sec. 2 that station calibration should track electronic drift if needed so that appropriate corrections can be applied, but that individual calibration measurements are subject to their own errors. We thus need to compare the magnitude of the (expected) calibration errors against electronic drift and the tolerance on beamforming errors.

As stated in Sec. 2, simulations assessing the impact of ignoring the EEP differences between elements indicate that this may already introduce relative errors that consume the entire error budget even if 2% sensitivity loss is acceptable. Note that, in the case of SKA1-LOW, this is the equivalent of losing 10 stations. Empirical evidence from LOFAR suggests that errors in the source model add an equally significant calibration error. This is an indication that the smoothness of the receive path gains over frequency should be exploited by combining calibration measurements at multiple frequencies to reduce the modelling errors. This assumes that the spectral variability of the errors in both the EEP model and the source model is more randomised than the spectral variability of the receive path gains. Although initial results obtained with a prototype stations look reassuring [7], ***this assumption warrants further validation in measurement and in simulation*** using the receive path gain model described in Sec. 3. Such an approach based on combining multiple measurements over frequency will also help to reduce the measurement noise and, likely, to reduce the impact of ionospheric phenomena as their effects diminish with increasing frequency.

Whether the calibration errors can be reduced even further by averaging over multiple instances in time depends on the electronic stability of the system. For a first assessment, the ambient temperature measured over a full day starting on November 21, 2019, at 6:00 PM local time by a weather station at the SKA1-LOW site was fed into the receive path gain model described above along with a model of the solar irradiance taking into account the elevation of the Sun and assuming a cloudless sky. To isolate the effect of electronic drift, all gains were corrected for their time average over the 24-hour period at each frequency. The standard

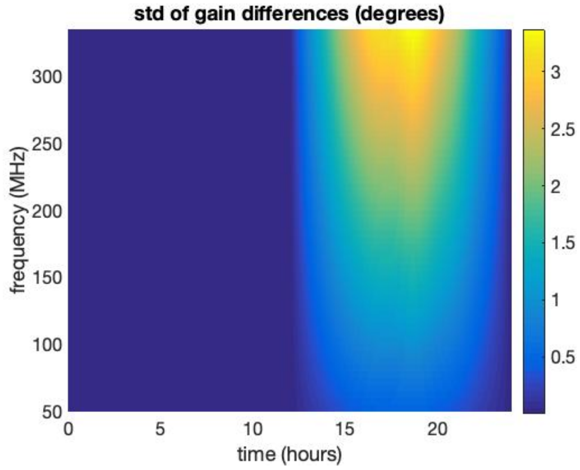

Figure 2. Standard deviation of the gain differences over time and frequency after correcting each gain with its average of the 24-hour period considered.

deviation of the corrected gains was calculated to assess the remaining receive path gain differences across the array as function of time since the start of the temperature measurement and frequency. The results are shown in Fig. 2. These results indicate that, even at the worst instance around noon on November 22, 2019, the dynamic gain differences remain small enough to achieve over 99% beamformer efficiency if no further errors are present. This suggests that a strategy using calibration tables that are fixed over a significant period of time, as is done in LOFAR, may be suitable for SKA1-LOW. This conclusion is corroborated by the fact that reasonable imaging results were obtained with a prototype station using calibration factors that were established one week earlier. ***This conclusion warrants further validation:*** the receive path gain model should be used to explore its limits by feeding it with extreme temperature variations or large/fast changes in solar irradiance. The model should also be used to explore the extremes of the model, for example, large differences in susceptibility to solar irradiation or extreme production errors.

In Sec. 2, an overview was given of various calibration strategies that are being explored to reduce the impact of source model errors. Combined with exploiting smoothness over time and frequency, this may preclude the need to model the EEPs of all 130k antennas. Since that would save a significant effort, ***further exploration in simulation and measurement should focus on strategies for which knowledge of the average EEP is sufficient.***

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